ENSURING SIMILITUDE OF RELATIVE DEPTH IN BED FORMATION PROCESSES SIMULATION

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ABSTRACT

Ensuring both qualitative and quantitative similitude in alluvial bed formation processes simulation has always been a rather difficult task. Partial similitude is usually achieved after many simplifications of the equations describing the process and even so, the results have been applicable to the local conditions.

An important factor in alluvial bed formation processes simulation is the depth-width ratio of the model and the prototype. A study of the depth-width relationship of natural rivers has shown that there is a general law governing the depth-width ratio as the discharge increases. Laboratory investigations [4, 10, 14] have shown that this law could be extended to our laboratory streams and could be a very useful tool in model calculation.

KEYWORDS: ***Movable bed, similitude, depth-width ratio, vertical distortion, model, prototype.***

INTRODUCTION

Man's activities on natural waterways can bring about changes in cross-sectional shape, discharge, sediment yield and river grades. These changes can disturb the delicate balance between the various variables involved in the establishment of an equilibrium and hence the stability of the waterways.

However, stability in alluvial channels is of prime importance when planning any irrigation schemes, river improvement projects for navigation and other purposes on natural waterways. The demand for such projects has increased progressively over the years and has consequently given great impetus to the search for a better understanding of the behaviour of streams and rivers.

THEORY

The flow of a stream may cause the scouring of the stream bed and banks, the subsequent transport and deposition of the scoured material depending on the hydraulics of the stream, its morphology and bed materials. In simulating this process, the basic equations used are the Navier Stokes' equation and the sediment mobility equations. Thus to ensure dynamic similarity of flow in the model and prototype, the Froude criterion is used;

$$Fr = \frac{V^2}{gh} = const. \text{-----------} (1)$$

Dynamic similitude of the interaction between the fluid and the bed material is achieved by using the generalized criterion of mobility [8].

$$D_s = \frac{V_*^2}{gd\rho^1} = \frac{hi}{d\rho^1} \text{--------------------} (2)$$

where V-velocity of flow

d-grain size

h-depth of flow

${}_{*} = \sqrt{ghi}$ -shear velocity

$$\rho^1 = \frac{\rho_s - \rho}{\rho}$$

Using these criteria the necessary scaling factors could be obtained for converting prototype parameters to model parameters and vice-versa.

Thus from the Froude criterion the velocity scaling factor is

$$\lambda_v = \lambda_h^{1/2}, \lambda_v = \text{-------------------}(3)$$

From the mobility criterion, we have

$$\lambda_h \lambda_l = \lambda_d \lambda_{\rho^1} \text{--------------------------}(4)$$

and if we should write $I = \frac{h}{l}$ **equation (4) becomes**

$$\lambda_h \lambda_l = \lambda_h^2 \lambda_l^{-1} = \lambda_d \lambda_{\rho^1} \text{-----------------}(5)$$

By applying Manning's formula $' = \frac{1}{n} h^{2/3} I^{1/2}$

and assuming a proportionality of $n \propto d^{1/6}$

$$\lambda_d = \frac{\lambda_h^4}{\lambda_l^3} \text{--------------------------}(6)$$

Substituting (6) into (5) we shall arrive at

$$\lambda_{\rho^1} = \left(\frac{\lambda_l}{\lambda_h}\right)^2 \text{----------------------------}(7)$$

The inferences that could generally be made from these scaling factors are;

i.-to accomplish dynamic similitude between model and prototype, the bed material of model must have sizes which conform to equation (6) and densities which conform to equation (5).

ii.-equation (7) shows that distortion of geometric scale is a necessary condition for dynamic similitude between model and prototype.

It must be added that the need for distortion of geometric scale arises in practically all cases of modelling with movable beds. It may even arise when modelling with immobile bed, when for qualitative similitude of flow, the Reynolds' number on the prototype is large, we rarely can ensure the necessary velocity on the model without distorting the geometric scale. For example, if $\lambda_h = 100$ and $h_p = 2m$, then the depth on the model $h_m = 2cm$ which will not be sufficient to ensure the necessary Reynolds' number on the model.

In mobile bed investigations, especially in bed formation processes simulation however, it is not easy to obtain bed materials to satisfy the above conditions i.e. eq. (5) or eq. (7). Researchers overcome this problem by substituting the bed materials of the model with materials having relatively larger sizes and lower densities. In this case, quantitative similitude is usually achieved (i.e. the required sediment transport is achieved), however we rarely can achieve qualitative similitude (for example where dunes are expected we may obtain ripples etc.) and this is very important in bed formation processes. What is usually done to overcome this problem is to maintain the same bed material on the model as it is in the natural conditions. In this case the distortion of the geometrical scale i.e. depth-width ratio of the model must take a definite value which must be related to the depth-width ratio of the prototype. This relationship which is the ***vertical distortion factor*** is given by

$$K = \frac{\left(H/B\right)_m}{\left(H/B\right)_p} \text{--------------------------------}(8)$$

where the subscripts "m" and "p" denote model and prototype respectively.

The depth-width ratio is an important factor in ensuring similitude on model and has consequently been investigated by different researchers [9, 12,1, 10, 11, 6]. Webber [13] writes that some justification for distortion of scale is to be found in nature, since the width-depth ratio for large rivers is much greater than for small ones, whereas the grain size of the bed material and the general configuration of bed forms may not be very different. Chang [6] indicates that since changes in channel width and channel elevation are closely inter-related, modelling of aggradation and degradation must take into account width changes. Znamenskaya [12] and Abalianse [2] use the criteria

$$\left(\frac{B}{H}\right)y = const. \text{ and } \left(\frac{B}{H}\right)^{x} = const$$

respectively to express this distortion; x,y being constants.

Below is a table of the basic scaling factors as obtained from the criteria and discussion above to calculate the model.

Table 1. Scaling factors for the computation of model

Quantity	a) $\lambda_d = 1$	b) K is known
Depth	$\lambda_h = \lambda_l^{3/4}$	$\lambda_h = \frac{1}{K}\lambda_l$
Vertical distortion factor	$K = \lambda_l^{1/4}$	K
Velocity	$\lambda_v = \lambda_l^{3/8}$	$\lambda_v = \sqrt{1/K}\lambda_l^{1/2}$
Density	$\lambda_{\rho'} = \lambda_l^{1/2}$	$\lambda_{\rho'} =$
Discharge	$\lambda_Q = \lambda_l^{17/8}$	$\lambda_Q = \frac{1}{K^{3/2}}\lambda_l^{5/2}$
Longitudinal slope	$\lambda_i = \lambda_l^{-1/4}$	$\lambda_i = \frac{1}{K}$
Time	$\lambda_t = \lambda_l^{5/8}$	$\lambda_t = \sqrt{K}\lambda_l^{1/2}$

Comments: a) -a substitute for bed material for model is made with

b) -bed material on model is the same as on prototype and the required vertical distortion factor K is assumed known

EXPERIMENTAL SET UP

The basic objective of the experimental work was to investigate the depth-width relationship in a movable channel and the deformation in the channel as a result of non-conformity of this relationship.

The experiments were carried out on a flume 25m long, 1.4m wide and 0.5m high. The model under investigation consisted of a straight river channel which had a trapezoidal cross-section with two symmetrical flood plains as shown in figure 1. The main working section of the model was 10m. The side slope of the channel was m=1:1 and the longitudinal slope of channel and flood plains was 1=0.0016. The bed material of model consisted of a homogeneous sand with mean grain sized d Investigations were carried out for three channel width ratios

B-total width of channel with floodplains,

b-mean width of channel

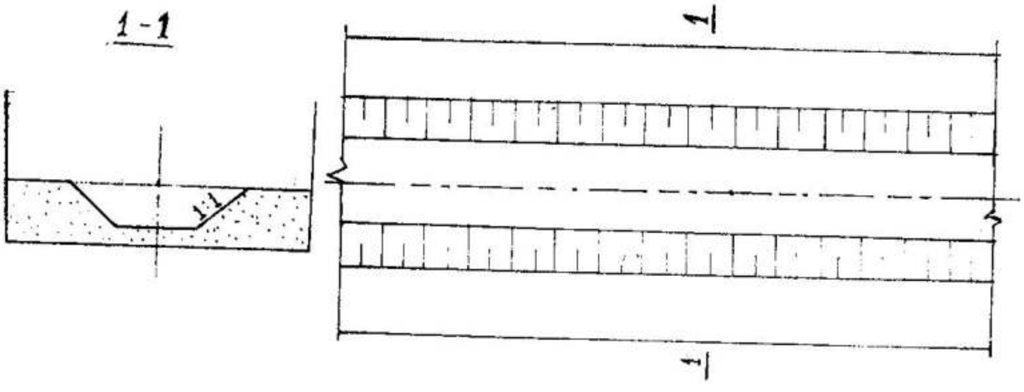

Figure 1. Typical section of model

For each channel width, three different depths (h=10.0cm, 16.5cm, 24.0cm) were maintained in the model. In all 27 experiments were performed. Discharge was measured by means of thin walled weir and velocities by micro-current meter.

EXPERIMENTAL PROCEDURE

After obtaining the right profile of the channel and floodplains (using prophylographs), the initial cross-section were carefully taken. The model was then gradually filled with water to the required depth with all gates and valves on the model closed. The pump was then started and by gradual manipulation of a system of gates and valves, the required regime on the model was established. The required regime of flow was considered that regime which ensured mass movement of sediments. The mean velocity on the model was 26.3cm/s. This regime was kept constant until a "relatively stable form" of channel was established. This form was achieved within 6 hours. This time was obtained by a series of preliminary experiments on the same model to find the time dependence of deformation by plotting volumetric deformation against time. After the 6 hours, the experiment was stopped, the water gradually drained and the new channel form taken.

EXPERIMENTAL RESULTS AND ANALYSIS

Results of typical deformation are presented in figures 2-4. Figure 2 shows the mean deformation when $\beta=1/8$ with varying depths of flow. Similar deformation patterns were obtained for $\beta=1/4$ and $\beta=1/2$. Figure 3. shows the mean deformation of model with varying relative width β of channel given in the absolute form. Figure 4 represents mean deformation of channel with varying width of channel given in the dimensionless form; i.e.

$$\frac{h}{H_m} \text{ against } \frac{h}{B_m}$$

where H_m, B_m -maximum depth and width of channel after deformation, h-depth of channel at any point in the cross-section and b-distance from axis of channel to point in question.

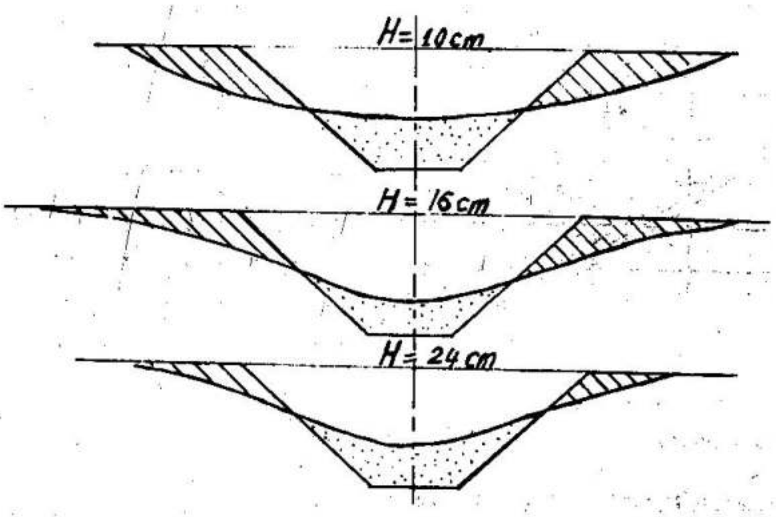

Figure 2. Channel profile with varying depth of flow given in absolute form

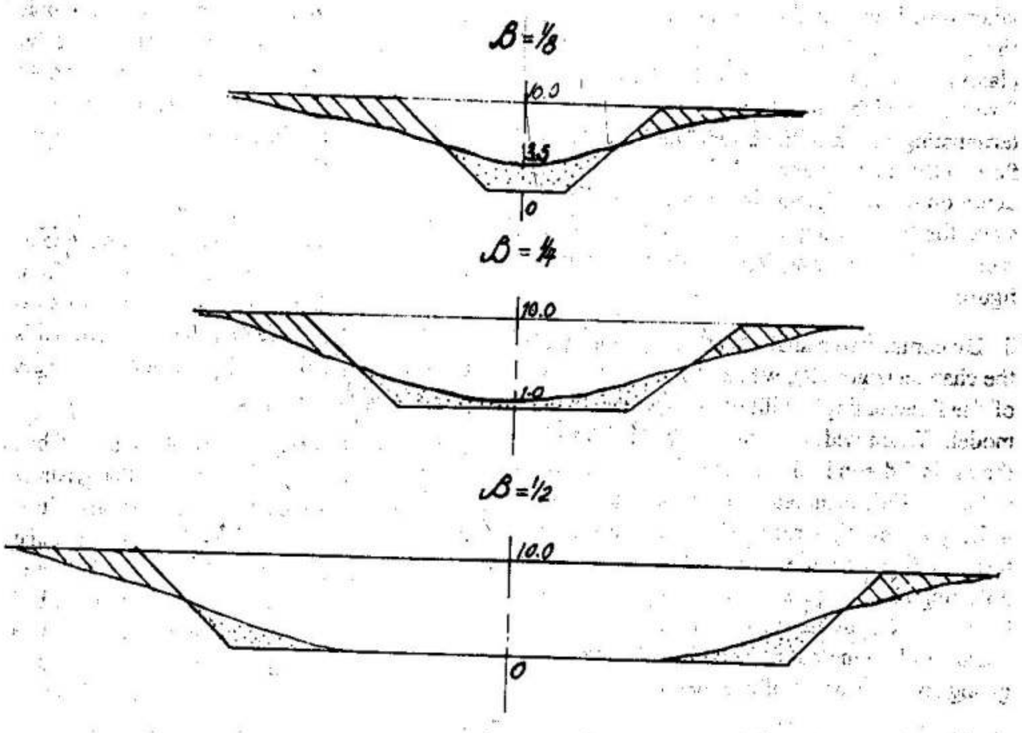

Figure 3. Channel profile with varying relative width given in absolute form. Depth of flow = 16.5cm.

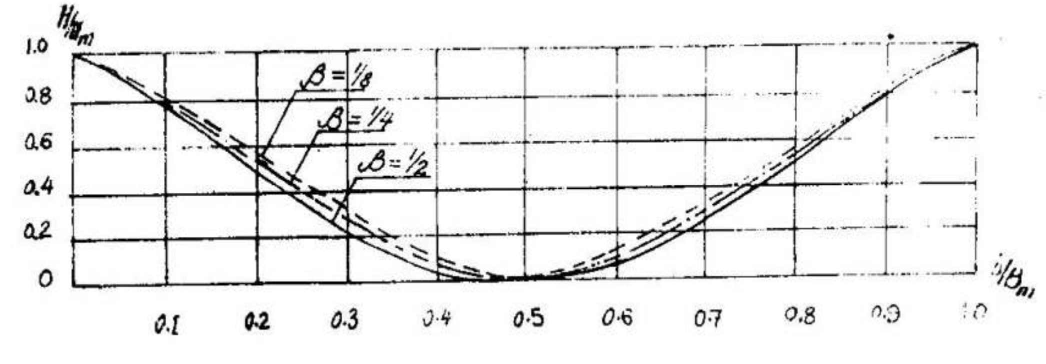

Figure 4. Mean channel profile given in dimensionless form.

Analysing these results, the following inferences could be made;

1) for a given channel width, the effect of depth of flow on the deformation of channel is relatively small. Deformations are relatively large when flow is confined in the channel (h =10cm). As the water flows into the floodplains (h=16.5cm) there is a reduction in the deformation which might be accounted for by the terminating action of the floodplain flow on the flow in the main channel. A further increase in depth on the floodplains increases deformation since the terminating action of the floodplains reduces with increasing depth on the floodplains, figure 2.

ii. By comparative analysis of the geometry of the channel (figure 3), we see an important law of "self regulating" abilities of streams in the model. When width of channel is small $\beta=1/8$, the orginal depth in the channel is considerably reduced. With an increase in channel width $\beta=1/4$ the given depth is reduced but to a lesser extent. A further increase in width $\beta=1/2$ does not bring about any reduction in depth. It may be predicted that any further increase in width could result in increase in depth of channel thus giving rise to erosion of channel bed.

iii. Changes in the geometry of the cross-section of channel are brought about by the fact that for every discharge (both fluid and sediment) in the model, a corresponding "relatively stable" form of channel capable of discharging the given load is established. This to ensure minimum stream power per unit channel lenght. As a result, in some cases the changes in channel geometry are confined only along the side slopes ($\beta=1/2$) and in other cases along the side slopes and bed of channel ($\beta=1/4$, $1/8$).

iv. The geometry of channel remains similar for all the investigated cases. Channel geometry represented in the dimensionless form virtually coincide (figure 4). The above results show that provided the bed material is the same, channels with different discharges produce similar shapes.

Investigating the geometrical shapes of bed forms Rzhanitsyn [10] has shown that changes in the relative geometrical dimensions of bed forms either local or general are constant and do not depend on the order (size) of stream, i.e. even though isolated dimensions of bed forms and channel geometry change with changes in stream order, however, the geometry expressed in the dimensionless form remain similar for the same bed material.

These results together with observation of natural rivers and artificial canals have shown that with a reduction in size of a channel, streams work out relatively deeper channels as if to give these channels a natural distortion in the vertical scale, while at the same time maintaining a form characteristic of the bed material in order to discharge the required load. "Wider channels are shallower" [5]. Therefore in simulating bed formation processes, it is important to know the extent of this distortion of vertical scale on the model.

OBTAINING THE RELATIVE DEPTH (H/B) OF MODEL

In trying to find the relative depth of model which will ensure similitude to prototype, it is necessary to understand the laws governing the vertical distortion of natural streams so as to apply them to the model since these models may be considered as very small streams, governed by the same physical laws. In his study of bed formation processes Rzhanitsyn [10] has acquired a lot of material on the characteristics of natural rivers. One of such characteristics is the changes of relative depth with discharge for alluvial and sand bed channel (Fig. 5) which expresses thelaw governing the vertical distortion of natural streams. This curve can be used to calculate the required vertical distortion factor in the model by the method of successive approximation.

Initially an arbitrary value of the vertical distortion factor is assumed and the discharge in the model is computed using table 1 (b). The depth-width ratio is then obtained from figure 5 and the vertical distortion factor com bputed. If the assumed value coincides with the computed value, then it means the value of the vertical distortion factor has been predicted correctly. On the contrary, a new value is assumed and the procedure repeated. Generally the third or fourth trial yields the correct value. It must be noted that the relationship given in Fig. 5 is valid for alluvial and sand bed materials. For bed material such as gravels, we may use the morphometric relationship of Altunin V.S. [3] to compute the relative depth of model.

$$\frac{B}{H}=\left(\frac{Q}{d^2\sqrt{gd}}\right)^{1/8}$$

where d-mean grain size, Q-maximum discharge.

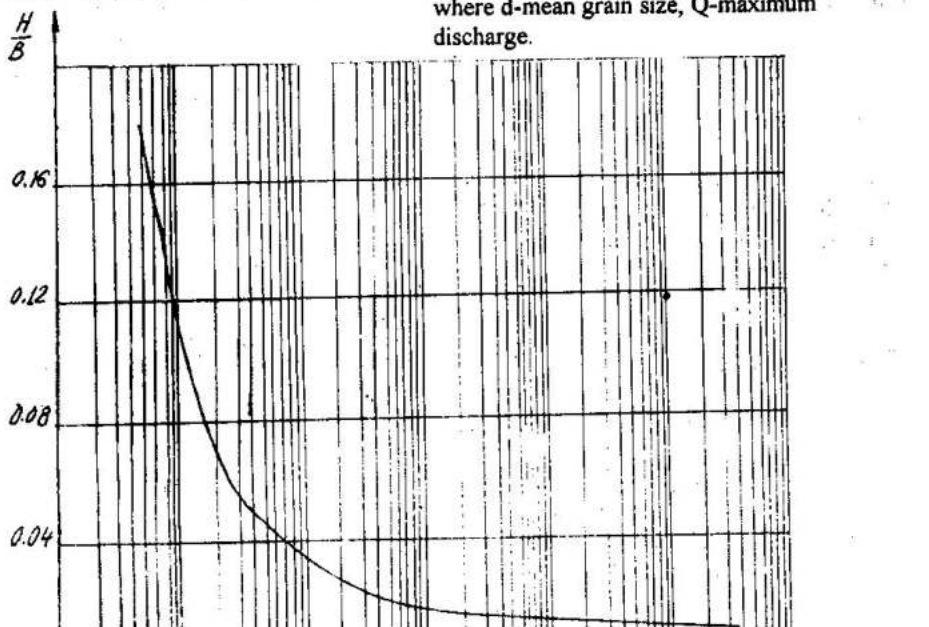

Figure 5. Changes of relative depth with discharge.

CONCLUSION

This paper examined the depth-width ratio of channels with different channel order (size). Deformations of channel geometry as a result of non-conformity of depth-width ratio were measured. it is generally observed and has been shown by our investigations that channels of different order (size) have different depth-width ratio and that wider channels are shallower. However, it has been shown that provided the bed materials are the same, the geometry of these channels expressed in the dimensionless form are similar. In other words, a channel of bigger order differs from a smaller one only by the depth-width ratio and the two channels can be related by a definite factor of vertical distortion. Therefore in the calculation of models, apart from the conditions of fluid flow (equation 1) and the interaction between fluid and bed material (equation 2), it is very necessary to observe the correct depth-width ratio given by the vertical distortion factor of the prototype and model. A method of model calculation has been suggested based on observed data on natural river channels.

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