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DECOMPOSITION OF INTERCONNECTED POWER SYSTEMS FOR DYNAMIC STUDIES

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ABSTRACT

The paper presents a method for decomposing interconnected power systems into subsystems. Each subsystem is associated with mechanical modes which represent intermachine oscillations within it. The resulting decomposition is shown to give correct grouping of generating units for the development of dynamic equivalents for dynamic studies. The capability of the method is illustrated by a 10-machine 39-bus power system.

Key words

Decomposition, interconnected power system, aggregation, dynamic studies, coherency, modal technique, modes, dynamic equivalent.

INTRODUCTION

With the increasing interconnection of power plants in modern large electric power systems, power system dynamic studies by routines with system components represented by comprehensive models become prohibitive in terms of computation time and storage [1,2]. To achieve reduction in computer storage requirements and computation times, a subsystem under study is represented in detail and external subsystems by dynamic equivalents [2-5]. A disadvantage in this approach is that dynamic equivalents developed for subsystems obtained by artificial partitioning of a power system can produce excessive conservative results [1]. Methods for decomposing power systems into subsystems must take into account the dynamic interaction among the system components. Decomposition schemes used to achieve this partitioning are said to be natural [1,6].

Proposed in this paper, is one such natural decomposition scheme. The model used for the scheme is the linearized electromechanical model of power system which has been found appropriate for the decomposition of power system into "coherent areas" [6,7]. The initial system is assumed to be decomposable into subsystems to each of which can be assigned mechanical modes which are not excited when the rotor speed or angle of any machine not contained in the subsystem is subject to small and gradual change. Using the notion of aggregation to analyse the electromechanical model, an algorithm which leads to such a decomposition is developed. A certain number of corollaries are also shown to result if the assumption is true. These corollaries allow us to verify quantitatively the correctness of a decomposition achieved when the method is applied to a system which will in general not be known to be decomposable a priori. It is also shown that dynamic equivalents can be effectively developed for these subsystems by using the modal reduction or coherency-based reduction.

SYSTEM MODEL

The electromechanical model of n-machine system is

$$p\delta_i = \omega_o(\omega_i - 1) \tag{1}$$

$$2H_i p\omega_i + D_i(\omega_i - 1) = P_{mi} - P_{ei} \tag{2}$$

$$i = 1,2,\ldots,n \tag{3}$$

where P_{mi} is the mechanical input power and P_{ei} the electrical output power. In this model the mechanical input power P_{mi} is assumed to be constant and the electrical output power is

$$P_{ei} = E_i^2 g_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^{n} E_i E_j b_{ij} \sin(\delta_i - \delta_j) \tag{4}$$

The constants b and g are the imaginary and real parts of the elements of **Y'**. The off-diagonal conductive terms of **Y'** are neglected and loads are represented by passive admittances. After linearizing the system equations about its operating point and neglecting the small damping coefficients D_i which have insignificant effect on the oscillatory modes of the system equations [2,6], the model used in the

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paper is obtained in the form

$$p^2 x = A x \tag{5}$$

where
$$A = -\frac{\omega_o}{2} H^{-1} K$$

and

$$x = [\Delta\delta_1, \Delta\delta_2 ..., \Delta\delta_n]^T$$

$$H = diag(H_1, H_2, ..., H_n)$$

$$K = (k_{ij})$$

$$k_{ij} = -E_i E_j b_{ij} \cos(\delta_{oi} - \delta_{oj}) \;\; j \neq i$$

$$k_{ii} = -\sum_{\substack{j=1 \\ j \neq i}}^{n} k_{ij}$$

$$A = (a_{ij})$$

The system matrix A has the following properties which have some bearing on the decomposition scheme:

(P.1) *The eigenvalues of* A *are distinct.*

proof. The matrix A is similar to the symmetric matrix

$$\bar{A} = -\frac{\omega_o}{2} H^{-1/2} K H^{-1/2} \tag{6}$$

(P.2) One of the eigenvalues is zero and its associated eigenvector is

$$w_1 = [1,1,...1]^T \tag{7}$$

proof. This follows from $\mathbf{A w_1} = 0$ which is due to the fact that

$$\sum_{j=1}^{n} a_{ij} = 0, \quad i = 1,2,...,n \tag{8}$$

(P.3) *All the eigenvalues which are not zero are negative.*

proof. According to the theorem of Gershgorin, for a given λ of A

$$|a_{kk} - \lambda| \leq |a_{k1}| + |a_{k2}| + ...$$
$$+ |a_{k,k-1}| + |a_{k,k+1}| + ... + |a_{kn}|$$

for some integer k ($1 \leq k \leq n$). Noting that the off-diagonal elements of A are all positive and that

$$a_{kk} = -\sum_{\substack{j=1 \\ j \neq k}}^{n} a_{kj}$$

we obtain

$$|a_{kk} + \lambda| \leq |a_{kk}|$$

which implies that $\lambda \leq 0$. And since the eigenvalues are distinct only one can take the value of zero.

(P.4) *There exists an eigenvector matrix* W *of* A *with inverse*

$$V = W^T H \tag{9}$$

which will diagonalize the matrix A.

proof. The symmetric matrix $\bar{A}$ can be diagonalized by a unitary matrix. Let the matrix be Q. Then from (5) and (6), there exists an eigenvector matrix

$$W = H^{-1/2} Q \tag{10}$$

which will diagonalize A. Its inverse according to (10) is

$$V = Q^T H^{1/2} = W^T H \tag{11}$$

(P.5) *Let*

$$W = [w_1 \, w_2 ... w_n] \tag{12}$$

and
$$V = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} \tag{13}$$

where w_1 *is the eigenvector of* A *associated with* $\lambda_1 = 0$. *Then*

$$\sum_{j=1}^{n} v_{ij} = 0 \;\; i = 2,3,..,n \tag{14}$$

proof. This follows from (7) and the fact that

$$\langle w_1, v_i^T \rangle = 0 \quad i = 2,3,...,n \tag{15}$$

The 2n eigenvalues of the system are given by the roots of the n eigenvalues of A. It follows that 2(n-1) eigenvalues of the system are imaginary. These modes represent low-frequency oscillations between machines of the power system. The frequency is of the order of a fraction of a HZ to a few HZ, typically 0.5 to 2 HZ [2,6].

REVIEW OF THE NOTION OF AGGREGATION

The analysis leading to the decomposition scheme proposed in this paper is based mainly on the concept of aggregation. We present therefore a brief review of the concept.

Consider the following system:

$$px = Ax + Bu \ , x(0) = x^0, \ x \in R^n \tag{16}$$

If it exists a reduced system

$$pz = Fz + Gu \quad , z \in R^m \ , m < n \tag{17.a}$$

which is such that

$$z = Cx \quad \forall t \geq 0 \tag{17.b}$$

the system (17.a) is said to be an aggregated model of system (16). The condition (17.b) is achieved if and only if [8]

$$FC = CA \tag{18.a}$$

$$G = CB \tag{18.b}$$

With the model used in this paper represented as in (5) the aggregated model is

$$p^2 z = Fz \tag{19.a}$$

$$z(0) = Cx(0) \ ; \dot{z}(0) = C\dot{x}(0) \tag{19.b}$$

The general form of the aggregation matrix C is

$$C = MC_o \tag{20}$$

where M is a nonsingular matrix. The matrix F retains m eigenvalues of A. If the values of the eigenvalues to be retained are specified then rows of C_o will consist of the transpose of the eigenvectors of A^T associated with them or the appropriate rows of the inverse of the eigenvector matrix of A [9]. The matrix M which is the eigenvector matrix of F is chosen arbitrarily. On the other hand if C is known, F is obtained by postmultiplying each side of (18.a) by the generalized or pseudo inverse of C given by

$$C^+ = PC^T(CPC^T)^{-1} \tag{21}$$

The matrix F for a given C is unique and does not depend on the nonsingular matrix P which is arbitrarily chosen.

With C_o as a row vector given by

$$c_{oi} = \frac{H_i}{\left(\sum_{j=1}^{n} H_j\right)} \quad , i = 1,2,\ldots,n$$

we obtain an aggregated model which retains the zero eigenvalue only. This eigenvalue is said to correspond to aggregate motion of machine angles and speeds of the n machines in the system.

HYPOTHESES FOR DECOMPOSITION AND THEIR CONSEQUENCES

We assume that the power system is decomposable into subsystems and that to every subsystem made up of m (m > 1) machines, we can assign (m - 1) non-zero mechanical modes which are not excited when rotor angles and speeds of machines contained in other subsystems are subject to small and gradual changes. The algorithm we propose seeks each group of mechanical modes and their corresponding set of machines.

Consider without loss of generality the equations of an n-machine system decomposable into two subsystems:

$$S_1 = \{x_1, x_2, \ldots, x_q\} \tag{22.a}$$

$$S_2 = \{x_{q+1}, x_{q+2}, \ldots, x_n\} \tag{22.b}$$

We partition the system equations as follows:

$$p^2\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \begin{matrix} x_1 \in R^q \\ x_2 \in R^r \end{matrix} \tag{23}$$

where

$$q + r = n \tag{24}$$

We suppose that the set of eigenvalues of this system is

$$\Lambda_s = \{\lambda_1, \lambda_2, \ldots, \lambda_n\} \tag{25}$$

where $\lambda_1 = 0$ and assign to subsystems 1 and 2 the intermachine modes

$$\Lambda_1 = \{\lambda_2, \lambda_3, \ldots, \lambda_q\} \tag{26.a}$$

$$\Lambda_2 = \{\lambda_{q+1}, \lambda_{q+2}, \ldots, \lambda_{n-1}\} \tag{26.b}$$

and λ_n as the intersubsytem mode.

Let an aggregated model of the initial system which retains all the modes Λ_1 be given as

$$p^2 z_1 = F_1 z_1 \ , z_1 \in R^{q-1} \tag{27}$$

Its matrix of aggregation has the form

$$C_1 = M_1 \begin{bmatrix} v_2 \\ v_3 \\ \vdots \\ v_q \end{bmatrix} \tag{28.a}$$

$$\begin{array}{cc} x_1 & x_2 \\ \downarrow & \downarrow \end{array}$$
$$= \begin{bmatrix} C_{11} & C_{12} \end{bmatrix} \tag{28.b}$$

where v_i is the transpose of the eigenvector of A^T associated with mode λ_i. Now the following statements can be made with respect to the initial and aggregated models if our hypotheses are true:

(S1) *If the modes Λ_1 are not to be excited when variables in the subsystem 2 are subject to change then C_1 given by (28.b) must have the form*

$$\begin{array}{cc} x_1 & x_2 \\ \downarrow & \downarrow \end{array}$$
$$C_1 = \begin{bmatrix} C_{11} & \vdots & 0 \end{bmatrix} \tag{29}$$

proof. This immediately follows from (19.b).

(S2) *The matrix of aggregation can be expressed also as*

$$C_1 = \begin{array}{c|c} x_1 & x_2 \\ \downarrow & \downarrow \end{array}$$
$$C_1 = \begin{bmatrix} 1 & 0 & 0 \ldots\ldots 0 & -1 & \vdots & \\ 0 & 1 & 0 \ldots\ldots 0 & -1 & \vdots & \\ \cdot & \cdot & \ldots\ldots & \cdot & \vdots & 0 \\ \cdot & \cdot & \ldots\ldots & \cdot & \vdots & \\ \cdot & \cdot & \ldots\ldots & \cdot & \vdots & \\ 0 & 0 & 0 \ldots 01 & -1 & \vdots & \end{bmatrix} \tag{30}$$

proof. The matrix of aggregation C_1 is a full rank matrix and therefore columns can be exchanged so that the first (q-1) columns form a nonsingular matrix. If we choose M_1 to be the inverse of that matrix and note also that the algebraic sum of the elements in each row is zero, the matrix C_1 is seen to take that form.

(S.3) *When the rotor variables in subsystem 2 are disturbed, we shall have*

$$\Delta\delta_i(t) = \Delta\delta_q \;, i = 1,2,\ldots,q-1 \tag{31}$$

proof. Since z_1 is not excited, we shall have according to (17.b) $C_1 x_1 = 0$ for all $t \geq 0$. So with C_1 defined as in (30) the results follow.

The above set of equations indicate that the machines constituting such an identified subsystem form a coherent group. The above coherency criterion has been used to decompose power systems into coherent areas so that effective dynamic equivalents can be developed for the system by coherency-based reduction [7]. In identifying the coherent areas however the author has to obtain a large number of simulated responses of the rotor angles and this tends to defeat the objective of deriving simplified models for portion of a large interconnected power system.

(S.4) *The set of modes Λ_1 are eigenvalues of A_{11}*

proof. From

$$F_1 [C_{11} \vdots\ 0] = [C_{11} \vdots\ 0] \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

we obtain

$$F_1 C_{11} = C_{11} A_{11} \tag{32}$$

$$0 = C_{11} A_{12} \tag{33}$$

Equation (32) indicates that the eigenvalues of F_1 which should be the set of modes Λ_1 are also the eigenvalues of A_{11}.

We note also that rows of C_{11} will consist of the transpose of eigenvectors of A_{11}^T associated with these modes and that (33) is verified when elements on each column of A_{12} are all equal. The elements of this submatrix are given by

$$a_{ij} = -\frac{1}{2}\frac{\omega_o}{H_i} E_i E_j b_{ij} \cos(\delta_{oi} - \delta_{oj}) \tag{34}$$

$$i = 1,2,\ldots,q;\; j = q+1, q+2,\ldots,n \tag{35}$$

Expressions similar to (34) called "acceleration distances" are used for decomposition of power system into coherent groups [4]. The machine i's to be grouped in subsystem 1 are required to have their acceleration distances with respect to machine j in subsystem 2 under study to be as close as possible. The analysis presented in this paper thus provide an alternative analytical basis for that method.

(S.5) From (29) the eigenvector matrix of A^T associated with Λ_1 is in the form

$$\hat{V} = \begin{bmatrix} \hat{V}_1 \\ 0 \end{bmatrix} \begin{array}{l} \leftarrow x_1 \\ \leftarrow x_2 \end{array} \tag{36}$$

Therefore according to (9) we shall have the eigenvectors of $\hat{A}$ corresponding to these eigenvalues in a similar form:

$$\hat{W}=\begin{bmatrix}\hat{W}_1\\0\end{bmatrix}\begin{matrix}-x_1\\-x_2\end{matrix} \tag{37}$$

From the relation

$$\begin{bmatrix}A_{11} & A_{12}\\A_{21} & A_{22}\end{bmatrix}\begin{bmatrix}\hat{W}_1\\0\end{bmatrix}=\begin{bmatrix}\hat{W}_1\\0\end{bmatrix}\Lambda_1$$

we obtain

$$A_{11}\hat{W}_1=\hat{W}_1\Lambda_1 \tag{38}$$

$$A_{21}\hat{W}_1=0 \tag{39}$$

It is proved in the appendix that (39) is also true if (33) is verified. Equation (38) indicates that $\hat{W}_1$ are eigenvectors of A_{11} and confirms also that Λ_1 are eigenvalues of A_{11}.

(S.6) Let the eigenvector matrix $\bar{W}_1$ of A_{11} be partitioned as

$$\bar{W}_1=\begin{bmatrix}\hat{W}_{11} & \hat{W}_{12}\\\hat{W}_{21} & \hat{W}_{22}\end{bmatrix} \tag{40.a}$$

and its inverse as

$$\bar{V}_1=\begin{bmatrix}\hat{V}_{11} & \hat{V}_{12}\\\hat{V}_{21} & \hat{V}_{22}\end{bmatrix} \tag{40.b}$$

where the column and row vectors

$$t_{01}=\begin{bmatrix}\hat{W}_{12}\\\hat{W}_{22}\end{bmatrix} \tag{40.c}$$

$$v_{01}=\begin{bmatrix}\hat{V}_{21} & \hat{V}_{22}\end{bmatrix} \tag{40.d}$$

are those associated with λ_{01} of A_{11} which is not an intermachine mode. Then application of the similarity transformation

$$\begin{bmatrix}z_{11}\\z_{12}\\x_2\end{bmatrix}=\begin{bmatrix}\hat{V}_{11} & \hat{V}_{12} & 0\\\hat{V}_{21} & \hat{V}_{22} & 0\\0 & 0 & I\end{bmatrix}\begin{bmatrix}x_{11}\\x_{12}\\x_2\end{bmatrix} \tag{41.a}$$

to (23) will yield

$$p^2\begin{bmatrix}z_{11}\\z_{12}\\x_2\end{bmatrix}=\begin{bmatrix}A_1 & 0 & \hat{A}_{12}\\0 & \lambda_{01} & \hat{A}_{13}\\\hat{A}_{21} & \hat{A}_{23} & A_{22}\end{bmatrix}\begin{bmatrix}z_{11}\\z_{12}\\x_2\end{bmatrix} \tag{41.b}$$

where

$$x_1=\begin{bmatrix}x_{11}\\x_{12}\end{bmatrix},\ x_{11}\in R^{r-1},\ x_{12}\in R^1$$

$$\hat{A}_{12}=\begin{bmatrix}\hat{V}_{11} & \hat{V}_{12}\end{bmatrix}A_{12}$$

$$\hat{A}_{21}=A_{21}\hat{W}_1$$

$$\hat{A}_{13}=v_{01}A_{12}$$

$$\hat{A}_{23}=A_{21}t_{01}$$

According to (33) and (39) $\hat{A}_{12}=0$ and $\hat{A}_{21}=0$. Therefore (41.b) consists of two decoupled subsystems. The reduced system

$$p^2x_E=F_Ex_E \tag{42}$$

where

$$x_E=\begin{bmatrix}z_{12}\\x_2\end{bmatrix}$$

$$F_E=\begin{bmatrix}\lambda_{01} & \hat{A}_{13}\\\hat{A}_{23} & A_{22}\end{bmatrix}$$

is a minimal representation for dynamic studies of the subsystem 2. The scalar variable z_{12} is the dynamic equivalent of the subsystem 1 obtained by the method of aggregation which is also known in power system literature as modal technique. The aggregated model of subsystem 1 conserves the eigenvalue λ_{01} of A_{11} which is not an intermachine mode. The matrix F_E is obtained by properly coupling the aggregated model with the subsystem 2.

The reduced model can also be seen as the aggregated model of the entire initial system with the matrix of aggregation as given below to preserve the physical variables of the subsystem 2:

$$
\begin{array}{cccc}
 & x_{11} & x_{12} & x_2 \\
 & \downarrow & \downarrow & \downarrow \\
C = & \multicolumn{3}{c}{\begin{bmatrix} \hat{V}_{21} & \hat{V}_{22} & 0 \\ 0 & 0 & I \end{bmatrix}}
\end{array}
$$

The matrix $\mathbf{F_E}$ is then obtained as

$$F_E = CAC^{+} \tag{43}$$

$\mathbf{F_E}$ retains all the eigenvalues of **A** except intermachine modes Λ_1. The above analysis indicates that decomposition based on our assumption lends itself also to equivalencing by the method of aggregation.

(S.7) *The vectors* $\mathbf{t_{01}}$ *and* $\mathbf{v_{01}}$ *are given by*

$$t_{01} = [1, 1, ..., 1]^T \tag{44}$$

$$v_{01,i} = \frac{H_i}{\left(\sum_{j=1}^{q} H_j\right)} \tag{45}$$

proofs. $\mathbf{t_{01}}$ can be thus defined because the algebraic sum of every row of $[\hat{V}_{11}\ \hat{V}_{12}]$ which has a rank of (q-1) is zero. Equation (45) follows from (9) which applies also to the submatrix $\mathbf{A_{11}}$ and the fact that $< \mathbf{t_{01}}, \mathbf{v_{01}}^T >$ must be unity.

The above theoretical expressions for $\mathbf{t_{01}}$ and $\mathbf{v_{01}}$ indicate that λ_{01} like the zero eigenvalue of the complete system corresponds to aggregate motion of machine angles and speeds in subsystem 1. We therefore expect this value to be the most dominant.

ALGORITHMS FOR THE DECOMPOSITION

Consider without loss of generality an ideal 5-machine system decomposable into two subsystems:

$$S_1 = \{1, 2, 3\} \tag{49.a}$$

$$S_2 = \{4, 5\} \tag{49.b}$$

An example of such a system is shown in Fig.1. The machines in each subsystem are supposed to be identical and their operations also identical.

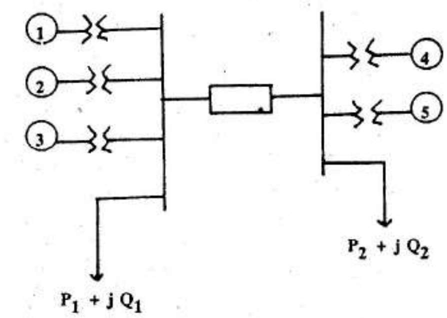

Fig.1 An ideal 5-machine power system

The equation of the system will be given by

$$
p^2 \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} \tag{50}
$$

We suppose the set of eigenvalues of this system to be $\Lambda_s = \{\lambda_1, \lambda_2, ..., \lambda_5\}$ where $\lambda_1 = 0$ and assign to subsystems 1 and 2 intermachine modes $\Lambda_1 = \{\lambda_2, \lambda_3\}$ and $\Lambda_2 = \{\lambda_4\}$. The eigenvalue λ_5 is then the intersubsytem mode.

An aggregated matrix **F** which retains all the intermachine modes Λ_1 and Λ_2 can be obtained using a matrix of aggregation

$$
\begin{array}{cccccc}
 & x_1 & x_2 & x_3 & x_4 & x_5 \\
 & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\
C = & \multicolumn{5}{c}{\begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}}
\end{array} \tag{51}
$$

We can also exchange certain columns of the matrix and obtain it in the form

$$
\begin{array}{cccccc}
 & x_3 & x_5 & x_1 & x_2 & x_4 \\
 & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\
C = & \multicolumn{5}{c}{\begin{bmatrix} -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0. & 0 & 1 \end{bmatrix}}
\end{array} \tag{52.a}
$$

$$\begin{array}{cc} x_r & x_a \\ \downarrow & \downarrow \end{array}$$
$$=[L_g \;\; I] \qquad (52.b)$$

where $\mathbf{x_r} = [x_3 , x_5]^T$ and $\mathbf{x_a} = [x_1, x_2, x_4]^T$.

Given $\mathbf{x_r}$, $\mathbf{x_a}$ and $\mathbf{L_g}$, the subsystems can still be determined. The machines $\mathbf{x_a}$ (ie the machines whose variables are $\mathbf{x_a}$) shall be called "reference machines" and the machines $\mathbf{x_r}$ (ie the machines whose variables are $\mathbf{x_r}$) "associated machines".

The matrix C of (51) for a real system which we shall denote by $\mathbf{C_R}$ will have to be constructed from the eigenvectors of $\mathbf{A^T}$ associated with the intermachine modes. Let the matrix whose rows are these eigenvectors be [See (20)]

$$\begin{array}{cc} x_r & x_a \\ \downarrow & \downarrow \end{array}$$
$$C_O=[V_{21} \;\; V_{22}] \qquad (53)$$

Then choosing $\mathbf{M} = \mathbf{V_{22}^{-1}}$, we obtain the true aggregation matrix in the form similar to C′ of (52.b) as

$$\begin{array}{cc} x_r & x_a \\ \downarrow & \downarrow \end{array}$$
$$C_R=[L_d \;\; I] \qquad (54)$$

where
$$L_d = V_{22}^{-1} V_{21}$$

The matrix $\mathbf{L_d}$ does not have the same structure as $\mathbf{L_g}$ because an intermachine mode in a subsystem is excited to some degree by a disturbance in other subsystems. Nevertheless, the algebraic sum of its elements on each row according to (P.5) is -1.

The algorithm which we propose for the grouping of the machines seeks $\mathbf{L_d}$ which will minimise the function

$$E_R = \| L_d - L_g \| \qquad (55)$$

and its associated variables $\mathbf{x_r}$ and $\mathbf{x_a}$.

An algorithm which seeks to decompose power systems into subsystems by minimising this function has been proposed by some authors [6]. Their proposed algorithm and what this paper presents however do not yield the same subsystems as our numerical example indicates. The difference arises from the fact that in their work, the decomposition is based on the assumption that the intermachine modes are the non-dominant modes and they are supposed to be known a priori. The assumption that the intermachine modes are the non-dominant ones enabled them to use the method of time scale separation for analysis which led to similar error function. As our numerical example indicates such an assumption can lead to artificial decomposition.

In our method the intermachine modes are not known a priori. We therefore set out on the assumption that all the non-zero eigenvalues of A are these modes and then construct $\mathbf{L_d}$ by stages. At each stage, we require that the error function which corresponds to $\mathbf{L_d}$ which may be incomplete at that stage to be as small as possible and suppose that $\mathbf{L_d}$ is complete if the error at the next stage is judged to be intolerable.

We need to know only the non-zero entries of the matrix $\mathbf{L_g}$ when evaluating the error function for a given $\mathbf{L_d}$. They can be obtained as follows: we examine each row of $\mathbf{L_d}$ and if the largest negative element is the jth in the row i, then the required non-zero entry of $\mathbf{L_g}$ is (i-j).

We shall illustrate the algorithm by a 5-machine real system. We first construct the matrix of aggregation $\mathbf{C_0}$ for the 4 negative eigenvalues which we define as

$$C^0 = \begin{array}{c} \\ \\ \lambda_2 \rightarrow \\ \lambda_3 \rightarrow \\ \lambda_4 \rightarrow \\ \lambda_5 \rightarrow \end{array} \begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \end{array} \\ \begin{bmatrix} v_{11}^0 & v_{12}^0 & v_{13}^0 & v_{14}^0 & v_{15}^0 \\ v_{21}^0 & v_{22}^0 & v_{23}^0 & v_{24}^0 & v_{25}^0 \\ v_{31}^0 & v_{32}^0 & v_{33}^0 & v_{34}^0 & v_{35}^0 \\ v_{41}^0 & v_{42}^0 & v_{43}^0 & v_{44}^0 & v_{45}^0 \end{bmatrix} \end{array} \qquad (56)$$

First stage:

We obtain $\mathbf{L_d}$ of the order 1 x 4. Since to minimize E_R, $\| \mathbf{L_d} \|$ should be necessarily small [6], a machine which corresponds to the largest element on a row is taken as the associated machine of the corresponding mode. With associated machine thus defined, $\mathbf{L_d}^0$ of each mode is constructed and the one which gives $\mathbf{L_d}$ that minimizes $\mathbf{E_R}$ is then declared once and for all an intermachine mode. The designation of a machine corresponding to the largest element on its row as an associated machine also becomes final.

Let λ_5 be this mode and machine 5 its associated machine, then we shall have

$$\begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \\ \downarrow & \downarrow & \downarrow & \downarrow \end{array}$$
$$L_d^0 = x_5 \rightarrow [L_{11}^0 \;\; L_{12}^0 \;\; L_{13}^0 \;\; L_{14}^0] \rightarrow \lambda_5 \qquad (57.a)$$

where

$$L^0_{1j} = v^0_{4j}/v^0_{45} \;, j = 1,2,3,4 \qquad (57.b)$$

Machine 5 and λ_5 are then eliminated in further search of intermachine modes by using a reduced matrix of aggregation

$$C^1 = \begin{array}{c} \\ \\ \lambda_2 \rightarrow \\ \lambda_3 \rightarrow \\ \lambda_4 \rightarrow \end{array} \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \\ \downarrow & \downarrow & \downarrow & \downarrow \end{array} \\ \begin{bmatrix} v^1_{11} & v^1_{12} & v^1_{13} & v^1_{14} \\ v^1_{21} & v^1_{22} & v^1_{23} & v^1_{24} \\ v^1_{31} & v^1_{32} & v^1_{33} & v^1_{34} \end{bmatrix} \end{array} \qquad (58.a)$$

where

$$v^1_{ij} = v^0_{ij} - v^0_{i5} \cdot v^0_{4j}/v^0_{45} \qquad (58.b)$$

$$i = 1,2,3 \;;\; j = 1,2,3,4 \qquad (58.c)$$

The algebraic sum of elements on each of row of $\mathbf{C}^1$ like that of $\mathbf{C}^0$ is zero.

Second stage

With the matrix $\mathbf{C}^1$, we determine the mode which in combination with λ_5 gives an $\mathbf{L_d}$ of the order 2 x 3 that minimizes $\mathbf{E_R}$ by trying all possible combinations. The choice of an associated machine is made as in the first stage. If the mode in question is 4 and the largest element on its row corresponds to machine 4, we shall obtain

$$L^1_d = \begin{array}{c} \\ \\ x_4 \rightarrow \\ x_5 \rightarrow \end{array} \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \\ \downarrow & \downarrow & \downarrow \end{array} \\ \begin{bmatrix} L^1_{11} & L^1_{12} & L^1_{13} \\ L^1_{21} & L^1_{22} & L^1_{23} \end{bmatrix} \end{array} \begin{array}{c} \\ \\ \leftarrow \lambda_4 \\ \leftarrow \lambda_5 \end{array} \qquad (59.a)$$

where

$$L^1_{1j} = v^1_{3j}/v^1_{34} \;, j = 1,2,3 \qquad (59.b)$$

$$L^1_{2j} = L^0_{1j} - L^0_{14} \cdot v^1_{3j}/v^1_{34} \quad ; j = 1,2,3 \qquad (59.c)$$

The expression (59.b) similar to (57.b) appears as a row of a partially-constructed $\mathbf{L_d}$ at each stage. This explains the choice of an associated machine as in the first stage.

The reduced matrix of aggregation for further search of intermachine modes after this stage is given as

$$C^2 = \begin{array}{c} \\ \\ \lambda_2 \rightarrow \\ \lambda_3 \rightarrow \end{array} \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \\ \downarrow & \downarrow & \downarrow \end{array} \\ \begin{bmatrix} v^2_{11} & v^2_{12} & v^2_{13} \\ v^2_{21} & v^2_{22} & v^2_{23} \end{bmatrix} \end{array} \qquad (60.a)$$

where

$$v^2_{ij} = v^1_{ij} - v^1_{i4} \cdot v^1_{3j}/v^1_{34} \qquad (60.b)$$

$$i = 1,2 \;;\; j = 1,2,3 \qquad (60.c)$$

Third stage

Taking the newly discovered intermachine mode to be λ_3 and assuming the largest element on its row to correspond to machine 3, the $\mathbf{L_d}$ at this stage will be defined as

$$L^2_d = \begin{array}{c} \\ \\ x_3 \rightarrow \\ x_4 \rightarrow \\ x_5 \rightarrow \end{array} \begin{array}{c} \begin{array}{cc} x_1 & x_2 \\ \downarrow & \downarrow \end{array} \\ \begin{bmatrix} L^2_{11} & L^2_{12} \\ L^2_{21} & L^2_{22} \\ L^2_{31} & L^2_{32} \end{bmatrix} \end{array} \begin{array}{c} \\ \\ \leftarrow \lambda_3 \\ \leftarrow \lambda_4 \\ \leftarrow \lambda_5 \end{array} \qquad (61.a)$$

where

$$L^2_{1j} = v^2_{2j}/v^2_{23} \; j = 1,2 \qquad (61.b)$$

$$L^2_{ij} = L^2_{i-1,j} - L^1_{i-1,3} \cdot v^2_{2j}/v^2_{23} \qquad (61.c)$$

$$i = 2,3 ;\; j = 1,2 \qquad (61.d)$$

The construction of $\mathbf{L_d}$, in a general case, to be pursued gradually as indicated up to this stage, is stopped when the error at some stage is judged to be no longer acceptable.

In an attempt to reduce the computation time a second algorithm is also suggested. This algorithm differs only in the definition of $\mathbf{E_R}$. At each stage $\mathbf{E_R}$ is obtain using strictly the error due to the row similarly defined as in (57.b), (59.b) and (61.b).

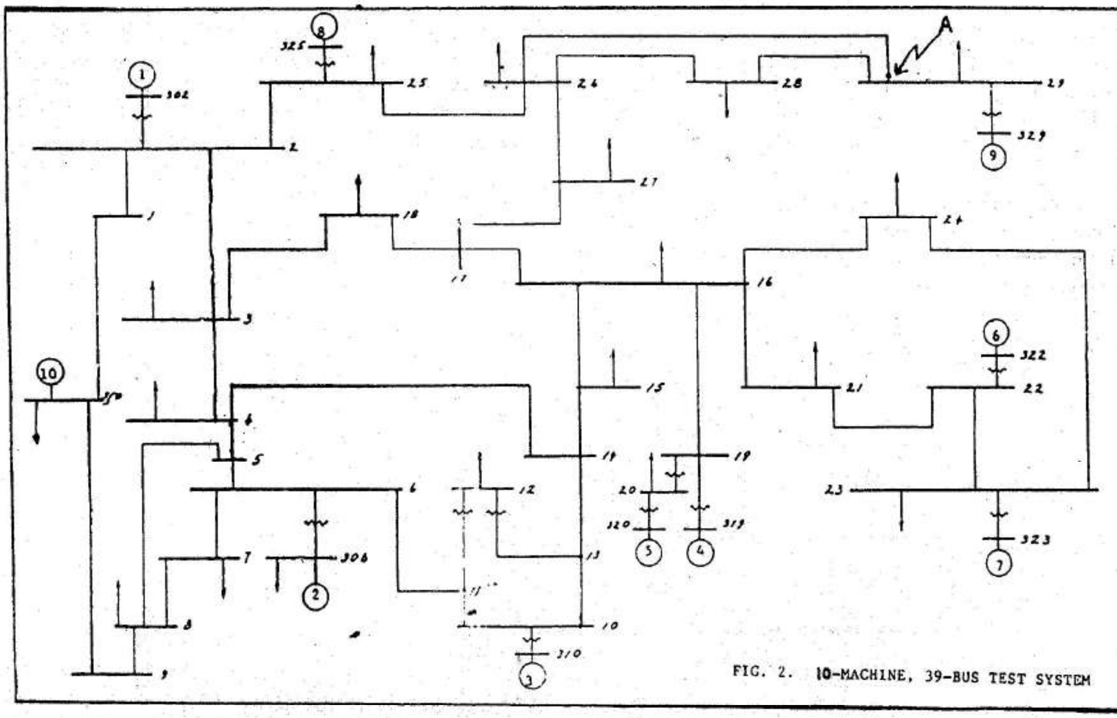

FIG. 2. 10-MACHINE, 39-BUS TEST SYSTEM

NUMERICAL EXAMPLE

The proposed decomposition scheme is applied to a 10-machine, 39-bus power system of Fig.2 referred to as "New England System". The complete system data can be found in reference 10. The eigenvalues of the A-matrix are as follows:

$\lambda_1 = 0.0000 \quad \lambda_2 = -17.5170$

$\lambda_3 = -42.3079 \quad \lambda_4 = -48.1119$

$\lambda_5 = -56.4506 \quad \lambda_6 = -65.2602$

$\lambda_7 = -69.9629 \quad \lambda_8 = -91.0559$

$\lambda_9 = -95.4398 \quad \lambda_{10} = -98.1179$

Considering all the negative eigenvalues as intermachine modes, we have obtained, using the intial algorithm, E_R by stages as indicated in Table 1. Using the simplified algorithm gives the same results up to the sixth stage.

We observe a jump in the value of E_R after the third stage. We suppose therefore that the eigenvalues λ_9, λ_{10} and λ_7 are the intermachine modes. The corresponding $\mathbf{L_d}$ is given in Table 2.

Table 1. Growth of error with stage

stage	1st	2nd	3rd	4th	5th	6th	7th	8th
mode	λ_9	λ_{10}	λ_7	λ_5	λ_8	λ_3	λ_4	λ_2
E_R	0.1923	0.2327	0.2835	1.1930	0.9700	0.9730	0.9984	0.8310

Table 2. grouping matrix Ld

Ass. m/cs	Reference machines							mode
	6	4	9	10	5	1	2	
8	-0.0049	-0.0201	-0.0692	0.0539	0.0033	-0.9695	0.0064	λ_9
7	-0.9079	-0.1006	-0.0026	0.0006	0.0236	-0.0072	-0.0058	λ_{10}
3	-0.0654	0.0000	0.0085	0.0399	-0.0056	-0.0707	-0.9067	λ_7

The largest negative elements on rows 1, 2 and 3 are associated respectively with machines 1, 6 and 2. Thus, the subsystem which contains two or more machines are

$S_1 = \{ 8, 1 \}$; $S_2 = \{ 7, 6 \}$
$S_3 = \{ 3, 2 \}$

To verify the correctness of the decompositon we have obtained the following:

(1) eigenvalues of the coeffient matrix of each subsystem (S4):

S_1 : (-55.6660, -93.3179)
S_2 : (-46.3737, -98.0559)
S_3 : (-43.9779, -69.9091)

The three underlined eigenvalues which are the non-dominant ones are found to be very close to the respective subsystem intermachine modes.

(2) eigenvalues of the aggregated matrix $\mathbf{F_E}$ which retains only the eigenvalues of **A** which are not intermachine modes (S6):

{-0.0088,-17.5807,-42.3596,-48.1342,
-56.4755,-65.2558,-91.0961}

These eigenvalues are found to be very close to those of **A** which are not intermachine modes.

(3) the vectors $\mathbf{v_0}$ and $\mathbf{t_0}$ and their ideal values for comparison in Table 3 (S7).

Table 3. Verification of S7 for subsystems 1 to 3

Subsystem	v_0		t_0	
	True	Ideal	True	Ideal
{ 8, 1 }	[1.0, 0.5781]	[1.0, 0.5786]	$[1.0, 0.9991]^T$	$[1.0, 1.0]^T$
{ 7, 6 }	[1.0, 0.7189]	[1.0, 0.7586]	$[1.0, 0.9488]^T$	$[1.0, 1.0]^T$
{ 3, 2 }	[1.0, 0.8897]	[1.0, 0.9090]	$[1.0, 1.0513]^T$	$[1.0, 1.0]^T$

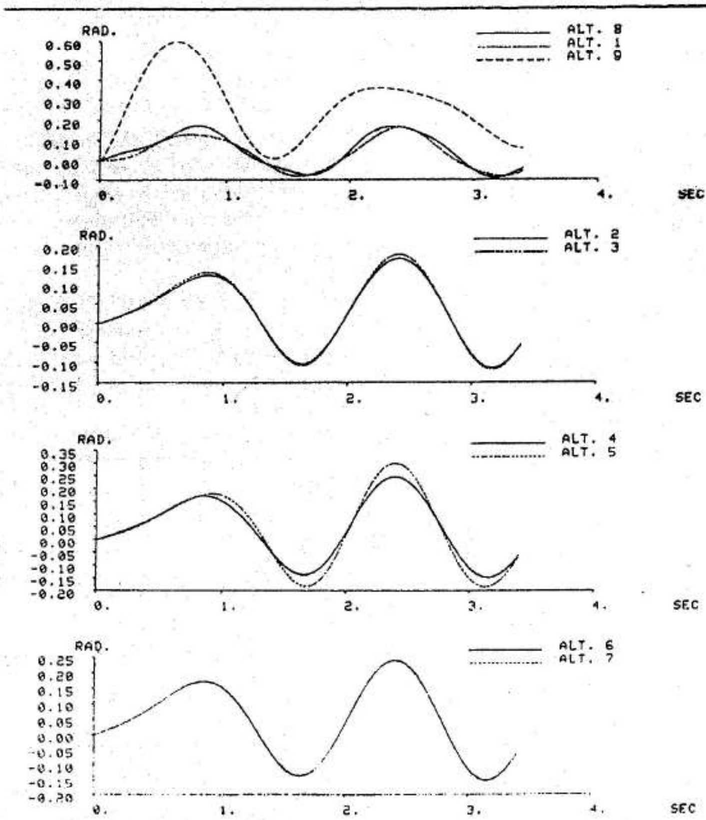

FIG. 3. DEVIATION OF ROTOR ANGLES FROM PREFAULT VALUES

(4). The response of rotor angles in the presence of a three phase solid short circuit at point A near generator 9 terminal and subsystem 1, cleared after 1.5 cycles by opening line 26-29 (S3):

The curves of Fig 3 represent the responses of $\Delta\delta_{i,s}$ which are deviations of the post-fault rotor angles from their prefault steady state values. The post-fault rotor angles were obtained by solving the nonlinear equations (1) - (3). The angle of machine 10 was used as reference thus bringing the system order to 2n-1 [11].

We notice that λ_7 is an intermachine mode but λ_8 which is larger is not. Considering this as an additional intermachine mode and using the method proposed in reference 6, we obtain a fourth subsystem S={ 4, 5 }. The eigenvalues of its coefficient matrix are {-39.0810, -87.7737} and the vectors $\mathbf{v_0}$, $\mathbf{t_0}$ and their respective ideal values are given in Table 4.

Table 4 Verification of S7 for subsystems 4

Subsystem	v_0		t_0	
	True	Ideal	True	Ideal
{ 4, 5 }	[1.0, 1.9228]	[1.0, 0.9090]	$[1.0, 2.1153]^T$	$[1.0, 1.0]^T$

The departure of the subsystem intermachine mode (taken to be - 87.7737), the true $\mathbf{v_0}$ and $\mathbf{t_0}$ from their theoretical values are relatively large. Thus, the assumption that the non-dominant eigenvalues are the intermachine modes according to our analysis can lead to artificial decomposition for equivalencing using the method of aggregation or coherency-based technique.

CONCLUSION

A method has been proposed for decomposition of power systems into subsystem which can be used for dynamic studies by equivalencing technique. Certain pieces of quantitative information have been derived to help us to ascertain the correctness of a decomposition achieved when the method is applied to a system which in general may not be known to be decomposable a priori. A numerical example has been presented to demonstrate the capability of the method to decompose interconnected power systems.

PRINCIPAL SYMBOLS

δ = rotor angle
ω_0 = rated angular speed
ω = rotor angular speed
H = inertia constant
D = damping coefficient
p = differential operator d/dt
Δ = prefix denoting a small change
E' = constant voltage behind transient reactance
Y' = reduced admittance matrix at internal machine nodes(E')
n = number of power systems synchronous machines
$\| L \| = \max_i \sum_j | L_{ij} |$
$<.,.>$ = dot product

Subscripts of machine quantities

i = refers to ith synchronous machine
o = refers to operating point quantities

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APPENDIX

proof of equation (39):

From (9), we can write

$$\hat{V}_1 = \hat{W}_1^T H_1 \tag{A.1}$$

where

$$H_1 = diag(H_1, H_2, \ldots\ldots, H_q)$$

Now according to (28), when (33) is satisfied then

$$\hat{V}_1 A_{12} = 0 \tag{A.2}$$

This also implies that

$$\hat{V}_1 H_1^{-1} K_{12} = 0 \tag{A.3}$$

where $\mathbf{K_{12}}$ is a submatrix of the symmetric matrix **K** in Equ.(5). Substituting (A.1) into (A.3) yields

$$\hat{W}_1^T K_{12} = 0$$

Taking transpose of each side of this equation gives

$$K_{12}^T \hat{W}_1 = K_{21} \hat{W}_1 = 0$$

and hence

$$A_{21} \hat{W}_1 = 0$$