AN EVALUATION OF THE UNIT-GRADIENT ASSUMPTION FOR THE UNSATURATED HYDRAULIC CONDUCTIVITY DETERMINATION IN UNIFORM SOILS

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ABSTRACT

The unsaturated hydraulic conductivity $K(\Theta)$ as a function of water content (Θv) was determined in some uniform Coastal Thicket Soils in Ghana using the unit-gradient assumption of the internal drainage method. The study showed an exponential relationship between $K(\Theta)$ and Θv for three locations investigated. It was also noted that the exponential $K(\Theta)$ - Θv relationships could be lumped together for the different soil depths up to the 0.4m depth, suggesting the uniformity of the soils within the zone of study. The obtained exponential ($K(\Theta)$ - Θv relationship confirmed the reliability of the unit-gradient assumption in uniform soils.

Key words: *unsaturated hydraulic conductivity; volumetric water content; unit-gradient; internal drainage.*

INTRODUCTION

The movement of water in soils when the soil pores are partly filled with water and air is termed unsaturated flow. Unsaturated flow takes place in the movement of water through the soil to the plant roots, and in the loss of water from the soil through evaporation. The hydraulic constant describing the ability of the soil to transmit water under unsaturated flow condition is termed unsaturated hydraulic conductivity $K(\Theta)$, which depends to a great extent on the soil water contents and pore geometry.

Both the field and the laboratory methods for determining $K(\Theta)$ are available. However, for most applications in field engineering, $K(\Theta)$ determined in the field is often preferred. The most promising field method for determining $K(\Theta)$ is the internal drainage method [8, 13]. In this method, the soil profile is deeply wetted and the water content and the matric potential are measured as a function of time and soil depth whilst evaporation of water from the soil surface is prevented. This method has been used extensively to determine $K(\Theta)$ in the field [9, 11].

The internal drainage method is not devoid of limitations. The inappropriateness of the method on sloping lands, the extraction of water by lateral roots and the potential problem of unequal wetting of the soil profile due to the possible occurrence of impermeable layers in the soil are some of the limitations. An important drawback of the internal drainage method that is often overlooked is the inconsistencies in tensiometric data that are obtained in the field.

To circumvent this problem, the unit gradient assumption that neglects tensiometric data has been devised as a modification of the internal drainage method [1, 3, 6, 12].

There is a paucity of information in $K(\Theta)$ in tropical soils south of the sahara. However, Bonsu and Lal (1982) have provided extensive data on $K(\Theta)$ in some Nigerian soils. Probably the paucity of information on $K(\Theta)$ in tropical soils south of the sahara could be due to the lack of simple field methods for $K(\Theta)$ determination. The use of the soil water flow theory to describe hydrologic processes such as infiltration, drainage, and plant-water extraction requires the determination of $K(\Theta)$ as a function of water content or matric potential. Therefore, this study was undertaken to test the usefulness of the unit-gradient assumption in the determination of $K(\Theta)$ - Θv relationship in some Coastal Thicket soils within the Coastal Savanna Ecological Zone of Ghana.

THEORY OF INTERNAL DRAINAGE METHOD

The one-dimensional flux density of water through the soil in a vertical direction can be expressed as:

$$q = - K(\Theta)\, d\Psi/dz \qquad (1)$$

where q is the flux density, Ψ is the hydraulic potential, z is the soil depth in the vertical direction, $K(\Theta)$ is the hydraulic conductivity which depends on the water content Θ, and $d\Psi/dz$ is the hydraulic potential gradient.

By evoking the continuity equation

$$d\Theta/dt = -dq/dz \qquad (2)$$

and combining it with eq (1), the general flow equation is obtained;

$$d\Theta/dt = d/dz\,[K(\Theta)\,d\Psi/dz] \qquad (3)$$

Integration of eq (3) as a function of depth z for the limits $z = o$, $z = z$ gives

$$d\Theta/dt \;\; z = K(\Theta)\;d\Psi/dz \qquad (4)$$

$$\text{and} \quad K(\Theta) = z\;d\Theta/dt\;/\;d\Psi/dz \qquad (5)$$

By the unit-gradient assumption, $d\Psi/dz = 1$, and eq (5) finally becomes

$$K(\Theta) = z\,d\Theta/dt \qquad (6)$$

A further assumption can be made that the change of water content with time during the internal drainage is represented as [6, 9]:

$$\Theta = at^{-b} \qquad (7)$$

where t is the drainage time, and a and b are parameters.

Taking the natural logarithm of both sides of eq (7) gives

$$\ln\Theta = \ln a - b\ln t \qquad (8)$$

Thus a plot of $\ln\Theta$ against $\ln t$ during internal drainage must yield a straight line with the intercept as $\ln a$ and the slope as the magnitude of b.

Taking the first derivative of eq (7) gives

$$d\Theta/dt = -\,a\,bt^{-b-1} \qquad (9)$$

Combining eq (6) and eq (9) gives

$$K(\Theta) = -\,z\,(a\,bt^{-b-1}) \qquad (10)$$

The parameters a, and b are simply obtained by regressing $\ln\Theta$ on $\ln t$.

MATERIALS AND METHODS

The experiment was carried out in the Research Farm of University of Cape Coast during the dry season from November 1991 through February 1992. Three flat plots measured 0.7m by 0.7m were cleared of all growing vegetation and roots. Brick dikes of 0.15m high were made around each plot. The surface of the plots was covered with dry grass mulch and then ponded with water until the profile was deeply wetted. The brick dikes prevented the water from moving out laterally. A raised bed would have been cheaper, but the soils were sandy in nature. As soon as the water had infiltrated the soil was sampled for the initial water content at depths 0-10cm, 10-20cm, 20-30cm and 30-40cm using soil moisture sampling tube. After taking the initial water content, the plots were covered with transparent polythene sheets to prevent evaporation of water from the plots. The top of the polythene sheets was covered with dry grass mulch to minimise temperature effects.

The soil was sampled daily for a period of 6 days for the changes in water content as internal drainage took place without evaporation from the soil surface. After 6 days, the soil was sampled once a week continuously for 4 weeks to monitor the changes in soil water content. After the 4th week the bulk densities of the soil at the various depths were determined by the core method. Using the bulk densities, the changes in water content was converted from mass basis to volume basis.

It must be emphasized that the soil within the dikes was not disturbed since diking was done while standing outside the plot.

Furthermore, small plot sizes were chosen so as to reduce the amount of water required to wet the soil profile as deep as possible. Also since gravimetric sampling was used, the depth of sampling was limited to 0.4m because there was no long sampling tubes to go beyond 0.4m.

RESULTS AND DISCUSSION

The texture and the bulk densities of the soils for the various depths are presented in Table 1. The texture varied from sandy loam to sandy clay loam. The bulk density increased directly with depth, and its value varied from 1.3 to 1.8g cm $^{-3}$. Sample plots for the variation of water content (Θ) with depth (Z) at different drainage times is presented in Fig.1. Fig, 1 shows that the water content decreased with drainage time. Also, irrespective of the drainage time, the water content increased with depth, giving the indication that the soil profile was deeply wetted. Sample plots of $\ln\Theta$ against $\ln t$ for the different depths is illustrated in Fig. 2.

In Fig. 2 approximate straight lines with negative slopes could be fitted through the points indicating a good agreement with the empirical eq 8.

The $K(\Theta)$ values calculated by the unit-gradient assumption were plotted as a function of Θv using a semi-log scale. The semi-log plots of $K(\Theta)$ as a function of Θv for plots 1, 2 and 3 are shown in Figs 3, 4, and 5 respectively. The scatter indicated that for each plot, $K(\Theta)$ and Θv relationships for the different layers could be lumped together and represented by a single exponential function. The correlation coefficients between ln $K(\Theta)$ and Θv for the three respective plots were highly positive and significant ($r = 0.9999$; $P < 0.1\%$). The fact that ln $K(\Theta)$ and Θv relationships for the different depths of the plots could be represented by single exponential functions indicated that the soil was uniform up to a depth of 0.4m, even though there was a slight change in the texture at the 0.4m depth (Table 1).

The exponential relationship between $K(\Theta)$ and Θv has been extensively documented, for example [2, 4, 5, 12, 14]. Therefore the exponential functional relationship between $K(\Theta)$ and Θv obtained in this study suggests that the unit-gradient assumption in determining $K(\Theta)$ in the field using the internal drainage method is valid for uniform soils. However, in complex stratified soil profiles the unit-gradient approximation may not be adequate. Studies carried out elsewhere [1, 11, 15] showed that the hydraulic potential gradients were strongly influenced by the complexity and nature of the soil profiles. The hydraulic potential gradients could vary with both time and soil depth in stratified soil profiles and could be either smaller or greater than unity [1]. The hydraulic potential gradient during the internal drainage process in uniform soil profiles has been noted to be nearly equal to unity [3, 7].

CONCLUSION

The study confirms that the unit-gradient assumption of using the internal drainage method for $K(\Theta)$ determination in the field is valid for uniform soils, and simple direct soil moisture sampling can be employed with the method for the determination of $K(\Theta)$ - Θv relationship which is required in modelling hydrologic processes in soils. In stratified soil profiles, tensiometric data may be required in order to determine $K(\Theta)$ - Θv relationship in the field, since the hydraulic potential gradient may not be unity. However, when the soil profile is sufficiently wet, the unit-gradient assumption is approached in non-uniform soils. Therefore, for modelling hydrologic processes in the field that do not require very accurate $K(\Theta)$ values, the unit-gradient method can be employed for $K(\Theta)$ determination because it is simple and sufficiently accurate.

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Table 1. Some pertinent physical properties of the soils.

Plot No.	Depth (cm)	Sand (%)	Silt (%)	Clay (%)	Texture class*	Bulk density (gcm^{-3})
1	0-10	77.88	4.00	18.12	S.L.	1.27
	10-20	75.88	4.00	20.12	S.L.	1.45
	20-30	75.88	4.00	20.12	S.L.	1.67
	30-40	74.24	3.64	22.12	S.C.L.	1.84
2.	0-10	78.24	3.64	18.12	S.L.	1.27
	10-20	75.88	4.00	20.12	S.L.	1.46
	20.30	75.88	2.00	22.12	S.C.L	1.70
	30-40	73.88	4.00	22.12	S.C.L.	1.82
3.	0-10	77.88	6.00	16.12	S.L.	1.29
	10-20	75.88	6.00	18.12	S.L.	1.50
	20-30	73.88	6.00	20.12	S.L.	1.69
	30-40	73.88	4.00	22.12	S.C.L.	1.83

*S.L. = sandy loam
S.C.L. = sandy clay loam.

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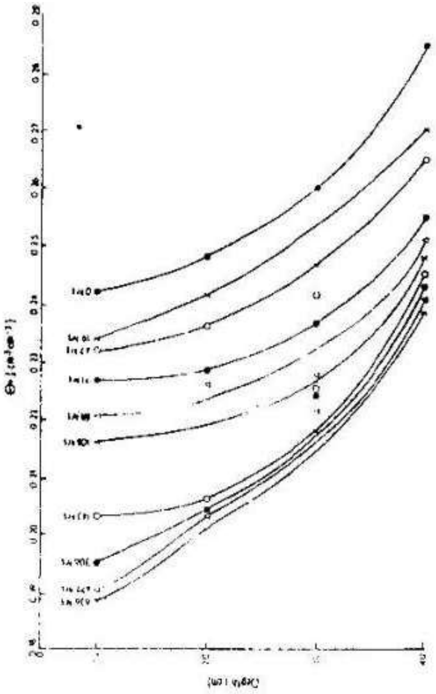
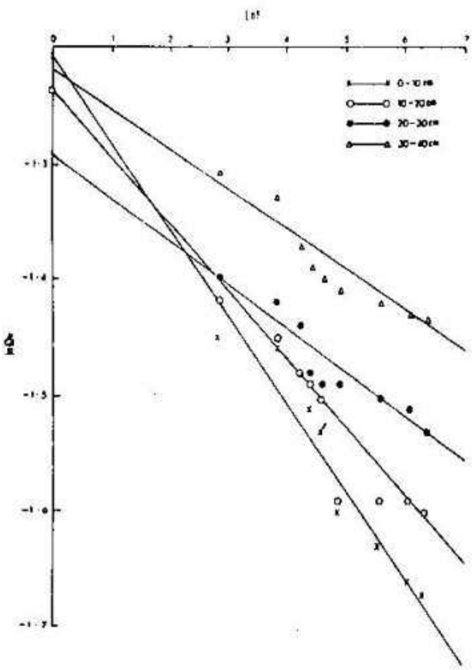
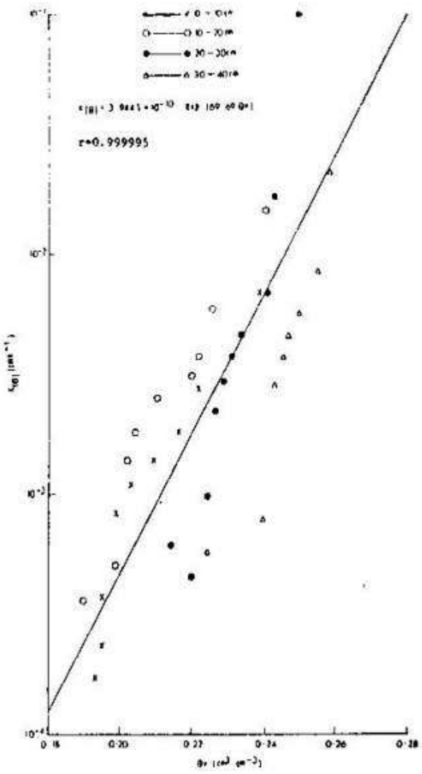
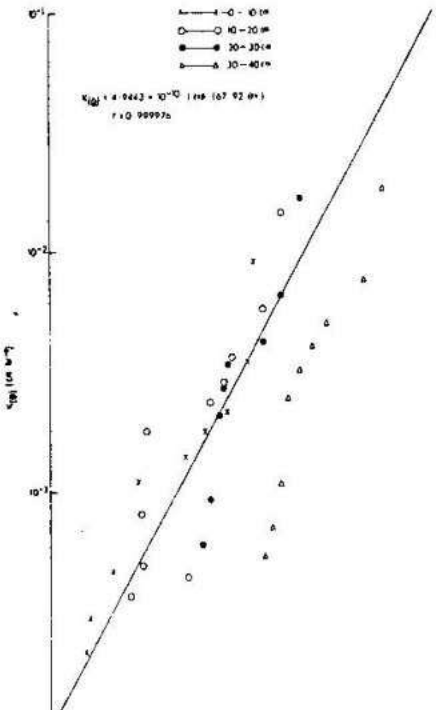
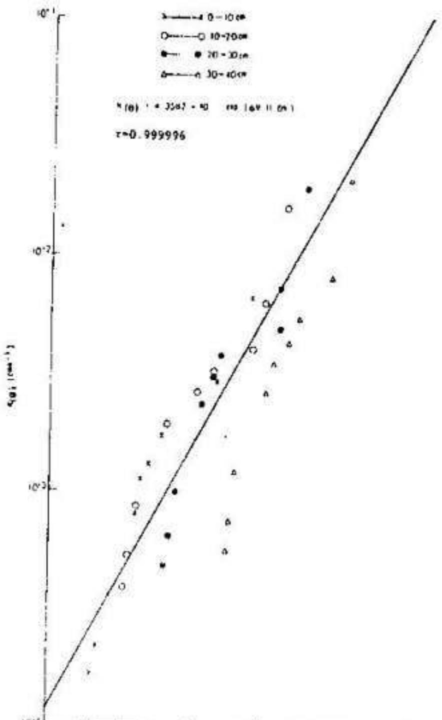



ln t
0
1
2
3
4
5
6
7
-1.3
-1.4
-1.5
-1.6
-1.7
0 - 10 cm
10 - 20 cm
20 - 30 cm
30 - 40 cm


0 - 10 cm
10 - 20 cm
20 - 30 cm
30 - 40 cm
K(θ) = 3.9443 × 10^-10 exp (69.69 θv)
r=0.999995
10^-1
10^-2
10^-3
10^-4
K(θ) (cm h^-1)
0.18
0.20
0.22
0.24
0.26
0.28
θv (cm^3 cm^-3)


0 - 10 cm
10 - 20 cm
20 - 30 cm
30 - 40 cm
K(θ) = 4.9443 × 10^-10 exp (67.92 θv)
r = 0.999976
10^-1
10^-2
10^-3
10^-4
K(θ) (cm h^-1)
0.18
0.20
0.22
0.24
0.26
0.28
θv (cm^3 cm^-3)


0 - 10 cm
10 - 20 cm
20 - 30 cm
30 - 40 cm
K(θ) = 4.3567 × 10 exp (69.11 θv)
r=0.999996
10^-1
10^-2
10^-3
10^-4
K(θ) (cm h^-1)
0.18
0.20
0.22
0.24
0.26
0.28
θv (cm^3 cm^-3)