A Computer Modelled Nuclear Design of a Subcritical Assembly Driven by Isotopic Neutron Sources

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ABSTRACT

A computer code SUNDES, which solves the one-dimensional multi-group neutron diffusion equation, was developed and used to provide a conceptual nuclear design of a facility. The device is capable of producing high thermal neutron fluxes using isotopic neutron sources in a multiplying medium. The neutronic calculations showed that thermal neutron fluxes ≥ 1 x 10^7n/cm^2-s could be produced making it suitable for neutron activation analysis (NAA) and for reactor physics experiments. The analysis also revealed that different thermal neutron fluxes could be produced depending on the strength of the driving isotopic neutron sources.

Keywords: *diffusion, equation, neutronic calculations, subcritical assembly, thermal neutron fluxes.*

INTRODUCTION

Neutron activation analysis (NAA) experiments are sometimes carried out using neutrons from radioisotope sources placed in non-multiplying medium. The thermal neutron fluxes emanating from such facilities are rather too low for trace element analysis and other reactor physics experiments. Since nuclear reactors are expensive and beyond the reach of many developing countries alternate facilities to provide high neutron fluxes must be provided. A stationary assembly, for which effective multiplication factor $k_{eff} < 1$ can be modified, and driven by isotopic neutron source(s), may be used to achieve this purpose.

In this study we provide a computer modelled nuclear design of a subcritical assembly driven by isotopic neutron sources in a multiplying medium which is capable of producing neutron fluxes of the order of 10^7n/cm^2-s to enhance NAA and other reactor physics experiments. Neutronic calculations for the design involve solving the one-dimensional multi-group neutron diffusion equation. A numerical scheme based on finite difference method is used to develop a code 'SUNDES' (an acronym for SUbcritical Nuclear DESign). The code is written in FORTRAN 77 programming language.

This paper discusses the mathematical model for the code and gives a description of the subcritical assembly and its operational characteristics. Finally, the results of the application of the code for the prediction of fast and thermal neutron fluxes are presented.

THEORY

For a steady state to be maintained in a subcritical assembly, external neutron-sources must be introduced. The diffusion equation is thus written as:

$$-D(r,E)\nabla^2\phi(r,E) + \Sigma_t(r,E)\phi(r,E)$$
$$= \int_0^\infty \Sigma_s(r,E'\rightarrow E)\phi(r,E')dE'$$
$$+ \int_0^\infty \chi(r,E')\nu\Sigma(r,E')\phi(r,E')dE'$$
$$+ \sum_{n=1}^{N}\int_0^\infty \eta^n(r,E')q^n(r,E')\delta(r-r_n)dE' \quad (1)$$

where

η(r,E) is the spectrum of neutrons emitted by isotopic sources

n = 1, ..., N

$$Q_{g'k} = \sum_{n=1}^{N} \sum_{g'=1}^{G} \eta^*_{g'k} q^*_{g'k}$$

Using equations (5) - (7), the simplified integrated equation becomes

$$\frac{D_{g,k} r^{\alpha}_{k-\frac{1}{2}}}{\Delta k}(\phi_{g,k}(r_k) - \phi_{g,k}(r_{k-1})) -$$

$$\frac{D_{g'k^-} r_{k+\frac{1}{2}}}{\Delta k^-}(\phi_{g,k}(r_{k+1}) - \phi_{g,k}(r_k)) +$$

$$\left(\frac{r_k^{\alpha+1} - r_{k-\frac{1}{2}}^{\alpha+1}}{\alpha+1}\right)\left(\Sigma_{ag,k}\phi_{g,k}(r_k) - \sum_{\substack{g'=1\\g'\neq g}}^{G} \Sigma_{ag'k}\phi_{g'k}(r_k)\right) + \quad (8)$$

$$\left(\frac{r_{k+\frac{1}{2}}^{\alpha+1} - r_k^{\alpha+1}}{\alpha+1}\right)\left(\Sigma_{ag,k}\phi_{g,k}(r_k) - \sum_{\substack{g'=1\\g'\neq g}}^{G} \Sigma_{ag'k}\phi_{g'k}(r_k)\right) =$$

$$\left(\frac{r_{k+\frac{1}{2}}^{\alpha+1} - r_{k-\frac{1}{2}}^{\alpha+1}}{\alpha+1}\right)(S_{g,k} + a_{g'k})$$

For further simplification of equation (8), the following terms are defined:

$$\tau_k = \frac{r^{\alpha}_{k-1/2}}{(r_k - r_{k-1})} \qquad \rho_k = \frac{r^{\alpha}_{k+1/2}}{(r_{k+1} - r_k)}$$

$$\mu_k = \frac{\left(r_k^{\alpha+1} - r_{k-1/2}^{\alpha+1}\right)}{(\alpha+1)}, \quad \beta_k = \frac{\left(r_{k+1/2}^{\alpha+1} - r_k^{\alpha+1}\right)}{(\alpha+1)}$$

and

$$\xi_k = \frac{\left(r_{k+1/2}^{\alpha+1} - r_{k-1/2}^{\alpha+1}\right)}{(\alpha+1)}$$

Thus equation (8) is rewritten in the form

$$\begin{aligned}
&-D_{g,k^-}\rho_k\phi_{g,k}(r_{k+1}) \\
&+(D_{g,k^-}\tau_k + D_{g,k^-}\rho_k + \mu_k\Sigma_{ag,k^-} + \beta_k\Sigma_{ag,k^+}) \\
&-D_{g,k^-}\tau_k\phi_{g,k}(r_{k-1}) = \xi_k(S_{g'k} + Q_{g'k}) \\
&+\mu_k\sum_{\substack{g'=1\\g'\neq g}}^{G} \Sigma_{ag'-g,k^-}\phi_{g'k}(r_k) \qquad (9) \\
&+\beta_k\sum_{\substack{g'=1\\g'\neq g}}^{G} \Sigma_{ag'-g,k}\phi_{g'k}(r_k)
\end{aligned}$$

If the material properties remain the same on the positive and negative sides of r_k which is chosen to be small, then

$\Sigma_{ag'-g,k^-} = \Sigma'_{ag'}-g,k+$ and is denoted as $\Sigma_{ag'-,k'}$

$D_{g,k^-} = D_{g,k^+}$ becomes $D_{g,k}$ and
$\Sigma_{ag,k^-} = \Sigma_{ag,k^+}$ is simply denoted as $\Sigma_{ag,k}$.

With the above notations, Eq.(9) can be rewritten as a finite difference equation of the form

$$\begin{aligned}
&-D_{g,k}\rho_k\phi_{g,k}(r_{k+1}) + \\
&(D_{g,k}\tau_k + D_{g,k}\rho_k + \xi_k\Sigma_{ag,k})\phi_{g,k}(r_k) \\
&-D_{g,k}\tau_k\phi_{g,k}(r_{k-1}) = \qquad (10) \\
&\xi_k\left(S_{g'k} + Q_{g'k} + \sum_{\substack{g'=1\\g'\neq g}}^{G} \Sigma_{ag'-g,k}\phi_{g'k}\right)
\end{aligned}$$

If we denote

$$\phi_{g,k}(r_{k+1}) = \phi_{g,k+1'}$$

$$\phi_{g,k}(r_k) = \phi_{g,k} \quad \textit{and}$$

$$\phi_{g,k}(r_{k-1}) = \phi_{g,k-1}$$

then equation (10) for energy group g and position k becomes;

$$\begin{aligned}
&-a_{k,k+1}\phi_{k+1} + a_{k,k}\phi_k \qquad (11) \\
&-a_{k,k-1}\phi_{k-1} = \sigma_k
\end{aligned}$$

where the coefficients are;

$$a_{k,k+1} = D_{g,k}\rho_k, \qquad a_{k,k-1} = D_{g,k}\tau_k$$

$$ak,k = a_{k,k+1} + a_{k,k-1} + \xi_k\Sigma_{ag,k} \qquad \textit{and}$$

$$\sigma_k = \xi_k\left(S_{g'k} + Q_{g'k} + \sum_{\substack{g'=1\\g'\neq g}}^{G} \Sigma_{ag'-g,k}\phi_{g'k}\right)$$

Collapsing the multi-group analysis to two (fast and thermal) groups only and further assuming that only "no up-scattering" of neutrons is allowed and all neutrons are born at fast energy then,

$$\sigma_{g,k} = \begin{cases} \xi_k(S_{1k} + Q_{1k}) & \textit{for fast group } (g=1) \\ \xi_k\Sigma_{1,2,k}\phi_{1k} & \textit{for thermal group } (g=2) \end{cases} \qquad (12)$$

where

$$S_{1k} = \sum_{g'=1}^{G} \chi_{g'g}(\nu\Sigma)_{f,g'k}\phi_{g'k} \quad and$$

$$Q_{g'k} = \sum_{n=1}^{N}\sum_{g'=1}^{G} \eta^{*}_{g'k} q^{*}_{g'k}$$

For point sources concentrated at the centre of a single region we assumed that

$$q_{1k} = \begin{Bmatrix} q_o & 0 \le r \le r_1 \\ 0 & r_k > r_1 \end{Bmatrix} \quad (14)$$

For a numerical solution of equation (11) two boundary conditions are applicable. First, it is assumed that the current is continuous across r_k and flux is constant between $r_{k-1/2}$ and $r_{k+1/2}$. Thus, the first equation of the series becomes

$$-D_{g,1}r^{\alpha}_{3/2}\frac{\phi_{g,2}-\phi_{g,1}}{r_2-r_1}+\Sigma_{\alpha g,1}\phi_{g,1}\frac{r^{\alpha+1}_{3/2}-r^{\alpha+1}_{1}}{\alpha+1}$$

$$= \frac{r^{\alpha+1}_{3/2}-r^{\alpha+1}_{1}}{\alpha+1}\left(\sum_{\substack{g'=1\\g'\neq g}}^{G} \Sigma_{sg'\to g,1}\phi_{g',1}+S_{g,1}+Q_{g,1}\right) \quad (16)$$

where the subscript 1 denotes the mesh point at the centre of the assembly. The values of $a_{1,1}$ and $a_{1,2}$ using equation (16) are simplified to the forms;

$$a_{1,1} = D_{g,1}r^{\alpha}_{3/2}+\Sigma_{\alpha g,1}\left(\frac{r^{\alpha+1}_{3/2}-r^{\alpha+1}_{1}}{\alpha+1}\right) \quad and$$

$$a_{1,2} = -D_{g,1}r^{\alpha+1}$$

The other boundary condition imposed was that the flux must be zero at the outermost boundary of the assembly and thus the last equation of the series for the last mesh point becomes

$$(D_{g,M}\tau_M + \Sigma_{\alpha g,M}\mu_M)\phi_{g,M} - \tau_M D_{g,M}\phi_{g,M-1}$$

$$\mu_M\left(\sum_{\substack{g'=1\\g'\neq g}}^{G} \Sigma_{sg'\to g,M}\phi_{g',M} + S_{g,M} + Q_{g,M}\right) \quad (17)$$

The coefficients $a_{M,M-1}$ and $a_{M,M}$ are accordingly fixed as $a_{M,M-1} = -T_M D_{g,M}$ and $a_{M,M} = D_{g,M}\tau_M + \Sigma_{\alpha g,M}\mu_M$ respectively and the resulting matrix equation is of the form

$$A_{g,k}\phi_{g,k}+\xi_k\left(B_{g,k}+\sum_{\substack{g'=1\\g'\neq g}}^{G} C_{g'\to g,k}\phi_{g',k}\right) \quad (18)$$

where

$$A_{g,k} = \begin{pmatrix} a_{11} & a_{12} & & & \bigcirc \\ a_{21} & a_{22} & a_{23} & & \\ & & a_{M-1,M-2} & a_{M-1,M-1} & a_{M-1,M} \\ \bigcirc & & & a_{M,M-1} & a_{M,M} \end{pmatrix}$$

$$\phi_{g,k} = \begin{pmatrix} \phi_{g,1} \\ \phi_{g,2} \\ \cdot \\ \cdot \\ \phi_{g,M-1} \\ \phi_{g,M} \end{pmatrix} \quad C_{g'\to g,k} = \begin{pmatrix} C_1 & & \bigcirc \\ & C_2 & \\ & & \ddots \\ \bigcirc & & C_{M-1} \\ & & & C_M \end{pmatrix} \quad B_{g,k} = \begin{pmatrix} b_1 \\ b_2 \\ \cdot \\ \cdot \\ b_{M-1} \\ b_M \end{pmatrix}$$

The matrix $C_{g'\to g,k}$ is the scattering term while $B_{g,k}$ accounts for the sum of the fission and source terms.

In the code SUNDES there exists an option for normalizing the neutron flux to pre-determined power levels produced in the assembly. The power is calculated from the relation

$$px = \xi\int_{V_A}\sum_{g'=1}^{G}\phi_{g,k}\Sigma_{fg,k}dV^{\alpha} \quad (19)$$

where V_A is the volume of the assembly, ξ = 3.2E-11 the number of watts/sec per fission and the elemental volume for a cylindrical geometry, $dV^{\alpha} = 2\pi r_k$. The fluxes were normalised to total power the subcritical assembly. The normalling constant is φ such that $\phi_{g,k} = \varphi\phi_{g,k}$. From equation (19) φ becomes

$$\varphi = \frac{p}{px} = \frac{p}{\xi\int_{V_A}\sum_{g'=1}^{G}\Sigma_{fg,k}\phi_{g,k}dV^{\alpha}} \quad (20)$$

The computational flowchart of the code SUNDES written for the reactor physics calculations of the assembly is presented in Fig.2. The details of the code are contained in ref [1]. Before the computed results are presented, the geometrical features of the nuclear device and its operational characteristics are briefly discussed in the next section.

DESCRIPTION OF THE ASSEMBLY

The design consists of nine distinct homogeneous regions hereby denoted as A, B, C, D, E, F, G, H and I. The dimensions of the respective regions are shown in Table 1. Fig 3 is a diagram of the assembly and its vertical cross section is shown in Fig 4.

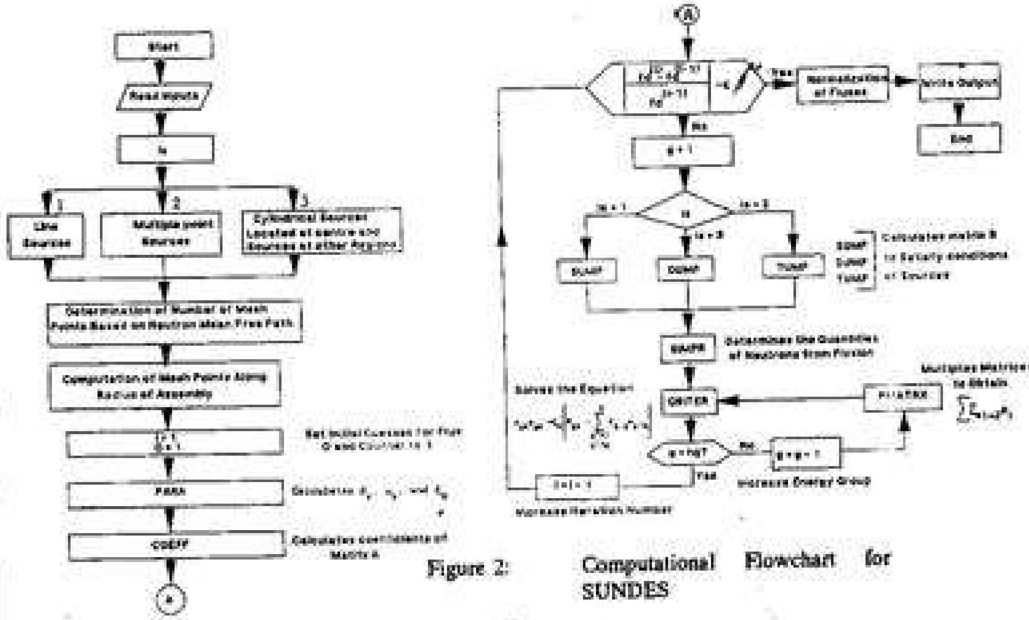


Figure 2: Computational Flowchart for SUNDES

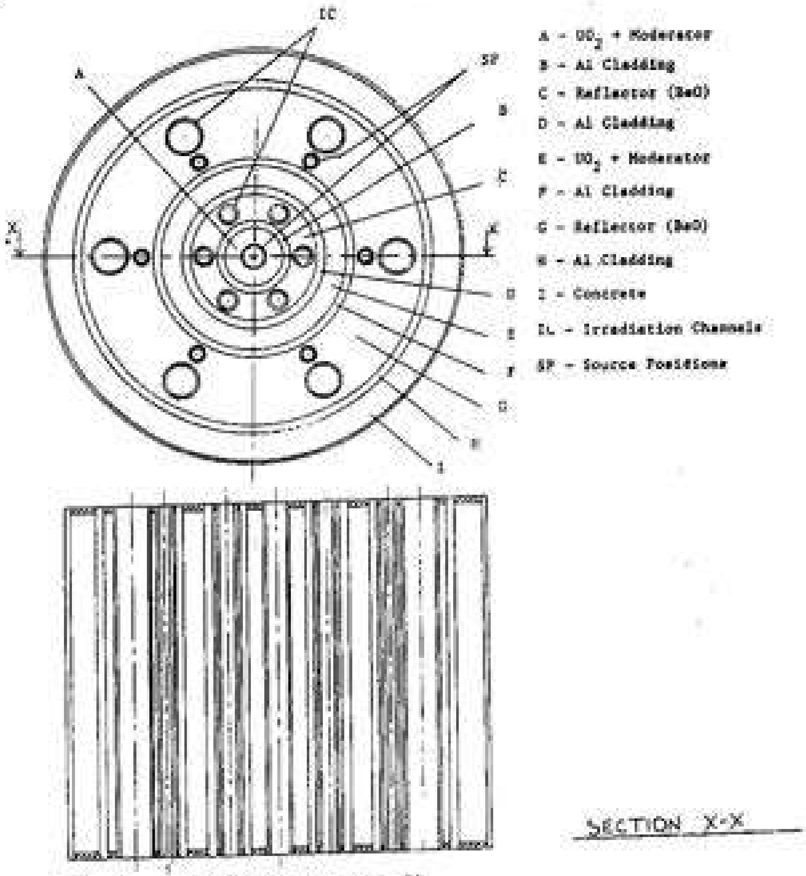
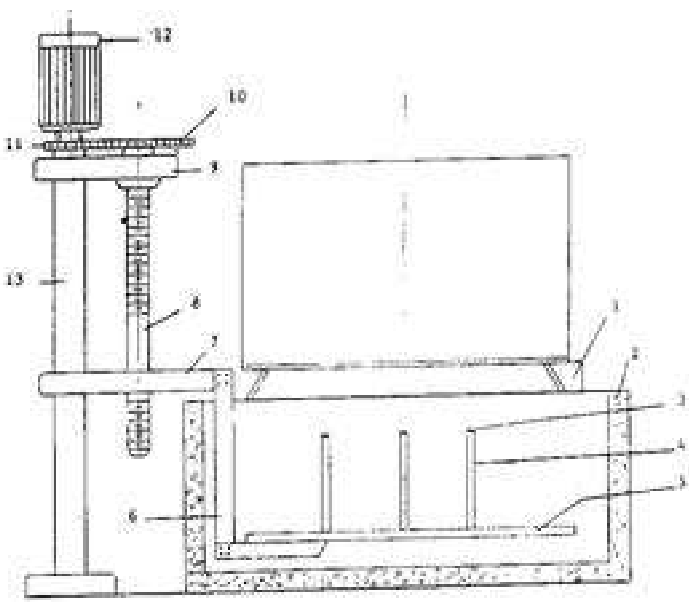


Fig. 3: Diagram of the Assembly

Table 1: Dimensions of Regions of the Assembly

Region	A	B	C	D	E	F	G	H	I
Radius (cm)	3	4	9	10	32	33	40	41	43



1. Supports
2. Concrete tank
3. Neutron source(s)
4. Thin Metal rods
5. Metal plate
6. Arm
7. Movable nut
8. Screw
9. Thrust bearing
10. Spur gear
11. Spur gear pinion
12. Electric motor
13. Steel pillar

Fig. 4: Vertical Cross-Section of the Assembly

The first concentric ring A is filled with a homogeneous mixture of fuel and moderator (20% enriched UO_2 + Be). Regions B, D, and F consist of Al material as cladding which separates regions A, C, and E from each other. Region C is occupied by a reflector (BeO). There is yet another homogeneous mixture of fuel and moderator in region E as in region A to cause more fission in the system. Regions G and H constitute the reflector (BeO) and shielding (Al) materials respectively. Finally, region I is ordinary concrete to act as biological shield.

The neutron multiplier has a lifting device for introducing or withdrawing neutron sources into and out of the multiplier. Four support for the multiplier (1) bear it from underneath. The support are bolted onto a concrete tank (2) which contains water into which the neutron sources (3) are submerged when outside the multiplying medium. The sources are mounted on thin metal rods (4) which are fitted vertically onto a metal plate (5). The rod and plate are made of Al which is resistant to heat, corrosion and radiation at low power.

The plate is bolted rigidly to an extension of a movable nut (7). A screw (8) runs through the nut and is supported in thrust bearing (9) at the upper end. The screen is keyed to a spur gear (10)at its upper end. The spur gear is meshed with a spur gear pinion (11) which is keyed to an electric motor (12) mounted against a vertically guide for the nut and provides a counter moment against that set up by the weight of the plate, arm and water above the arm.

A projection on the movable nut actuates an electrical switch at the two extreme ends of its movements up and down the screw. This switch is one of two alternative switches for switching the motor (12) 'on' or 'off'. The second switch is located in the control room. The two switches work in consonance and are capable of reversing the polarity of the motor. By their action neutron sources are introduced into or withdrawn from the multiplying medium.

RESULTS AND DISCUSSIONS

Nuclear properties of macroscopic group constants for the design were generated using WIMSPC [2], a PC version of lattice code WIMS [3] for the multi-region described above.

An isotopic neutron source of strength 8.45×10^8 n/s is located at the centre to drive the assembly. The variation of the fast and thermal fluxes with radius of the assembly is shown in Fig 5. It can be seen from the figure that the maximum flux levels for both fast and thermal groups occurred at the centre of the assembly where the isotopic neutron sources is located.

The advantage of selecting BeO as reflector is that reactivity will increase with the presence of fast neutrons available from Be(n,2n) and photo neutrons from Be(γ,n) reactions. Pure Be metal could be used but it was found out that BeO is less expensive and easier to fabricate. The shielding materials considered in the analysis were Al, Pb, and stainless steel (SS) for region H. Al being light in weight and less expensive was suitable for such a low power assembly.

Multiple source effects were investigated in the multi-region assembly by placing sources of different strengths in various zones. It was observed that different source strengths have different effects on the neutron fluxes and so were their positions in the assembly. These are presented in Figs. 6 and 7 respectively.

CONCLUSION

A computer code was successfully developed for subcritical reactor analysis. The computational techniques applied in solving the 1-D multi-group diffusion equation have been described in the paper. It has been demonstrated that it is technically feasible to design a subcritical assembly driven by isotopic neutron source(s) to produce thermal neutrons for NAA and other reactor physics experiments. Using an isotopic neutron source of strength 8.45×10^8n/s, thermal neutron fluxes as high as 1×10^7n/cm^{2}s were achieved. Comprehensive thermal and structural analysis are yet to be carried out. The preliminary conceptual design is very encouraging and detailed two - dimensional analysis needs to be carried out. In addition, thermal and structural analysis are necessary before the actual assembly could be constructed.

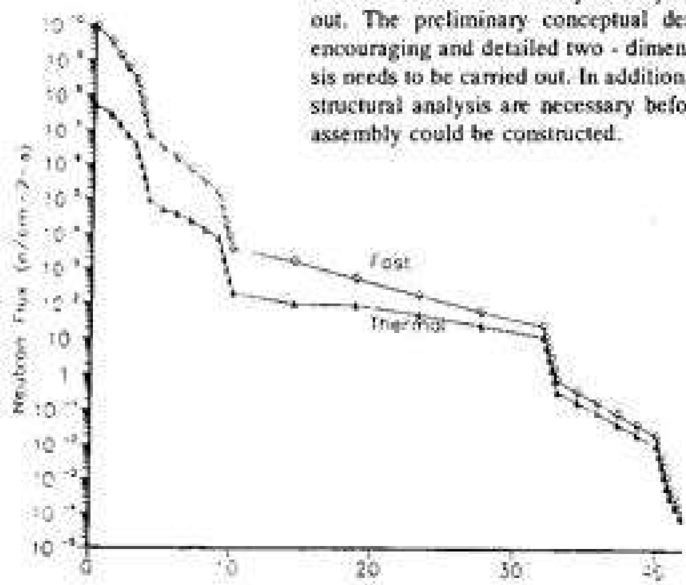


Fig. 5: Variation of Neutron Fluxes with Radius

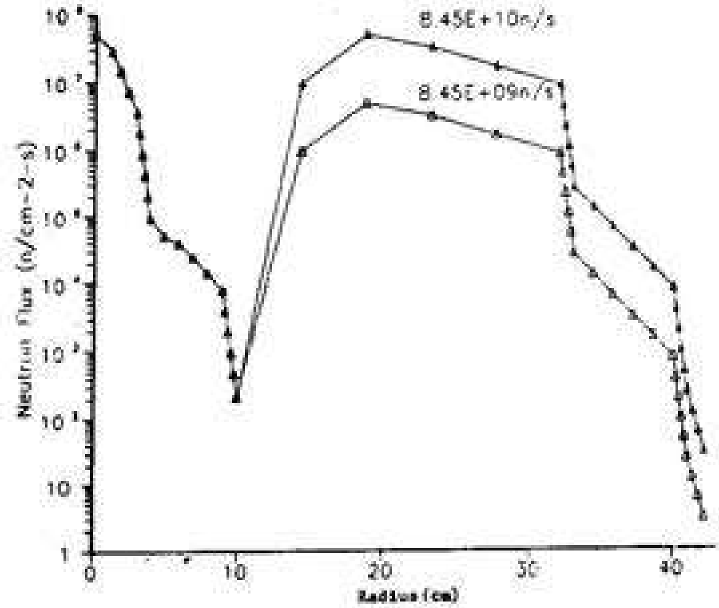


Fig. 6: Effect of Source Strengths on Thermal Flux in Region E.

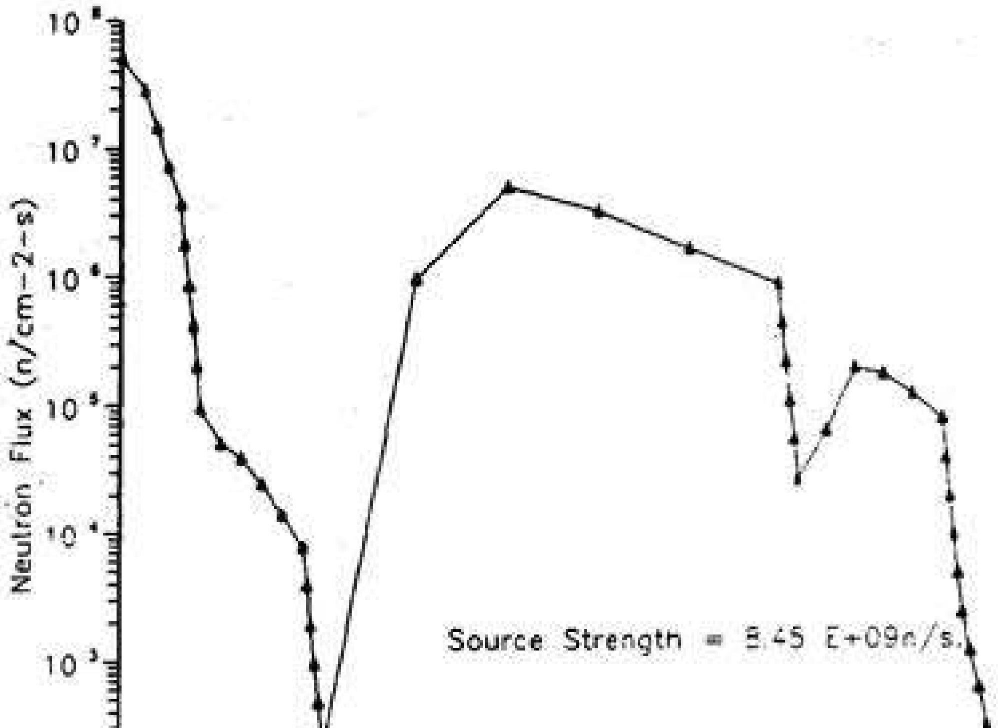


Fig. 7: Effect of Source Strengths on Thermal Neutron Flux in Regions E and C.

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