AN ADAPTIVE AUTO PILOT FOR A SAILING YACHT

F.K. OMANI, MSc., PhD, DIC
Department of Electrical Engineering
School of Engineering, U.S.T., Kumasi

ABSTRACT

The paper reports the design and software implementation of an adaptive autopilot for a sailing yacht. The method adopts an extended Kalman filter to estimate time varying system parameters. The estimator is combined with a weighted control minimum variance controller to ensure both efficient course keeping and optimal response to course alterations. Simulation results are presented to demonstrate the performance of the autopilot on a number of simulated models. The results presented show that the proposed autopilot works well for time varying system models subjected to sea disturbances.

KEYWORDS: *Autopilot, adaptive control, yacht dynamics, sea dynamics, computer simulation.*

INTRODUCTION

Adaptive control techniques can be divided into two distinct algorithms, namely model reference adaptive control (MRAS) and self tuning control (STC). In general STCs are designed from a stochastic point of view, thus they are preferable to MRAS systems in a highly noisy environment. The landmark paper on STCs was by Aström and Wittenmark [1]. Their method combined a recursive least squares algorithm with a minimum variance control law. However, the minimum variance controller of Aström [1] would lead to unbounded control for inverse unstable (non-minimum phase) systems. This is a severe restriction, since such systems often occur in sampled data systems even when the underlying continuous time system is inverse stable Aström [2]. Clarke and Gawthrop [3], extended the minimum variance controller by weighting the control variable, thus making the STC more widely applicable.

In this paper the weighted minimum variance controller is combined with an extended Kalman Filter type algorithm to design an adaptive autopilot for a sailing yacht subjected to sea disturbances. Simulation results show that the adaptation algorithm proposed, was able to track both slow variation in system parameters and also single sudden jumps in system parameters typical of taking down the jib sail, main sails, or sudden shift in wind direction and speed.

MODELLING OF YACHT AND SEA DYNAMICS

The pioneering work on modeling ship dynamics was done by Bech [9], in which a nonlinear model was derived for surface ships. For the purposes of this research, the yacht dynamics was approximated by a second order linear model. From this point of view the nonlinear underlying system may be regarded as piecewise linear, any changes in set point would lead to relinearisation about a new set point. This can be accommodated by allowing the linearised model to be time varying.

Defining y(t) as the yacht heading and u(t) as rudder angle leads to the yacht model.

$$\frac{y(s)}{u(s)} = \frac{a}{s^2 + bs + C} \qquad (1)$$

where a, b, c represent system parameters and t the time variable. The descretized version of (1) using zero order hold equivalence gives the model.

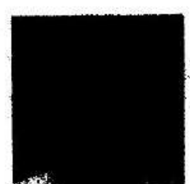
$$\frac{y_t}{u_t} = \frac{z^{-1}(b_1 + b_2 z^{-1})}{1 + a_1 z^{-1} + a_2 z^{-2}} = \frac{z^{-1}B(z^{-1})}{A(z^{-1})} \qquad (2)$$

The sea dynamics may be modeled by the well known Pierson-Moskowitz sea spectrum which has the form

$$\Phi(\omega) = \frac{A}{\omega^5} e^{-\frac{\beta}{\omega^4}} \qquad (3)$$

A = Constant, β = function of significant wave height ($H_{1/3}$). By sketching the frequency response plots, equation (3) maybe approximated by the spectrum

$$\Phi(\omega) = |G(\omega)|^2 \qquad (4a)$$



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where

$$G(s) = \frac{(K)^{0.5}S^2}{[S^2 + (2)^{0.5}ES + E^2]^2} \quad (4b)$$

E is a function of β hence a function of the significant wave height, K is a constant. Thus using the signal representation theorem (Box-Jenkins [4]) the sea disturbances was approximated by filtered white noise, where the filter dynamics is given by equation (4b). The descretized version of the noise filter has the general form

$$\frac{\theta_t}{n_t} = \frac{L(1 + C_1z^{-1} + C_2z^{-2})}{1 + d_1z^{-1} + d_2z^{-2} + d_3z^{-3} + d_4z^{-4}} = \frac{LC(z^{-1})}{D(z^{-1})} \quad (5)$$

The complete model of the yacht and sea disturbances may be represented by the block diagram shown in fig. 1.

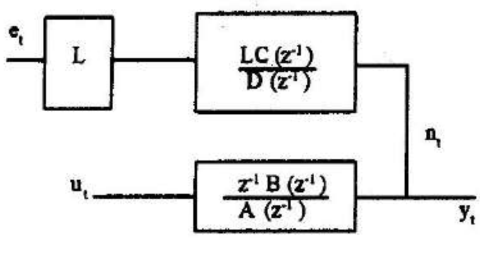


Fig. 1 Yacht plus noise model

MINIMUM VARIANCE CONTROLLER WITH WEIGHTED CONTROL ACTION

In reference [3], Clarke and Gawthrop proposed a weighted control action minimum variance controller. The algorithm minimizes the cost function

$$J = E[(Py_{t+k} - RW_t)^2 + (QU_t)^2] \quad (6)$$

where P, R and Q are weighting polynomials, W_t is a set point and E denotes the expectation. The main advantages of using this generalized cost function over that proposed in [1] are:-

i. The controller provides optimal response to set point changes. This would allow for efficient alteration of course by the autopilot.

ii. Control effort is penalized: This would help reduce overall drag on the yacht since it would avoid excessive rudder action.

iii. By careful selection of weighting polynomials P, Q and R, the controller can be made closed-loop stable for a variety of open-loop unstable and non-minimum phase systems. Thus Clarke's [3] algorithm may be interpreted in the pole placement mode.

iv. An important case arising from (iii) above is to choose polynomial Q so as to penalize the rate of change of control. This cost function was applied to ship steering by Horigome et al [6]. Such a cost function is generally desirable as the rate of change of rudder angle in a fast flowing liquid medium affects the power required by rudder servo and hence affects the overall control effort.

Thus performing the factorisation (leaving out the z^{-1} terms)

$$\frac{C^*}{A^*} = F + Z^{-k}G \quad (9)$$

The optimal control law is then given by

$$EU_t = C^*W_t + Gy_t \quad (10)$$

where

$$E = BF + QC^* \quad (11)$$

PARAMETER ADAPTATION BY EXTENDED KALMAN FILTERING

Following techniques developed in [5], the extended Kalman filter can be used to estimate time-varying parameters of a linear system by regarding system parameters as extra state variables in a state space model. This results in a nonlinear estimation problem which can be recursively re-linearised about each sampling point.

Equation (8) can be rewritten in the state space form

$$X_{t+1} = A_tX_t + B_tU_t + ge_t \quad (12a)$$

$$y_t = h^TX_t \quad (12b)$$

where

$$X_t = \begin{vmatrix} y_t \\ y_{t-1} \\ n_{t+1} \\ n_t \\ n_{t-1} \\ n_{t-2} \\ e_{t-1} \\ e_t \end{vmatrix} \quad g = \begin{vmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{vmatrix} \quad h = \begin{vmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{vmatrix} \quad (12c)$$

$$A_t = \begin{vmatrix} -a_1 & -a_2 & 1 & -a_2 & -a_3 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -d_1 & -d_2 & -d_3 & -d_4 & -c_1 & -c_2 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{vmatrix} \quad (12d)$$

$$B_t = \begin{bmatrix} b_1 & b_2 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad U_t = \begin{bmatrix} U_t \\ U_{t+1} \end{bmatrix} \qquad (12e)$$

For the purpose of parameter tracking the state vector X_t may be extended with the parameter vector as.

$$X_{t+1} = f(X_t, U_t) + ge_t \qquad (13a)$$

where

$$f[x(t), u(t)] = \left[\frac{A_t \mid O}{O \mid I}\right]\left[\frac{X_t}{S_t}\right] + \left[\frac{B_t}{O}\right] U_t \qquad (13b)$$

$$g = \begin{bmatrix} g \\ o \end{bmatrix}, \; S_t = \begin{bmatrix} a_1 \\ a_2 \\ b_1 \\ E \end{bmatrix} \qquad (13c)$$

The extended Kalman filter may then be implemented as follows:

$$K_t = P_{t|t-1} h[h^T P_{t|t-1} h + V_e]^{-1} \qquad (14a)$$

$$X_{t|t} = X_{t|t-1} + K_t[y_t - h^T X_{t|t-1}] \qquad (14b)$$

$$P_{t|t} = [P_{t|t-1} - K_t h^T P_{t/t-1}] \qquad (14c)$$

$$P_{t+1|t} = F_t P_{t|t} F_t + gL^2 g^T \qquad (14d)$$

$$X_{t+1|t} = A_t X_{t|t} + B_t U_t \qquad (14e)$$

where P_t = Covariance matrix

$P_{o|-1} = 1000I$

I = unit matrix

K_t = time varying Kalman gain

$$F_t = \frac{d}{dX_t}[f(X_t, U_t)]$$

λ_t is variable forgetting factor, Foretescue et al [7], which keeps the covariance matrix open to new data, but prevents the 'blow up' to the covariance matrix. λ_t may be generated as follows:

$$\varepsilon_t = y_t - h^T X_{t|t-1} \qquad (15a)$$

$$N_t = \frac{[1 + h^T P_{t|t-1} h]\, \Sigma o}{\varepsilon^{2t}} \qquad (15b)$$

$$\lambda(t) = 1 - \frac{1}{N_t} \qquad (15c)$$

Σ_o can be interpreted as the nominal memory length. In reference [8], Mayne et al, have shown that when the variable forgetting factor is incorporated into the Kalman filter, a modification to ensure that the covariance matrix remains bounded, is required to ensure asymptotic convergence of estimated parameters.

The overall algorithm thus reduces to

Estimator Algorithm

$$K_t = P_{t|t-1} H[1 + h^T P_{t|t-1} h]^{-1} \qquad (16a)$$

$$\varepsilon_t = y_t - h^T X_{t|t-1} \qquad (16b)$$

$$X_{t|t} = X_{t|t-1} + K_t \varepsilon_t \qquad (16c)$$

$$N_t = \frac{[1 + h^T P_{t|t-1} h]\, \Sigma o}{\varepsilon^2_t} \qquad (16c)$$

$$\lambda_t = 1 - \frac{1}{N_t} \qquad (16e)$$

$$W_t = [I - K_t h^T P_{t|t-1}] \qquad (16f)$$

$$\text{If } \left|\frac{1}{\lambda_t}\right| \text{trace } [W_t] \leq 10 \text{ E } 10 \qquad (16g)$$

$$\text{then} \quad P_{t|t} = \frac{1}{\lambda_t} W_t \qquad (16h)$$

$$\text{else} \quad P_{t|t} = W_t \qquad (16i)$$

$$\text{End if} \quad P_{t+1|t} = F_t P_{t|t} F_t + gL^2 g^T \qquad (16j)$$

Controller Algorithm

$$C^* = A^* F + z^* G \qquad (16k)$$

$$E = B^* P + QC^* \qquad (16l)$$

$$EU_t = C^* W_t + Cy \qquad (16m)$$

where

Σo, $P_{o|-1}$, $X_{o|-1}$ are supplied as data

The overall closed loop system is shown in Fig.2

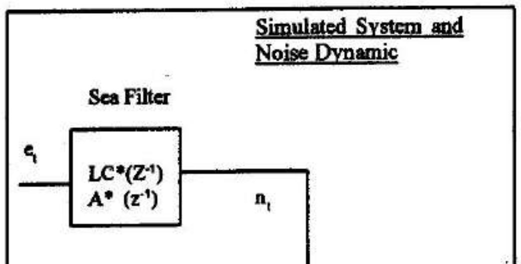


Fig. 2 The closed Loop System of Yacht and Self Tuning Controller

CLOSED LOOP SIMULATION RESULTS

$a_1 = -1.478$, Sea parameter $E = 0.64$

$a_2 = 0.619$

$b_1 = 0.304$, $\Sigma_o = 100$

$b_2 = 0.259$

Two series of tests were conducted on this model. For test No. 1, the controller was set to optimise the cost function

$$J_1 = E\{(y_{t+1} - W_t)^2 + q_0(U_t - U^2_{t-1})\}$$

This corresponds to the weighting polynomials

$$P = R = 1 \ , \ Q = q_0$$

For the test No.2 the controller was set to optimize the cost function

$$J_2 = E\{(y_{t+1} - W_t)^2 + q_0(U_t)^2\}$$

This correspondence to the weighing polynomials

$$P = R = 1 \ , \ Q = q_0(1 - z^{-1})$$

The effect of this cost function is to penalize rate of change of rudder angle. This gives smoother control, thus minimizing work done by the rudder servo and reducing drag.

Table 1 shows the effect of variation of q_o on rudder angle and yaw angle for test run No.1, while Table 2 shows the corresponding results for test run No. 2.

Table 1: Simulation Results, Test Run No. 1 ($P = R = 1$, $Q = q_o$)

q_o	S_u	S_y
0.0	15.4	10.1
0.25	6.6	10.6

S^2_u = rudder angle variance

S^2_y = yaw angle variance

Table 2: Simulation Results, Test Run No. 2 ($P = R = 1$, $Q = q_o(1-z^{-1})$)

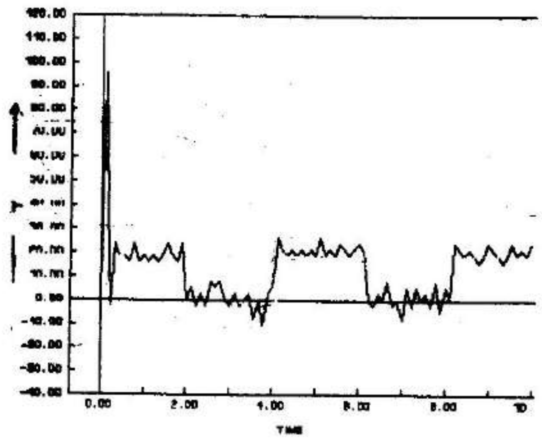
q_o	S_u	S_y
0.0	15.4	10.1
0.25	10.61	9.3
0.5	8.1	20.0

Note variation shown in Tables 1 and 2 include variations in set point changes.

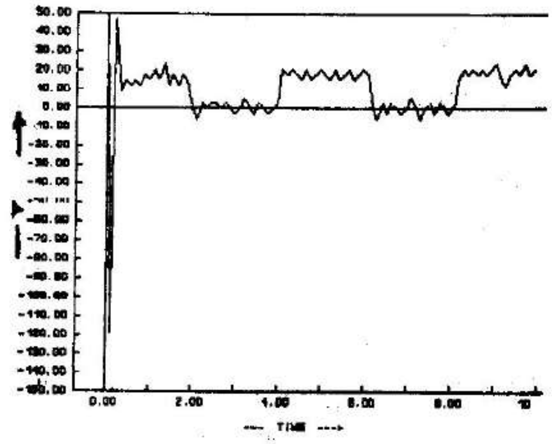
To demonstrate the ability of the autopilot to respond optimally to set point changes W was arranged to switch between 0 and 20 every 200 sample times as shown in figs. 3 and 4.

Comparison of Tables 1 and 2 show that for $q_0 = 0.25$ cost function J_1' gave a much smaller S_u than cost function J_2 for similar S_y. However, it can be readily seen on comparing plots of U_t in the two test runs (figs 3 and 4) that J_2 gives a much smoother control and hence less drag on the yacht.

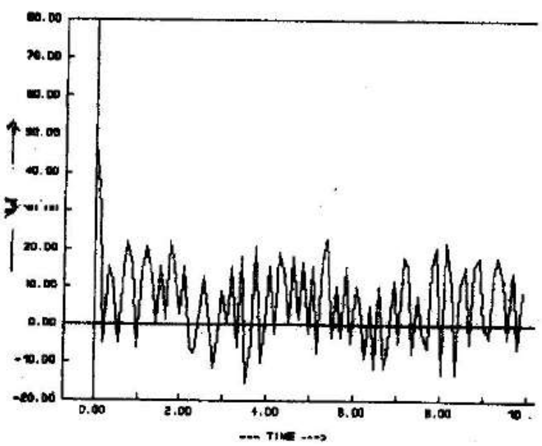
To test the ability of the estimator to adapt to sudden changes in system parameters, two arbitrary systems, models 1 and 2 were simulated. The program was arranged to switch from model 1 to model 2 after a set number of time steps. The parameters of the simulated model were



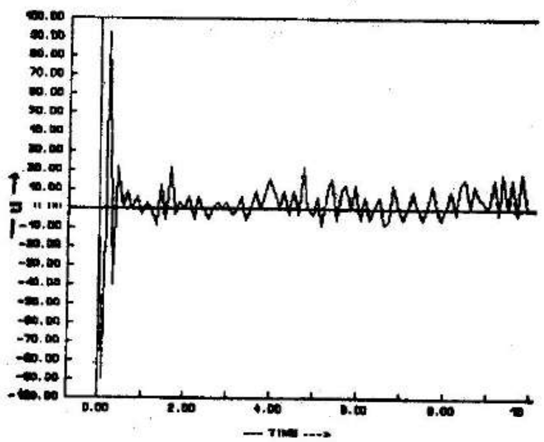
Yaw output showing set point following



Yaw output showing set point following



Clossed loop control effort reflecting effect of weighting control



Control signal showing effect of penalising control rate of change

Fig. 3: Closed loop simulation results with Clarke's S.T.C. Test no. 1 $Q = 0.25$

Fig. 4: Closed loop simulation results with Clarke's S.T.C. Test no. 2 $Q = 0.25(1-z^{-1})$

Model 1.

Continuous parameters $a = 2.0$, $b = 0.8$, $c = 0.5$, choosing sampling interval $\tau = 0.6$, corresponds to discrete model

$$a_1 = -1.48, \quad a_2 = 0.62$$

$$b_1 = 0.30, \quad b_2 = 0.26$$

Model 2

Continuous model $a = 0.5$, $b = 0.7$, $c = 0.2$ descretized model

$$a_1 = -1.35, \quad a_2 = 0.5$$

$$b_1 = 0.27, \quad b_2 = 0.2$$

$$\Sigma_o = 100$$

Sea dynamics parameters were kept constant.

The minimum variance control was used, i.e. minimize

$$J = E(y_{t+1})^2$$

So $P = 1$, $R = 0$, $Q = 0$, and hence no control weighting. This was to ensure a quick recovery of course heading from a sudden change in system parameters. This could simulate a situation when say the main sail is suddenly let loose. Figs. 5 and 6 show the controller adapting rapidly to the sudden jump in system dynamics. Also Fig. 6 shows plots of the estimated parameters.

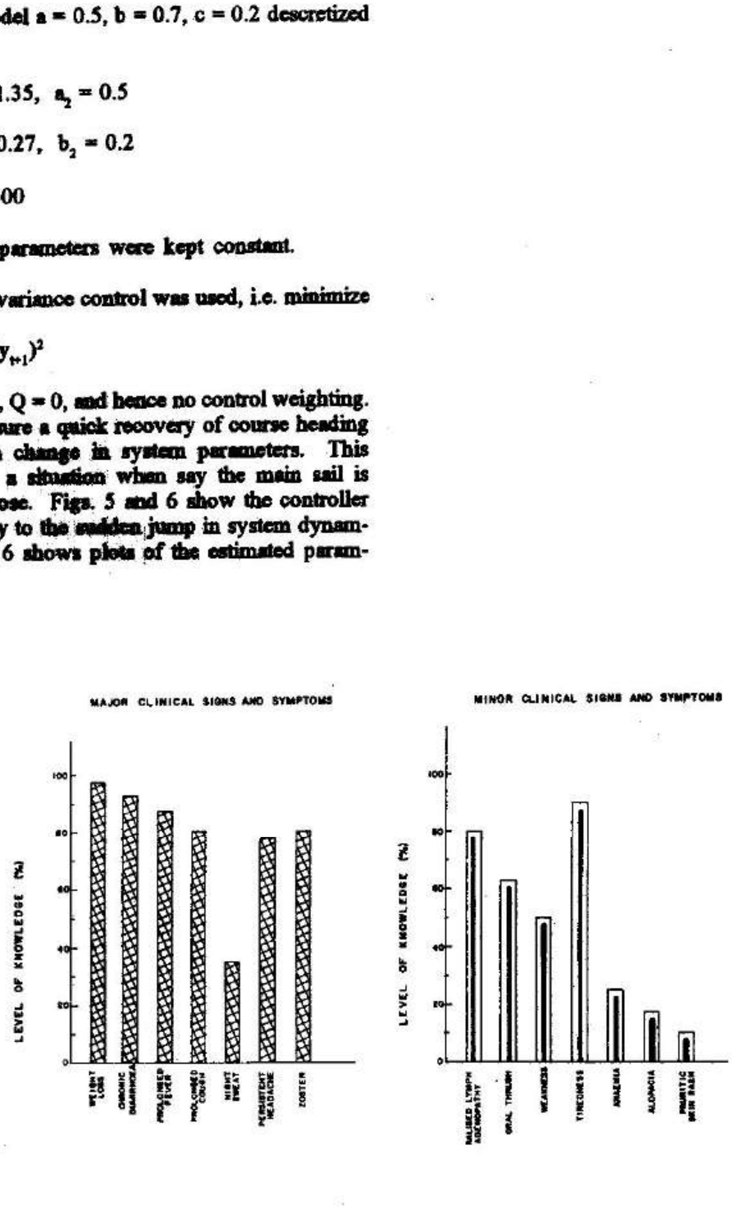
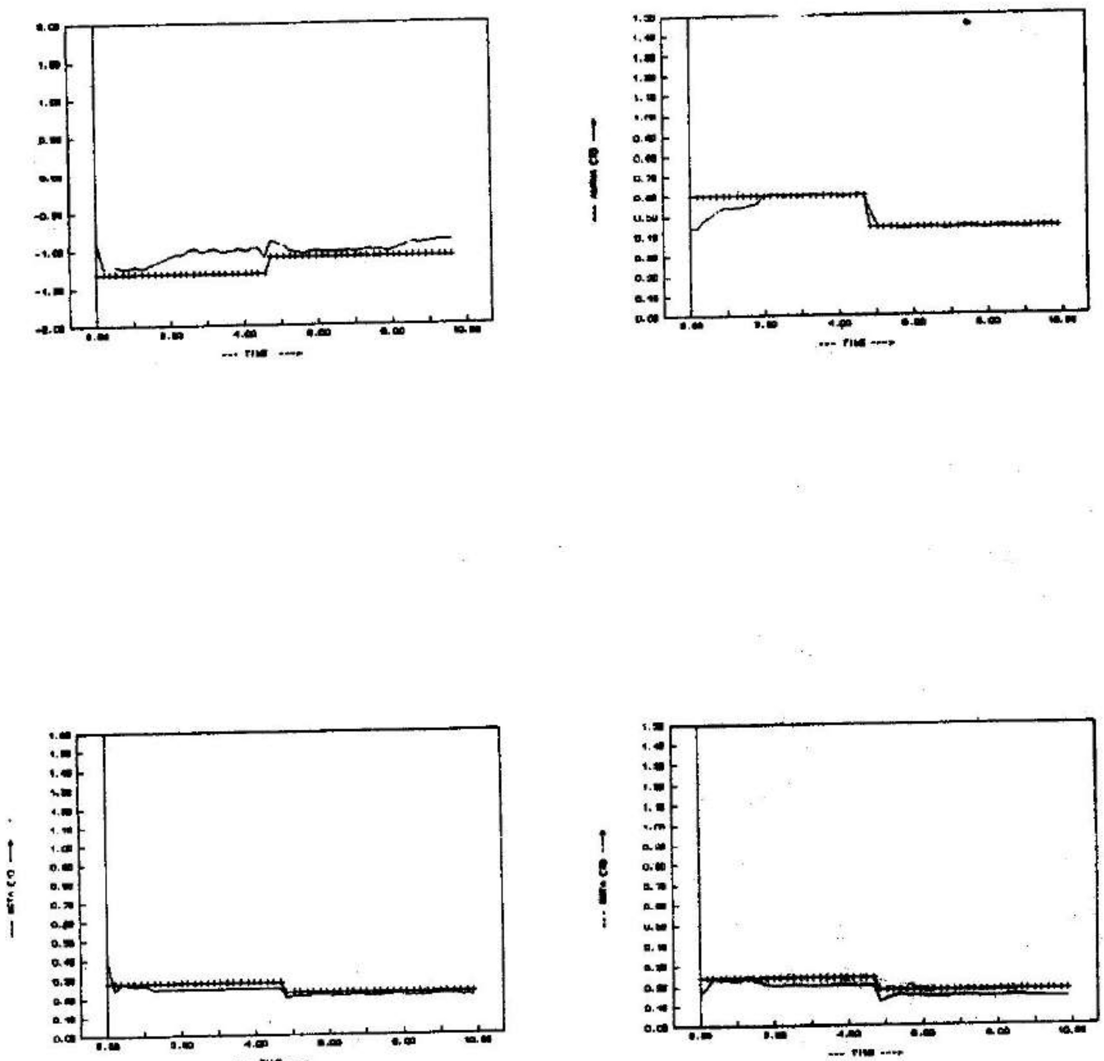


Fig. 5: Fortescue's variable forgetting factor $d_o = 100.0$



System Parameters

Fig. 6: Closed loop simulation results of adaptive controller with Fortescue's variable forgetting factor = 100.0

The third simulation run was to test the ability of the autopilot to adapt to continually varying system parameters. For this investigation the following models was simulated

$$a_1 = -1.48 \text{ Cos } (0.0015t)$$

$$a_2 = 0.62 \text{ Cos } (0.0015t)$$

$$b_1 = 0.30 \text{ Cos } (0.0015t)$$

$$b_2 = 0.26 \text{ Cos } (0.0015t)$$

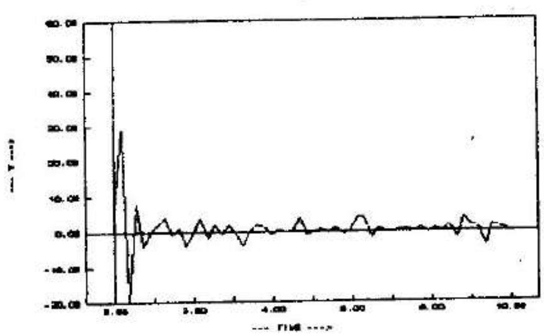
$$\Sigma_o = 100$$

Control parameters were set at $P = 1, \quad R = 0, \quad Q = 0$

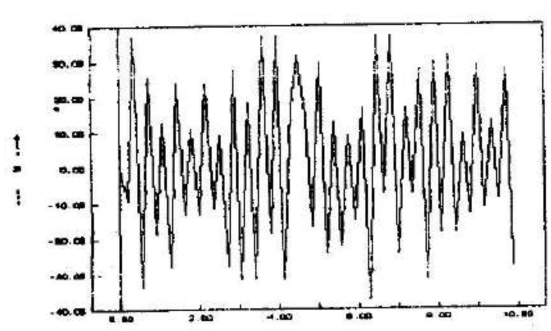
i.e.

$$J = E(y_{t+1}^2)$$

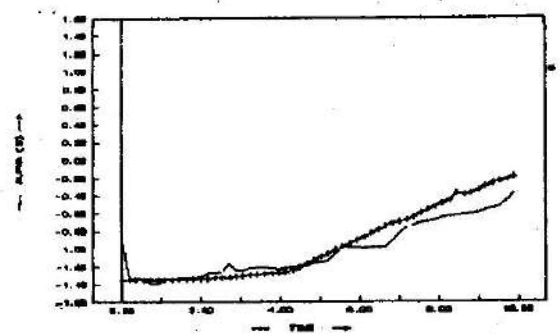
The results from this simulation run are shown in Fig. 7. Also these plots show that the controller is able to adapt to and track gradual changes in system parameters whilst maintaining efficient course-keeping control.



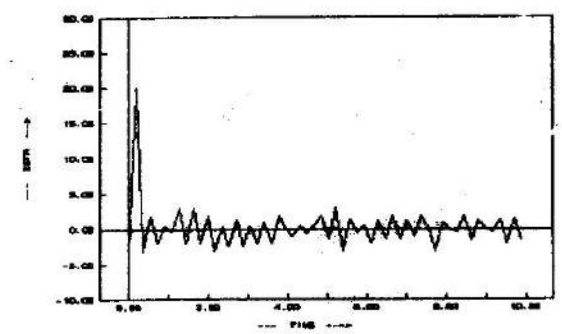
Yaw output



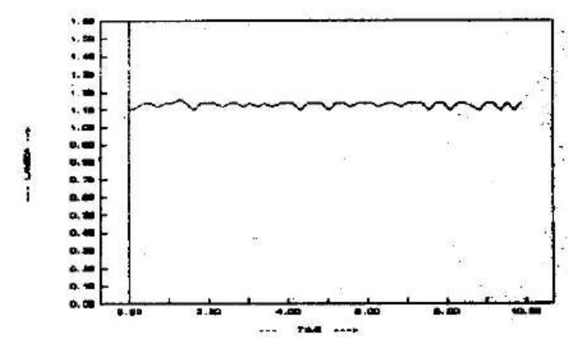
Control signal



+++++++ True values of paramter
——— Estimated value of parameter



Innovations



Fortescue variable forgetting factor Σ_0=100.0

Fig.7 Closed loop performance of adaptive controller for a system with time varying parameters

CONCLUSION

The control of sailing vessels is a challenging task even for skilled sailors. This is mainly due to sudden changes in wind directions, and also for the need to go from tack to tack. There is also, the possibility of letting go jib and main sails to prevent capsizing. This is thus a demonstration that an adaptive control algorithm can be applied successfully to sailing yacht dynamics.

Two important cases were investigated. Firstly, the case of single sudden jumps in system parameters was simulated and successfully controlled. Secondly, the case of continuously varying system parameters was simulated and successfully tracked. In all cases very good course keeping and servo action was observed in the presence of significant sea disturbances.

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