TWO-GROUP MACROSCOPIC CROSS-SECTION DATA BASE FOR THE PROTOTYPE MINIATURE NEUTRON SOURCE REACTOR

E.H.K. Akaho, BSc MSc PhD DIC

S. Anim-Sampong BSc

ABSTRACT

A two-group macroscopic cross-section data base has been generated using a simplified model for the Miniature Neutron Source Reactor (MNSR). The correction terms to the basic macroscopic cross-sections due to varying effects of temperature along the axis of the fuel and burnup were expressed in the form of polynomials. The error analysis shows that the data base is accurate enough to be used for neutronic calculations for the reactor.

Keywords: two-group, macroscopic crossection, homogeneous zones, burnup

INTRODUCTION

In any operating reactor there may be some spatially varying power feedback phenomenon that bring about changes in reactor composition or temperature. These affect reaction rates which lead to incremental changes on few-group constants over the range of expected computation.

In general, a change in any group constant can be caused by changes in burnup, U-238 temperature which in turn causes the Doppler broadening of U-238 neutron absorption resonances, the concentration of Xe-135 and the concentration of boron etc.

Core thermo-hydraulics calculations of the MNSR [1] revealed significant changes in fuel and coolant temperatures along the flow channel and this is represented graphically in Fig.1. The plot suggests that the changes in coolant temperature and density along the coolantchannel will definitely affect macroscopic cross-sections. An investigation of the variation of neutron spectrum with burnup of-the U-Al fuel element as shown in Fig. 2. reveals that up to 10,000 MWd/tU (1.1%) there

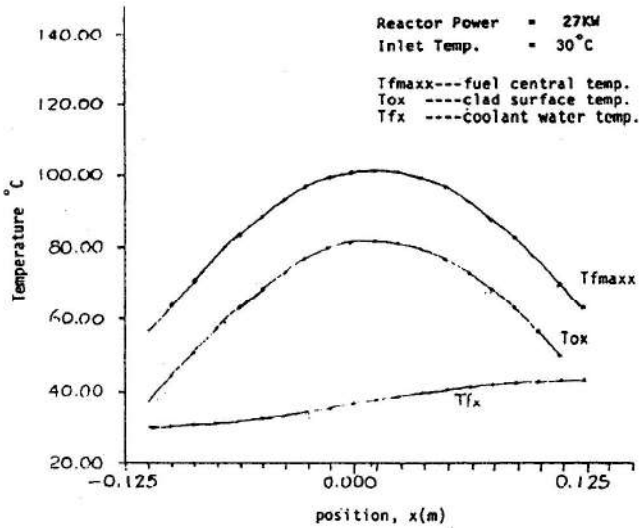


Fig. 1: Temperatures in Fuel, Clad, Coolant along the Flow Channel

is no significant changes. However, the burnup curve for the fuel as illustrated in Fig.3. clearly shows that the infinite multiplication factor k is strongly dependent on burnup. The k can be computed using the correlation equation fitted to our data:

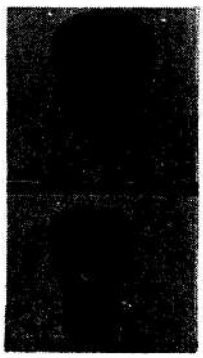
$$k_{\infty}(\tau) = 1.8019 - 2.4114E\text{-}06\ 1 \quad - 7.2635E\text{-}11\tau^2 \quad (1)$$

where τ is burnup (MWd/tU).

Based on these observations it has been decided to examine the effects of these parameters on group constants which will lead to creation of a two-group data base for the prototype MNSR. This data will be helpful in the analysis of the core. In particular, reactor physics calculations must use the reactor's data base to calculate

Dr. E.H.K. Akaho
Dept. of Nuclear Engineering
National Nuclear Research Institute
(GAEC), Accra, Ghana

S. Anim-Sampong
Dept. of Nuclear Engineering,
National Nuclear Research Institute
(GAEC), Accra



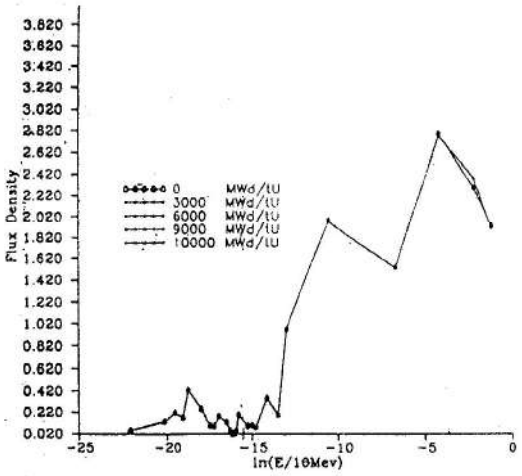


Fig. 2: Variation of Neutrum Spectrom with Burnup in MNSR Fuel Element

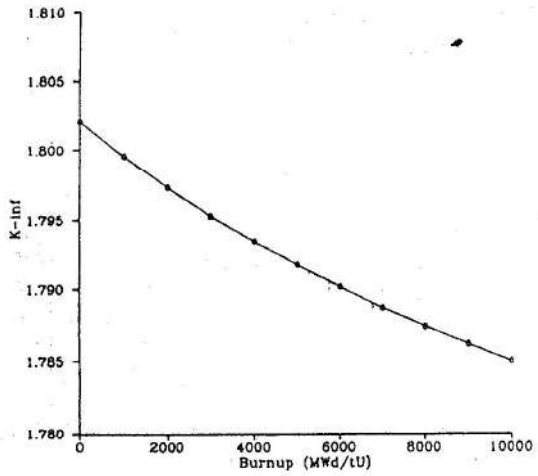


Fig. 3: Burup Curve for MNSR Fuel Element

the fast and thermal fluxes at any point within the reactor.

In this presentation, a brief description of the core of the prototype MNSR and the model for the preparation of the data base are presented. The generation of group constants is a complex process and is performed with sophisticated transport codes which takes into consideration the influence of neutron spectrum material composition and geometry. The code WIMSPC [2], a PC version of the lattice code WIMS [3] was used to generate the two-group constants (D_1, D_2, Σ_{a1}, Σ_{a2}, $\nu_1\Sigma_{f1}$, $\nu_2\Sigma_{f2}$ and $\Sigma_{s1\to 2}$). Cross-sections referred to as basic were calculated for the various homogeneous zones of the reactor at a reference level and their dependance on burnup was correlated. Cross-section terms to these basic cross-sections were then introduced to account for the spatially varying thermo-hydraulic effect and operating power of the reactor. Thermal macroscopic cross-section which is likely to be affected by the presence of xenon-135 was also corrected.

DESCRIPTION OF PROTOTYPE MNSR

The Miniature Neutron Source Reactor (MNSR) is a small, lightwater moderated tank-in-pool type reactor developed by China Institute of Atomic Energy. Cooling is provided by natural convection. The reactor core comprises of a fuel cage with fuel elements in triangular arrangement with average pitch 10.9 mm. The configuration of the prototype MNSR is shown in Fig.4 and the list of lattice number in each row is provided in Table 1. The fuel cage is enclosed by an annular Be reflector. It has an outer diameter of 435 mm, inner diameter of 231mm and of height 238.5mm. Fig.5 shows the MNSR fuel element. The content of uranium in U-Al is 26.1 wt% of enrichment 90.12% and the porosity is 3%.

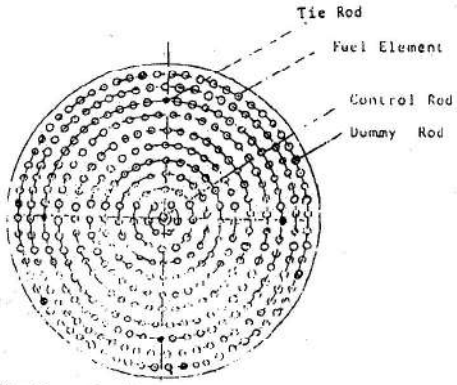


Fig.4: The Core Configuration of MNSR

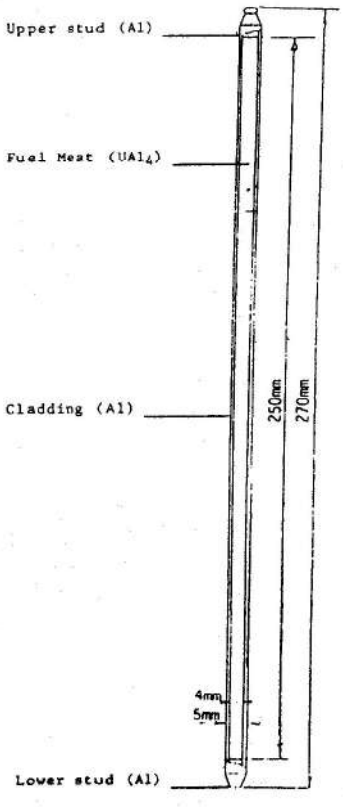


Fig. 5: NHSR Fuel Element

Table 1: Positions of Rings with Number of Roads

Ring No.	Diameter (mm)	No. of Elements	Radial Pitch (mm)
A	0	1	0
B	21.9	6	11.47
C	43.8	12	11.47
D	65.7	19	10.86
E	87.7	26	10.98
F	109.5	32	10.78
G	131.4	39	10.58
H	153.5	45	10.70
I	175.2	52	10.58
J	197.1	58	10.68
K	219.0	65	10.58

Compositions of materials used to construct the core and the operating conditions of the coolant density, temperature were used for the creation of the data base. Before the methodology for preparation of data base is discussed the mathematical model used for the derivation of the group constants is presented.

MODEL

Macroscopic cross-sections for any reactor depend upon neutronic and thermo-hydraulic parameters. The group constants for a lattice cell of the reactor core can be represented in terms of certain physical parameters:

$$\Sigma_1^{xg} = \Sigma_1^{xg}\left(\tau,\ \rho_z,\ T_{fz},\ T_{clz},\ T_{mz},\ P,\ Xe\right) \qquad (2)$$

where

Σ_1^{xg} = group constant of type x at energy g for zone 1
τ = burn-up
z = axial position
ρ_z = moderator/coolant density
P = reactor power
T_{fz} = fuel temperature at the axial position z
T_{clz} = cladding temperature at the axial position z
T_{mz} = moderator/coolant temperature at axial position z
Xe = xenon-135 concentration

If we consider a lattice cell as shown in Fig.6 ;then the group constants can be classified into two parts:

(i) the basic cross-section determined at a selected reference level at nominal reactor power (27 kW); for this work the reference level is the middle of the core.

(ii) the correction terms to the basic cross-sections due to axial variations of temperature along the flow channel, operating power and xenon effect.

Basic macroscopic cross-section, (BMC) were determined for a fresh core at nominal power Pø. It is also assumed that the group constants for the homogeneous zones of the reactor core at the reference level are functions of burn-up only. These cross-section will change from the reference level as the fuel and moderator temperatures change along the axis. When precalculating the cross-sections it is first assumed that no xenon-135 is present and it is only the thermal absorption cross-section that could be affected. A quantitative relationship between change in group constants as the power density varies is not easily developed. Such a calculation of the relationship needs analysis of series of neutron spectrum and evaluation of fuel-to-coolant heat transfer.

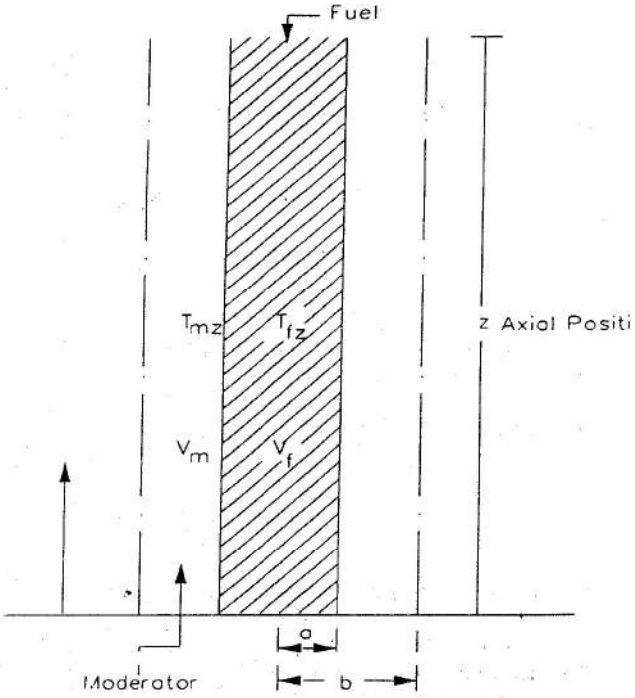


Fig. 6: Lattice Cell for Macroscopic Crossections

We have assumed that the fuel, cladding and coolant temperatures will vary linearly with reactor power as stated by Mele et al [4].

On the basis of these assumptions Eq. (2) is written for any power different from the nominal power for the cross-sections:

$$\Sigma_1^{xg}\left(\tau,\ z,\ p\right) = \left(\Sigma_{r,1}^{xg}(\tau) + \Delta\Sigma_1^{xg}(z)_1\right)\left(\frac{p}{P\phi}\right) \qquad (3)$$

where

$\Sigma_{r,1}^{xg}(\tau)$ = cross-section at reference level which is burnup dependant

$\Delta\Sigma_1^{xg}(z)$ = axial incremental change caused by changes in fuel temperature, T_{fz} cladding temperature, T_{clz}, moderator temperature T_{mz} and coolant density, z from reference level conditions

P = operating reactor power

$P\phi$ = nominal power (27kW)

The group constant Σ_1^{a2} determined using Eq. (3) is corrected for xenon-135 effect:

$$\Sigma_1^{a2}\left[\tau, z, p\right] = \left[\Sigma_{r,1}^{a2}(\tau) + \Delta\Sigma_1^{a2}(z) \right] \left(\frac{P}{P_\phi} \right) + \Delta\Sigma_1^{a2}(Xe) \quad (4)$$

where $\Delta\Sigma_1^{a2}(Xe)$ -= incremental change due to xenon-135 concentration

The methods used for the basic macroscopic cross-sections and the correction terms needed for the evaluation of Eqs. (3) and (4) are presented in the next section.

Basic Macroscopic Cross-sections

Macroscopic cross-section $\Sigma_{r,1}^{a2}(\tau)$ as stated earlier is to be determined using reference conditions. Assuming that the burnup can be separated from the variables and its influence described by function $b(\tau)$ then can be expressed in the form

$$\Sigma_1^{xg}(\tau) = \Sigma_{r,1,0}\left(1 + b^{xg}(\tau) \right) \quad (5)$$

where $\Sigma_{r,1,0}^{xg}$ is the group constant at the reference level for any zone, 1 for zero burn-up.

The axial effect $\Delta\Sigma_1^{xg}(z)$ in Eq. (3) is expressed as a function of z. Thus the parametrized cross-section for Eq. (3) can be written in the form.

$$\left[(z)_{\delta x}{}^{I_w} + \left((\lambda)_{\delta x}{}^{q} + \tau \right)^{0'I'\lambda}_{\delta x^{\Sigma}} \right] = \left(d\,'z\,'\lambda \right)_{\delta x^{\Sigma}}{}^{I_{\Sigma}} \quad (6)$$

The functions $b^{xg}(\tau)$ and $a_1^{xg}(z)$ are considered as polynomials of the second order:

$$b^{xg}(\tau) = \sum_{n=1}^{2} b_n^{xg}\,\tau^n = b_1^{xg}\,\tau + b_2^{xg}\,\tau^2 \quad (7)$$

$$a_1^{xg}(z) = \sum_{n=0}^{2} a_{1,n}^{xg}\,z^n = a_{1,0}^{xg} + a_{1,1}^{xg}\,z + a_{1,2}^{xg}\,z^2 \quad (8)$$

Correction for Xenon Effect

The basic cross-section for calculating the thermal macroscopic absorption cross-section is affected by the presence of xenon. This effect is taken into account by first calculating the correction term $\Delta\Sigma_1^{xg}(Xe)$ in Eq.(4). The term is calculated using the relationship [4]

$$\Delta\Sigma_1^{xg}(Xe) = \Sigma_{1,\phi}^{a2}\left(\frac{1 - \frac{P}{P_\phi}}{1 + (P/P_\phi)(\sigma_{Xe}^{a2}\phi_{2\phi}/\lambda_{Xe})} \right) \quad (9)$$

where $\Sigma_{1,\phi}^{a2}$ is thermal macroscopic cross-section corresponding to the buildup of the equilibrium poisoning which can be expressed as

$$\Sigma_{1,\phi}^{a2} = \frac{\chi Y_{Xe}(P/P_\phi)}{1 + \lambda_{Xe}/(\sigma_{Xe}^{a2}\phi_2)} \quad (10)$$

where

χ = fission/watt-sec (3.1×10^{10})

Y_{Xe} =total chain yield of xenon-135 (0.066)

P = power density at operating power (kW/cm^3)

P_ϕ = nominal power density fuel (kW/cm^3)

λ^{Xe}= xenon-135 decay constant ($2.1\times10^{-5}s^{-1}$)

σ_{Xe}^{a2}= thermal absorption microscopic cross-section for xenon-135 (1.7×10^{6} b)

A brief description is presented on the method of obtaining group constants from the parametrised equation $b^{xg}(\tau)$ and $a_1(z)$ for variation in burnup and axial effects respectively. A brief account is first given on the method of preparation of the input data using the lattice code WIMS.

METHOD OF ANALYSIS

Transport Calculations for Group Constants

Effective group constants which are needed for diffusion calculations were computed using the WIMSPC the PC version of WIMS This version is adapted to 286/386 AT PC from WIMSD/4 [6]. The code provides solution to the 1-D multi-group transport equation. We have restricted the present analysis to two-group only, thus only the cross-sections D_1, D_2, $\nu_1\Sigma_{f1}$, $\nu_2\Sigma_{f2}$ and $\Sigma_{s1\text{-}2}$ were computed for the various homogeneous zones of the core.

Details of the physics and the models are available in reference [3]. The 69-group structure of WIMS library consist of 14 fast groups (10 MeV-9.118 keV), 13 resonance groups between the range 9.118 keV - 4 eV and 42 thermal groups (4 eV- 0 eV). These group divisions are sufficient to analyse a wide range of reactors. However, the 28-group structure recommended by Fayers et al [7] for light-water reactors was selected because core calculations for the MNSR using this group structure yielded control rod worth of 7.1 mk [8] which

compared favourably with the measured value of 6.8 mk [9].

The types of unit-cells that are involved in the analysis are

(a) fuel-element + water
(b) Al dummy/tie rod + water
(c) control rod guide tube + water
(d) Be reflectors

Instructions for the preparation of standard input for WIMSD/4 was followed. Material composition for the unit cells of the core listed above and physical properties as density and temperature form part of the WIMS input data.

Material card requires density and temperature of fuel, cladding material and coolant/moderator at the position of interest (reference level or axial position z). This is followed by a weight fractions of the various elements consisting the material. The conditions prevailing at the middle of the core as shown in Fig.1 (ρ_0 = 0.99329 g/cc, T_{f0} = 374 ^oK, T_{cl0} = 355.2 ^oK, and T_{m0} = 309.77 ^oK) were used to obtain the reference level macroscopic cross-section $\Sigma^{xg}_{r,1,o}$ for zero burn-up condition.

The operating conditions corresponding to the position z from the reference point (ρ_z, T_{fz}, T_{clz} and T_{mz}) were also used to obtain the group constants. The densities of the coolant/moderator were computed as a function of coolant temperature at the position z using the correlating equation.

$$\rho_z(T_{mz}) = \frac{\sum_i \alpha_i T_{mz}^i}{\sum_j \beta_j T_{mz}^j} \qquad (11)$$

The coefficients are listed in ref [1]. The composition of the various materials of the reactor core such as SS 304 for cladding of control rod, Al for dummy/tie rod and fuel cladding and for the Be reflector are listed in Tables of Appendix A. The content of U-235 in grams in a fuel rod is calculated from the equation

$$g_{U\text{-}235} = \rho_U v(1 - e\%) X f_m \qquad (12)$$

X is the weight of uranium, ρ_U is the density of the fuel, e is the porosity and f_m is the weight enrichment of uranium in the U-Al alloy. The volume of the fuel is

$$V = (\pi d_o^2/4)h \qquad (13)$$

where h is the active height of the fuel rod.

$$f_m = \frac{m_{U\text{-}235}\,\varepsilon}{m_{U\text{-}235}\,\varepsilon + m_{U\text{-}238}\,(1 - \varepsilon)} \qquad (14)$$

where ε is the enrichment. The density of the fuel is ρ_U and is calculated using the expression [8]

$$\rho_U = \frac{3.3596(1 - e\%)}{1.2443 - X} \qquad (15)$$

In the determination of the polynomial for $b^{xg}(\tau)$, burn-up values were determined using the POWERC card in the WIMS input data. The card is specified by specific power, P_s and the residence time t_r. The specific power or sometimes called power rating is defined as the thermal power produced per unit of fuel loading:

$$P_s = \frac{10^3 P}{n_f g_U} \qquad (16)$$

where

P = reactor power (kW)
n_f = total number of fuel elements
g_U = fuel loading (U-235 + U-238) (g)

The burn-up of the core, B MWd/tU is needed for calculating the residence time tr. According to neutronic design calculations of the core is 1% [9]. From burn-up physics calculations

$$1\ \text{MWd/tU} \equiv 1\ \text{g U-235} \qquad (17)$$

thus, it can be estimated that the maximum burn-up of the MNSR fuel element. Using Eqs. (12 -15) we calculated an amount of 2.5 g of U-235 out of the total weight of 2.75 g which is equivalent to burn-up of 9090.9 MWd/tU corresponding to the maximum value of 1% for the core to be replaced with fresh fuel. The core was then depleted from 0 to 9090.9 MWd/tU in n_s steps. The residence time of fuel is determined using the relationship

$$t_r = \frac{B/n_s}{P_s} \text{ days} \qquad (18)$$

where

B = specific burn-up (MWd/tU)
n_s = number of burn-up steps

With the various specifications for the cards of the WIMS input, macroscopic cross-sections were obtained from which the parametrized equations are derived.

Parametrized Polynomials

In order to evaluate Eq.(6) the parametrized polynomials $a_l^{xg}(z)$ for axial effect and $b^{xg}(\tau)$ for burn-up variations at reference condition must be determined. For the nominal power of 27 kW, the conditions of position z were used to obtain the group constants. The difference between values at reference level and position z gives

$$\Delta\Sigma_l^{xg}(z) = \Sigma_{l,z,0}^{xg} - \Sigma_{l,r,0}^{xg} \qquad (19)$$

where

$\Sigma_{l,xz,0}$ = cross-section at position z for zero burn-up.

The computed values were fitted using DEM4 curve fitting program to obtain coefficients $a_{l,0}$, $a_{l,1}$ and $a_{l,2}$ of Eq.(8).

For burn-up variation, the polynomial $b^{xg}(\tau)$ was correlated using data within the range 0 - 10,000 MWd/tU of comprising 10 burnup steps. The percentage burn-up is within the range 0 - 1.1% The two-group constants were also fitted with DEM4 for the coefficients b_1 and b_2 for evaluation of Eq. (7).

RESULTS AND DISCUSSIONS

The results of the evaluation of the various equations of the model are used to create a library which can be used for solving either 2-D, two-group transport or diffusion equation in r-z dimension. The basic macroscopic cross-section for the various zones at reference position and zero burn-up $\Sigma_{r,1,0}$ and coefficients b_1^{xg} and b_2^{xg} are contained in a file BMC.LIB. The data for the coefficients $a_{i,0}^{xg}$, $a_{i,1}^{xg}$ and $a_{i,2}$ accounting for axial effect are contained in a file AXIS.INP.

The computed values of macroscopic cross-sections under conditions prevailing at nominal power for different burnup values at fixed positions along the flow channel are plotted in Figs.7-13. The variation of the various group constants with position of channel are represented graphically in Figs. 14-20. The correction term due to xenon-135 computed using the operating characteristics for the MNSR in Eq.(9) was found to be very small in comparison with axial and burnup effect on the value of the thermal macroscopic absorption cross-section.

In order to check the accuracy of the polynomial representation of the group constants calculated by the present method, the corrected forms of the basic macroscopic cross-sections $\sum^{xg}$(cal) were compared with the accurate values obtained using the WIMSPC code and designated Σ_i^{xg}(acc). The accuracy is indicated by the relative error:

$$\text{error} = \frac{\Sigma_i^{xg}(\text{cal}) - \Sigma_i^{xg}(\text{acc})}{\Sigma_i^{xg}(\text{acc})} \qquad (20)$$

Within a wide range of burnup, operating power at different positions along the flow channel the value of the maximum errors are listed in Table 2. The maximum errors were found to be below 1% .

CONCLUSIONS

. The two-group data base of the MNSR has been created using a simplified model. The error analysis has shown that the data base is accurate enough. The data could be used to solve a 2-dimensional (r,z) transport or diffusion code for fast and thermal fluxes produced at any point in reactor. The predicted values are to be compared with those obtained from experiments as a further confirmation of the present analysis.

Table 2: Maximum Error in Parametrized Group Constants.

Group Constant	D_1	D_2	Σ^{a1}	Σ^{a2}	$\nu\Sigma^{f1}$	$\nu\Sigma^{f2}$	Σ_{s1-2}
Maximum Error(%)	0.022	0.141	0.073	0.032	0.207	0.01	0.013

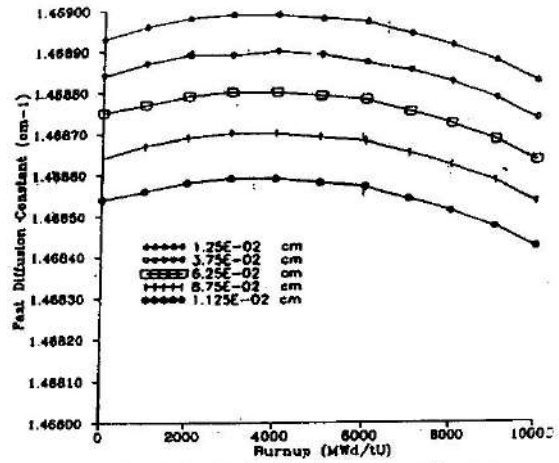


Fig. 7: Fast Diffusion Constant vrs. Burnup

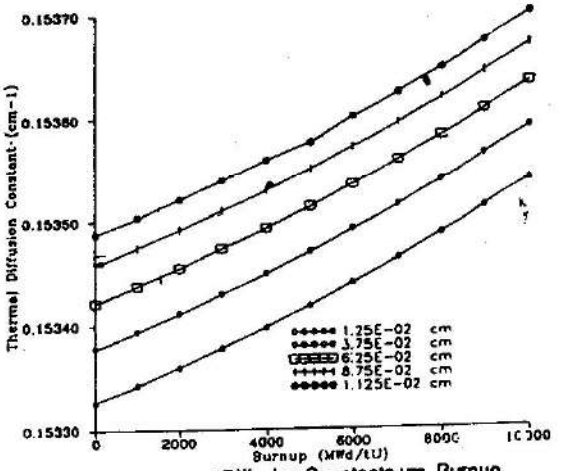


Fig. 8: Thermal Diffusion Constants vrs. Burnup

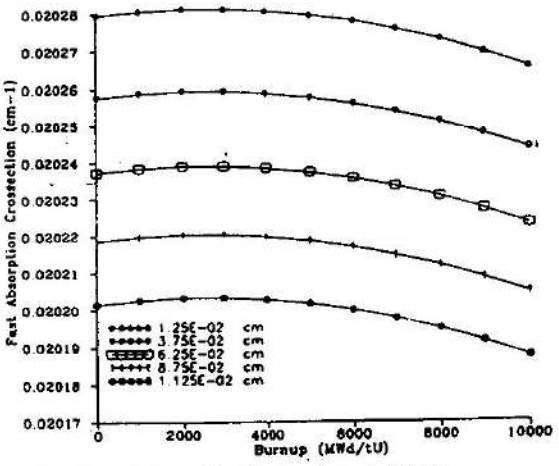


Fig. 9: Fast Absorption Crossection vrs. Burnup

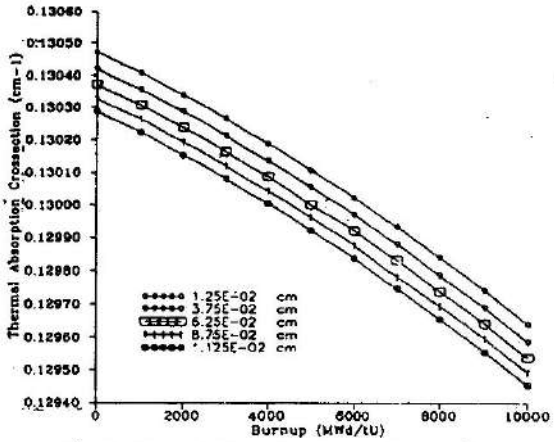


Fig.10: Thermal Absorption Crossection vrs. Burnup

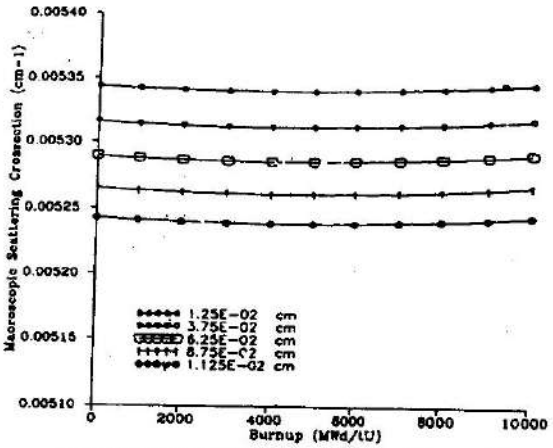


Fig. 13: Macroscopic Scattering Crossection vrs. Burnup

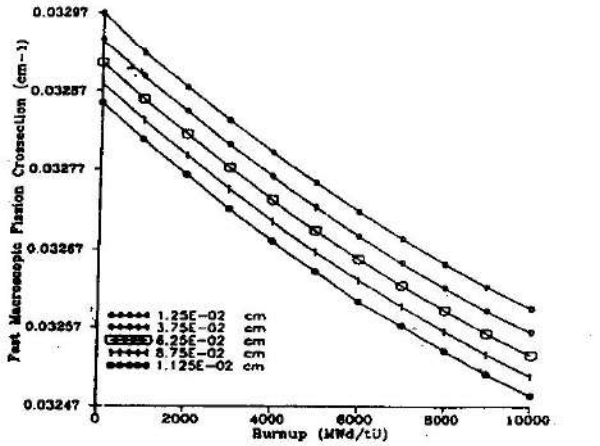


Fig. 11: Fast macrocospic Fission Crossection vrs. Burnup

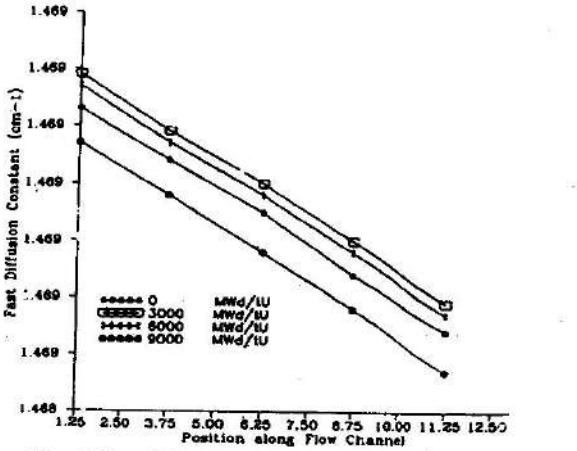


Fig. 14: Fast Diffusion Constant vrs. Position of Flow Channel

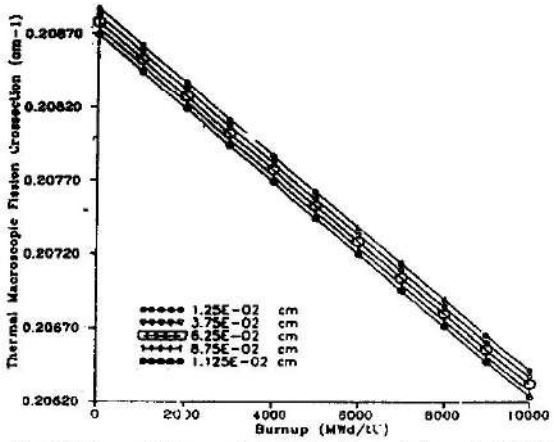


Fig. 12: Thermal Macroscopic Fission Crossection vrs. Burnup

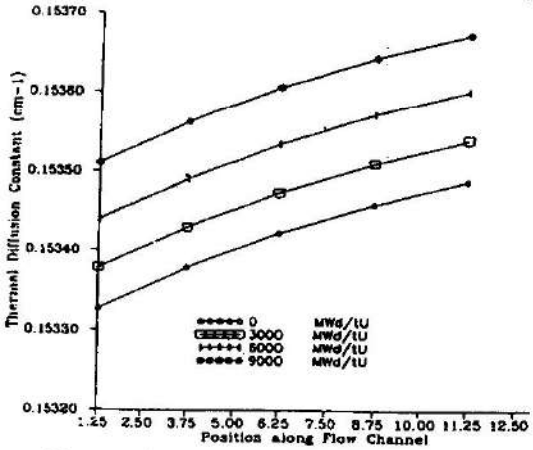


Fig. 15: Thermal Diffusion Constant vrs. Position of Flow Channel

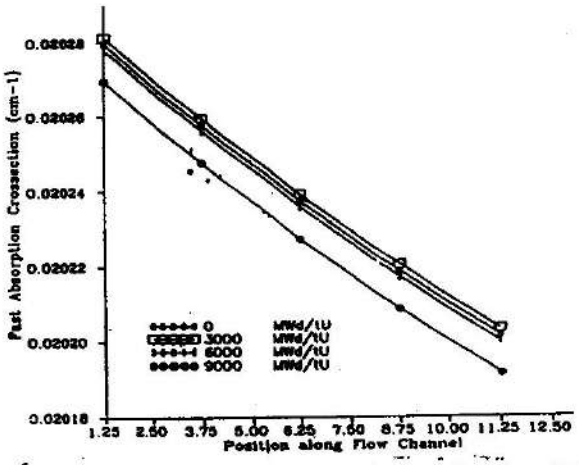


Fig. 16: Fast Absosption Crossection vrs. Position of Flow Channel

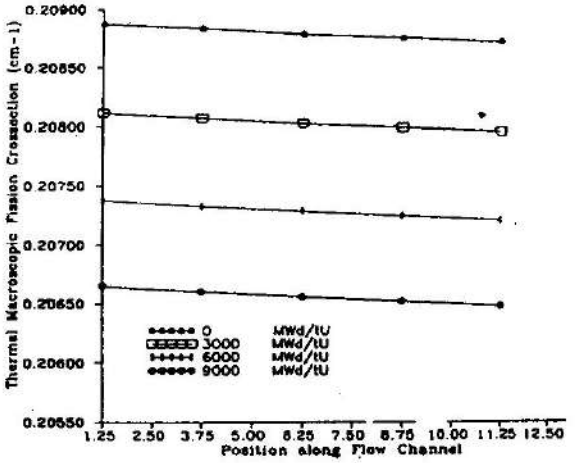


Fig. 19: Thermal Macroscropic Fission Crossection vrs. Position of Flow channel.

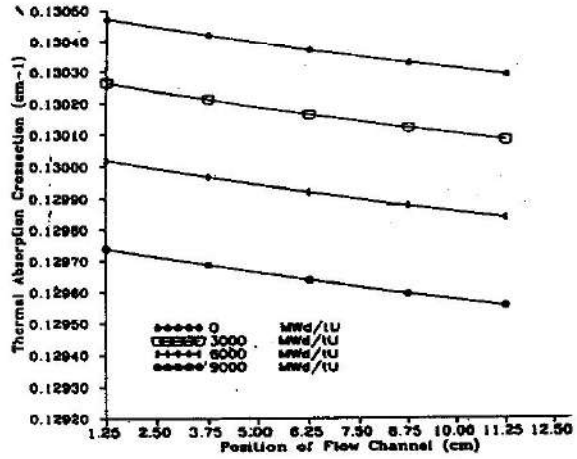


Fig. 17: Thermal Absorption Crossection vrs. Position of Flow Channel

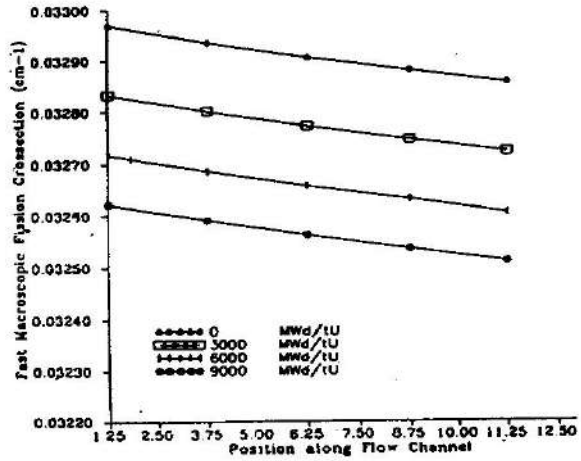


Fig. 18: Fast Macroscopic Fission Crossection vrs. Position of Flow channel

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