Sensitivity Analysis of some Equations used in Urban Stormwater Drainage Design in Ghana

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ABSTRACT

Sensitivity analysis was carried out for the Rational and Manning formulae - the most popular equations used for the design of stormwater drainage systems in Ghana. The Return Period and the Runoff Coefficient were the input parameters for the Rational Formula; the Channel Slope, Manning's Roughness Coefficient,the Depth and Width of a concrete U-drain formed the input parameters for the Manning's Equation. The results revealed that the Rational formula is more sensitive to changes in the Runoff Coefficient than equivalent percentage changes in the Return Period. For the Manning's Equation it was found out that channel depth and channel width have a more pronounced effect on the carrying capacity of the channel than the slope and roughness coefficient. Nevertheless, changes in the performance of a channel due to changes in the roughness coefficient and the slope were found to be very significant. Based on these results a strong recommendation is made for regular maintenance of drains.

Keywords: *Rational Formula, Manning's Equations, Sensitivity Analysis.*

INTRODUCTION

As part of Ghana's Structural Adjustment Programme (SAP), provision has been made for the expansion and upgrading of the necessary infrastructure in the urban areas of the country. Of particular importance to this study is the recognition given to the necessity to improve upon the stormwater drainage systems in the major cities and towns in Ghana. This has been demonstrated by the projects executed under the Urban I, Urban II and Urban III projects. The upgrading has become necessary because drains and sewers built several decades ago have broken down, or their capacities have become inadequate for the expanding urban environment.

It is an undeniable fact that the cost of upgrading the performance of the existing stormwater drainage systems is a significant proportion of the total urban upgrading expenditure. The development of an appropriate strategy for solving these drainage problems is therefore very necessary, and requires careful planning at all levels of the project if maximum cost savings are to be realized. This calls also for a corresponding serious research into the subject of stormwater drainage systems at the model formulation stages to the operation and maintenance levels.

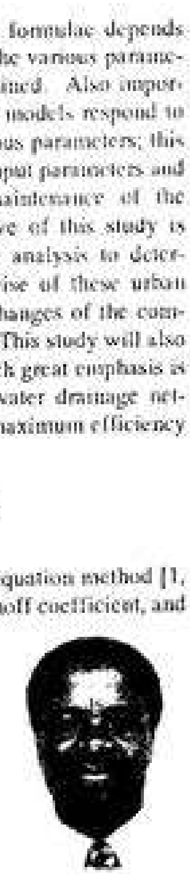
A study of stormwater drainage designs in Ghana reveals that, for most of these designs, the hydrological model employed in estimating the design flows is the "Rational Formula", while the hydraulic model used for sizing the channels is the "Manning Equation". These equations will continue to be used for urban stormwater drainage systems for a long time to come.

The effective use of these two formulae depends upon the accuracy with which the various parameters in each formula are determined. Also important is a knowledge of how the models respond to the relative changes of the various parameters; this will guide in the choice of the input parameters and help in the operation and maintenance of the drainage system. The objective of this study is therefore to employ sensitivity analysis to determine the robustness or otherwise of these urban drainage formulae to relative changes of the common parameters used as inputs. This study will also reveal those parameters on which great emphasis is to be directed in urban stormwater drainage network rehabilitation to achieve maximum efficiency and savings.

THE DESIGN EQUATIONS

The Rational Formula.

Basically, the rational method-equation method [1, 2] relates rainfall intensity, a runoff coefficient, and



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drainage area size to the peak runoff rate. This relationship is commonly expressed by the equation:

$$Q = KCIA \quad (1)$$

where Q = the peak runoff rate (m^3/S)

K = constant which depends on the units of I and A.

C = runoff coefficient

I = rainfall intensity (mm/h)

A = total area contribution runoff (hectares).

The Manning's Equation

Robert Manning presented a formula [3], which was later modified to its present well-known form:

$$Q = \frac{A}{n} R^{2/3} S^{1/2} \quad (2)$$

here Q = Discharge in channel (m^3/s)

A = Cross-sectional area of channel (m^2)

R = The hydraulic radius (m)

S = Slope of the energy line (m/m)

n = the coefficient of roughness, specifically know as Manning's n.

Owing to its simplicity of form and to the satisfactory results it gives in practical application, the Manning formula has become the most widely used uniform - flow formula for open-channel flow computation.

SENSITIVITY ANALYSIS

Definition:

By definition, a sensitivity analysis shows the effect of change of one factor on another [4]. That is, sensitivity analysis examines the effect of changes in model inputs and parameters on the values of the model outputs (and the revenue function where monetary values are attached). If the outputs change very little with perturbations in the inputs and parameters, then at the planning and design stages the coefficients and inputs in the design equation for the system may be quite approximate. It also implies that during the operation stage, any small changes in the input parameters will not lead to significant changes in the outputs. On the other hand, if the response of the model shows a high degree of sensitivity to changes in its parameters and inputs, these coefficients and inputs must be determined with much greater accuracy. Again changes in the design parameters during the operation stage will lead to alarming outputs. There is no point in carefully and accurately obtaining data for relationships that can be shown to play little part in the final analysis, and vice versa.

Sensitivity Analysis applied to the Rational Formula

In using the Rational Formula for design, the parameters which have a high degree of subjectivity in their determination are the Rainfall Intensity, I, and the runoff coefficient, C. It is always possible to determine the catchment area with a very high degree of accuracy.

The rainfall intensity is dependent on the Return Period T, and the time of concentration, t. This:

$$I = f(T,t) \quad (3)$$

where T and t have the units of years and minutes respectively. Analysis of the Ghana Meteorological Service Rainfall Intensity - Duration Frequency curves popularly used for design in Ghana had revealed that these curves are relatively flat for valves of t=0.1 hr to 1 hour - the range of t values commonly encountered for the urban catchments in Ghana. This flatness shows that I is relatively insensitive for $0.1 \text{ hr} \leq t \leq 1$. It is again worth stating that no studies appear to have been conducted to determine the best equation for t that is most suitable for our local conditions. Hence we do not know that rough estimates oft will result in underdesign or overdesign of our urban drainage structures. Thus, it is the determination of T, the return period, which is exposed to the designer's judgement. This is so because T is dependent on the designer's assessment of the utility or economic value of the catchment, and this at present, is very subjective.

The runoff coefficients dependent on the degree of imperviousness and the slope of the catchment. Even though this parameter has been studied for some catchments in the developed countries and widely quoted in the appropriate books, estimating this value for Ghanaian catchments is very subjective because of the absence of any comprehensive studies on the subject in the country.

In what follows, sensitivity analysis of the Rational Formula will be carried out using only the Return Period and the Runoff coefficient because of the inherent uncertainties in their determination. The Subin Valley Drain of the city of Kumasi will be used for the analysis.

The Subin Valley drain receives surface runoff and waste water from about two-thirds of Ashanti-New-Town area, half of the Methodist Mission area, parts of Mbrom and Manhyia/ Ahinboboano areas of Central Kumasi. It is a fully built-up area of mostly residential houses. The topography can be described as moderately steep. The total area of the catchment is estimated as 40 hectares. The design area is the "Subin Valley" which is to be converted into a Lorry Park and Light Commercial activities. A drain of adequate carrying capacity is to be provided to cope with high discharges that are expected.

Substituting the value of the catchment area, A = 40 hectares, and K = 1/360 in the Rational Formula as in equation (1), the following simplified general equation was obtained:

$$Q = \frac{CI}{9} \qquad (4)$$

For a given return period T_r and the estimated time of concentration t from the catchment, it was possible to determine the corresponding value of I from the Ghana Meteorological Services Rainfall Intensity - Duration Frequency Curves for Kumasi. The analysis was subsequently carried out using the following equations based on equation (4)

$$Q_{C_i} = \frac{C_i I}{9} \qquad (4a)$$

$$Q_{I_i} = \frac{CI_i}{9} \qquad (4b)$$

where Q = Catchment discharge for a particular value of coefficient C_i where ($0.1 < C_i < 0..9$) and keeping I constant.

QI_i = Catchment discharge for a particular value of return period, I_i, keeping C constant.

T = 5, 10, 20, 30, 50,

The catchment area and the time of concentration for the catchment were kept constant during the calculation of the above discharges. Table 1 shows the expected discharges from the Subin Valley Catchment into the Subin Valley drain with respect to changes in the Return Period and the Runoff Coefficient.

TABLE 1. Expected Discharges (m^3/s) from the Subin Valley Catchment into the Subin Drain due to changes in C and T values.

C	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
T YEARS	CATCHMENT DISCHARGES (m^3/s)								
5	1.7	3.4	5.1	6.8	8.4	10.2	11.8	13.6	15.2
10	2.0	4.0	5.9	7.9	9.9	11.8	13.8	15.8	17.8
20	2.2	4.5	6.8	9.0	11.3	13.6	15.8	18.0	20.3
30	2.5	5.1	7.7	10.2	12.8	15.4	17.9	20.4	23.0
50	2.8	5.6	8.5	11.3	14.1	17.0	19.8	22.6	25.4

Sensitivity Analysis applied to Manning's Equation

The commonest type of channel used in transporting urban storm water in Ghana is the concrete U-drain. This is either covered or open. The Manning's Equation is used in determining the dimensions of these drains. The carrying capacities of the channels are therefore fixed by the chosen dimensions for the drains. The input parameters are the slope of the channel, roughness coefficient, the depth of the channel and the width of the channel represented by the symbols s, n, d, and w respectively. See Fig. 1 for a typical section of a U-drain.

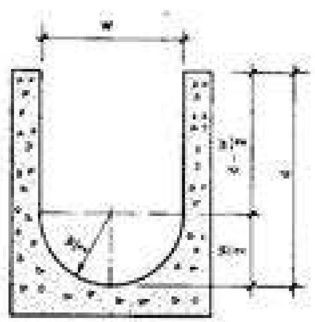

Fig. 1: A Typical Section of a Concrete U-Drain.

The discharge, Q, of the U-drain, using Manning's Equation, is given as:

$$Q = \frac{1}{n}\frac{[w(d-0.11w)]^{5/3}}{(2d+0.57w)^{2/3}} S^{1/2} \qquad (5)$$

The sensitivity analysis was performed on equation *(5)* by means of the following equation:

$$\frac{Q_i}{Q_o} = \frac{Q_i - Q(1\pm x)_i}{Q_o} \qquad (6)$$

where

Q_o = Design discharge (m^3/s)

$Q_{(1\pm x)i}$ = Discharge resulting from a change of parameter i by a fraction x, (m^3/s). $0 < x < 1$ in this study

Q_i = Loss or gain in channel carrying capacity as a result of change of magnitude of i by x while other parameters are kept constant.

The parameters used in the analysis are n,s,w and d as defined already. For the sake of this study typical design values were given to input parameters. These are n = 0.013, s = 0.01, d = 0.8m and w = 0.9m. Fig. 2 represents, in a graphical form, the result of the analysis. It shows the relative loss in the carrying capacity of the channel as a result of fractional changes in each of the parameters while the others remain constant.

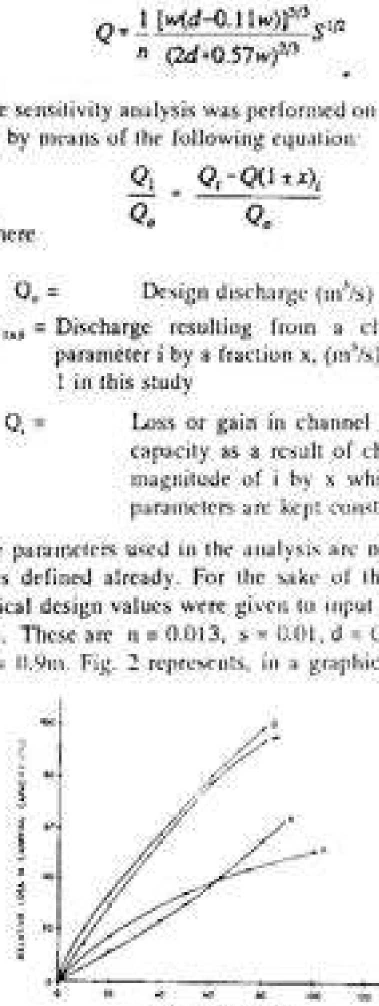

Fig. 2: Relative Loss in Channel carrying capacity W.R.T. Relative Chnges (%) in Channel Parameters. n, s, w and d (5).

DISCUSSION OF RESULTS

The results displayed in Table 1 show that the Rational formula is more sensitive to changes in the runoff coefficient than the return period. This result is important because, in practice, much more importance is often paid to the return period than it deserves.

From the results it can be seen that a 100% increase in the return period will not result in an average increase of catchment discharge of more than 20%. On the other hand a 100% increase in the coefficient leads to a corresponding 100% increase in the catchment discharge. The high sensitivity of catchment discharge to changes in C calls for efforts to determine accurately the C values for our catchments, especially urban catchments which are prone to frequent floods these days. Most of the main storm drainage channels serving our cities were designed when the contributing catchments were not urbanized as they are today. Rapid urbanization accompanied by an increase in the paved areas has resulted in corresponding increase in the runoff coefficient which is more than 100% in many cases. This partly explains the inability of the existing drains to cope with the storm runoff.

To reverse this trend in the increase in runoff coefficient in the urban areas, it is recommended that the surroundings of many residential areas and open spaces be grassed instead of being paved with concrete. Other measures which will encourage the infiltration of surface runoff into the ground should be researched into.

Fig.2 represents in a graphical form the response of the Manning's Equation to changes in the main parameters, n,s,d, and w.

The results show that changes in the cross-sectional area of the drain represented by d and w lead to a high decrease in the carrying capacity the channel. For example, a 40% decrease in d results in a 50% decrease of the carrying capacity. In practice these decreases in d and w occur in the form of siltation of drains, caving in of sides of drains along road sides, etc. which are common sites in our urban centres.

Even though smaller in magnitude, the loss in carrying capacity of the concrete U-drain due to changes in S and n are very significant. An increase of the value of n by 40% that is from n = 0.013 to n = 0.018, will result in a decrease of about 30% in the design carrying capacity of the channel. A value of n = 0.018 represents the n value of a very rough scoured concrete surface - a common condition in our U-drains. With the various items commonly dropped into drains and weeds growing in them, the value of n has often doubled in many drains, thus resulting in the decrease of performance, even for drains whose cross-sections have not been reduced.

Flattening of invert gradients of channels due to the deposition of silt and rubbish in drains is a common scene. This phenomenon can also happen when there is scouring and erosion at the upstream end of the channel, where gradients are steep, and subsequent deposition at the downstream ends where slopes are flat. The relative loss in performance of the channel is shown by the curve in Fig.2 labelled S.

It is worth noting that the relative loss of the carrying capacity of the channel due to the combined effect of changes in all the parameters n,s,d and w by 20% each is as high as 82%. When it is realized that this combined effect is common in practice, it is not surprising that many drainage systems in our urban areas are performing below their design capacities.

This analysis shows that regular maintenance of drains is a very effective means of draining our urban centres. Regular desilting of choked drains, good refuse management (to prevent refuse from entering drains), patching of scoured inverts and sides of drains, removing of weeds, and repairing broken or caved-in drains are very cost-effective ways of maintaining very good performance of storm drains in our cities.

Despite the fact that the analysis was performed using a particular catchment and locality for rational formula, and a chosen U-drain with given values of n, and s, for the Manning's Equation, it must be noted that the results follow the same pattern of response when other catchments and drain sections are subjected to sensitivity analysis.

CONCLUSION

It is no secret that even the ordinary man on the street knows that choked and silted drains constitute major drainage problems in our cities. What is not known is the relative magnitudes of the contributions of these defects to the poor performance of the drains. This study has tried to analyse the two main equations used in the design of Ghana's storm drainage systems and to bring to light the quantitative response of these equation to relative changes in the input parameter.

This study also has revealed that routine maintenance is a very effective way of operating an efficient storm drainage system in our cities.

Finally the relative importance of the runoff coefficient and the return period in the Rational formula has been brought to light.

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