POWER SYSTEMS

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INVERSION OF Y-MATRIX OCCURRING IN A LINEARIZED MULTIMACHINE POWER SYSTEM MODEL

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ABSTRACT

When constructing a linearized multimachine power system model, expressions for current components describing the interaction among machines must be sought. Explicit expressions for the current components can be derived in terms of the elements of a Y-matrix which because of saliency of machines is obtained by two successive inverse operations. This paper presents a method which circumvents the two inverse operations for efficiently computing the Y-matrix which can be very large. The method obtains the Y-matrix initially in a partially-constructed form and then obtains the final required form from it by just changing its elements. The proposed method when compared with the normal approach which makes use of standard routine, reduces the computation time by about half. The method also saves computer storage space because it does not require the extra working space which is equivalent to a real vector of dimension 2n necessary in the normal approach.

Key Words:
Linearized model, saliency, Y-matrix, multimachine power system, block diagonal matrix, dynamic problems

INTRODUCTION

Linearized power system models usually formulated in state-space form are used to analyse certain power system dynamic problems [1-6]. To obtain a linearized model for the multimachine power system, we need to derive expressions for current components expressed with respect to the d-q reference axes of the machines as functions of state variables. The derivation of these expressions using equation which describes the constraint imposed by the interconnecting network and transformation equations constitute the main computational task when constructing these models. The equations for the current components can be expressed explicitly in terms of the elements of a Y-matrix and state variables.

Depending on the dynamic problem analyzed, a sychronous machine may be represented by a fifth- [1,5], third- [2,3,5] or second-order [5,6] model. Each order has its own Y-matrix. Whereas the Y-matrix of the second-order model is straight-forward to compute, those of the third - and fifth - order models which have a common form require significant computation when the number of machines in the system is large. This paper presents a method which reduces the amount of computation for large multimachine power systems. Examples of such systems are the North American WSCC system and the Northeastern and Michigan system which, considering major machines alone, contain 300 machines each [5].

THE Y-MATRIX

The Y-matrix of the order 2n which results when the linearized current components are stacked up in a vector form can be written in the form

$$Y = [Z + T_O{}^T Y_N{}^{-1} T_O]^{-1} \qquad (1)$$

The matrix Z which is a function of the synchronous machines parameters has the form

$$Z = \text{Block diag}(D_1, D_2, \ldots, D_n) \qquad (2.a)$$

where

$$D_i = \begin{bmatrix} r_{ai} & -x_{1i} \\ x_{2i} & r_{ai} \end{bmatrix} \qquad (2.b)$$

The matrix T which is also a block diagonal is orthogonal. For a third-order model $x_1 = x_q$ and $x_2 = x'_d$ and for a fifth-order model $x_1 = x''_q$ and $x_2 = x''_d$. Linearized multimachine power system equations used for the design of Power System Stabilizers and other dynamic studies [3,4] are given in the appendix B with detailed information on its Y-matrix for illustration. In this linearized model, a synchronous machine is represented by a third-order model but it is extended to include the excitation system dynamics.

To avoid the two inverse operations when computing Y [see (1)], x_2 can be approximated by x_1, thus neglecting transient saliency in the third-order model and subtransient saliency in the fifth-order model [1]. In some work [2], the linearized model was derived by neglecting transient saliency just to simplify the computation of Y. Since x_q can be about ten times larger such a simplifying assumption can lead to conservative results when the model is used for studies.

When saliency is neglected the Y-matrix assumes a form which can be constructed as follows:

$$Y = T_O^T Y_q T_O \tag{3}$$

where Y_q is the real expanded form of the node admittance matrix of the extended power system which results by appending the quadrature impedance $r_a + jx_1$ of each machine to its node and eliminating all synchronous and non-synchronous machine buses, thus leaving only the fictitious buses.

PROPOSED METHOD FOR COMPUTING Y

The matrix Z can be expressed in the form

$$Z = Z_S + \sum_{i=1}^{n} Z_{Ai} \tag{4}$$

where

$$Z_S = \text{Block diag}(D_{S1}, D_{S2}, \ldots, D_{Sn}) \tag{5.a}$$

$$D_{Si} = \begin{bmatrix} r_{ai} & -x_{1i} \\ x_{1i} & r_{ai} \end{bmatrix} \tag{5.b}$$

and Z_{Ai} is a matrix with the only element (2i, 2i-1) which is not equal to zero being equal to $(x_{2i} - x_{1i})$.

Let

$$B_O = (Z_S + T_O^T Y_N T_O)^{-1}$$

$$= T_O^T Y_q T_O \tag{6.a}$$

Then

$$Y = (B_O^{-1} + \sum_{i=1}^{n} Z_{Ai})^{-1} \tag{6.b}$$

If we let

$$B_1 = (B_O^{-1} + Z_{A1})^{-1} \tag{7.a}$$

we obtain

$$B_1 = (I + B_O Z_{A1})^{-1} B_O \tag{7.b}$$

Similarly

$$B_2 = (B_O^{-1} + Z_{A1} + Z_{A2})^{-1}$$

$$= (B_1^{-1} + Z_{A2})^{-1} \tag{8.a}$$

is given by

$$B_2 = (I + B_1 Z_{A2})^{-1} B_1 \tag{8.b}$$

Proceeding in this manner, we obtain Y as

$$Y = (I + B_{n-1} Z_{An})^{-1} B_{n-1} \tag{9}$$

An example is given in appendix A to illustrate the computation of the matrix of the form

$$B_i = (I + B_{i-1} Z_{Ai})^{-1} B_{i-1} \tag{10}$$

An algorithm which results from the recursion formulae for computing Y is outlined as follows:

Step 1: Construct $Y = T_O^T Y_q T_O = (y_{ij})$

Step 2: Correct saliency error due to the ith machine if not done already following these steps:

a) Calculate $a = x_{2i} - x_{1i}$

b) Calculate $g = ay_{2i-1,2i} + 1$

c) If g happens to be zero go to step 2 and consider another machine

d) Correct the elements on row 2i-1 of Y partially constructed:

$y_{2i-1,j} = y_{2i-1,j} / g$, $j = 1, 2, \ldots, 2n$

e) Correct elements on other rows:

For each $k = 1, 2, \ldots, 2n$, $k \neq 2i-1$, calculate

(i) $h = ay_{k,2i}$

(ii) $y_{k,j} = y_{k,j} - hy_{2i-1,j}$, $j = 1, 2, \ldots, 2n$

Step 3: Stop when the saliency error due to all machines have been corrected.

EVALUATION OF PROPOSED ALGORITHM

The time required for the execution of the proposed algorithm for large n where efficiency is of greatest concern is determined mainly by the computation of Y_q as part of step 1 and then the execution of steps 2 and 3. The matrix Y_q is derived in its complex form from the matrix Y_N which is also initially obtained in its complex form. For large n, the derivation of complex Y_q from complex Y_N requires $4n^3$ real multiplications and divisions and $4n^3$ real additions and subtractions when advantage is not even taken of the fact that complex Y_q is symmetric to reduce the operations involved. The execution of steps 2 and 3 of the algorithm also for a large n involves similar number of counts for each category of operations. Thus the proposed algorithm requires a total of $8n^3$ multiplications and divisions and $8n^3$ additions and subtractions.

Without the proposed algorithm, the normal procedure would be to invert two square matrices of the order 2n one after the other. This approach requires $16n^3$ counts for each category of operations if the Gaussian elimination method (the recommended method for matrix inversion [7]) is used. These counts are two times those obtained with the proposed algorithm. Thus the proposed algorithm makes a time saving of about fifty per cent.

The number of columns of the matrix to be inverted using Gaussian elimination is augmented by one to allow for extra computer storage space. Since the proposed method obtains the final required matrix by just changing the elements of an initially constructed matrix of the same order in both the computation of complex Y_q and the execution of steps 2 and 3, this extra storage space which is equivalent to a real vector of dimension 2n becomes unnecessary.

CONCLUSION

A method for computing Y-matrix occurring in a linearized multimachine power system has been proposed. This method avoids two inverse operations which would otherwise be performed to obtain it. The method reduces the normal computation time by about half and has an additional advantage of reducing the time still further when the saliency of certain machines can be neglected as it is in the case of fifth-order model. The method also saves computer storage space for not requiring the working vector of dimension 2n used in the normal approach.

PRINCIPAL SYMBOLS

x_q = machine quadrature-axis reactance
x'_d = machine direct-axis transient reactance
x''_d = machine direct-axis subtransient reactance
x''_q = machine quadrature-axis subtransient reactance
r_a = armature resistance
δ = the phase angle difference of the d axis with respect to the D-axis
n = number of synchronous machines
T = a block diagonal matrix of the order 2n which is a function of δ's
Y_N = reduced synchronous machine node admittance matrix with all the non-synchronous machine nodes eliminated in its real expanded form
Δ = prefix denoting a small change
V = voltage vector
V = voltage
I = current vector
I = current
ω_o = rated angular frequency
ω = instantaneous rotor angular frequency in p.u.
T_e = electrical torque
H = inertia constant
E_{fd} = field voltage referred to armature circuit
v_t = machine terminal voltage
D_e = damping coefficient
p = differential operator
T'_{do} = armature open-circuited time constant
T_A = excitation system time constant
K_A = excitation system gain

Subscripts

d,q,M,m = machine reference d and q axes
D,Q,N,n = machine reference D and Q axes
i = ith synchronous machine
o = operating point quantities

Superscripts

' = synchronous machine transient parameters
T = transposition

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APPENDIX A

We consider without loss of generality the following example to illustrate the computation of the expression (9):

$$D = [I + BC]^{-1} B \qquad (A.1)$$

where $B = (b_{ij})$ is 4 x 4 matrix and $C = (c_{ij})$ which is also 4 x 4 matrix has c_{21} = a and all other elements being equal to zero. In this case

$$
[I + BC]^{-1} = \begin{bmatrix} + \dfrac{1}{ab_{12}+1} & 0 & 0 & 0 \\ - \dfrac{ab_{22}}{ab_{12}+1} & 1 & 0 & 0 \\ - \dfrac{ab_{32}}{ab_{12}+1} & 0 & 1 & 0 \\ - \dfrac{ab_{42}}{ab_{12}+1} & 0 & 0 & 1 \end{bmatrix}
$$

which on substituting into (A.1) gives

$$
D = \begin{bmatrix} \dfrac{b_{11}}{ab_{12}+1} & \dfrac{b_{12}}{ab_{12}+1} & \dfrac{b_{13}}{ab_{12}+1} & \dfrac{b_{14}}{ab_{12}+1} \\ b_{21} - \dfrac{ab_{22}b_{11}}{ab_{12}+1} & b_{22} - \dfrac{ab_{22}b_{12}}{ab_{12}+1} & b_{23} - \dfrac{ab_{22}b_{13}}{ab_{12}+1} & b_{24} - \dfrac{b_{22}b_{14}}{ab_{12}+1} \\ b_{31} - \dfrac{ab_{32}b_{11}}{ab_{12}+1} & b_{32} - \dfrac{ab_{32}b_{12}}{ab_{12}+1} & b_{33} - \dfrac{ab_{32}b_{13}}{ab_{12}+1} & b_{34} - \dfrac{b_{32}b14}{ab_{12}+1} \\ b_{41} - \dfrac{ab_{42}b_{11}}{ab_{12}+1} & b_{42} - \dfrac{ab_{42}b_{12}}{ab_{12}+1} & b_{43} - \dfrac{ab_{42}b_{13}}{ab_{12}+1} & b_{44} - \dfrac{b_{42}b_{14}}{ab_{12}+1} \end{bmatrix}
$$

APPENDIX B

Linearized multimachine power system equations with a synchronous machine represented by a third order model are given below with its Y-matrix detailed for illustration:

A) Differential Equations

$$p\Delta\delta_i = \omega_o \Delta\omega_i$$

$$p\Delta\omega_i = \frac{1}{2H_i}[- D_{ei}\Delta\omega_i - \Delta T_{ei}]$$

$$p\Delta E'_{qi} = \frac{1}{T'_{doi}}[- \Delta E'_{qi} + \Delta E_{fdi} - (x_{di} - x'_{di})\Delta I_{di}]$$

$$p\Delta E_{fdi} = \frac{1}{T_{Ai}}[- \Delta E_{fdi} + \frac{K_{Ai}}{T_{Ai}}(\Delta V_{REFi} - \Delta V_{ti})]$$

B) Algebraic Equations

$$\Delta T_{ei} = I_{qoi}\,\Delta E'_{qi} + [\,E'_{qoi} + (x_{qi} - x'_{di})I_{doi}\,]\Delta I_{qi} + [(x_{qi} - x'_{di})I_{qoi}\,]\Delta I_{di}$$

$$\Delta V_{di} = - R_{ai}\Delta I_{di} + x_{qi}\Delta I_{qi} \qquad (B.1)$$

$$\Delta V_{qi} = - x'_{di}\Delta I_{di} - R_{ai}\Delta I_{qi} + \Delta E'_{qi} \qquad (B.2)$$

$$\Delta V_{ti} = \left[\frac{V_{doi}}{V_{toi}}\right]\Delta V_{di} + \left[\frac{V_{qoi}}{V_{toi}}\right]\Delta V_{qi}$$

C) Current Components In Matrix Form

The equation relating the current components, state variables and the matrix **Y** is given in (B.3). Its proof is also added below.

$$\Delta I_M = [S_I - Y\,(C\,S_I + S'_V)]\,\Delta D_\delta + Y\,\Delta V'_M \qquad (B.3)$$

where

$$\Delta I_M = [\Delta I_{d1}, \Delta I_{q1}, \Delta I_{d2}, \Delta I_{q2}, \cdots, \Delta I_{dn}, \Delta I_{qn}]^T$$

$$C = \text{Block diag}(C_1, C_2, \text{——}, C_n)$$

$$C_i = \begin{bmatrix} 0 & x'_{di} - x_{qi} \\ x'_{di} - x_{qi} & 0 \end{bmatrix}$$

$$S_I = \text{Block diag}(\,S_{I1}, S_{I2}, \text{—}, S_{In}) \qquad (B.4)$$

$$S_{Ii} = [I_{qoi}, -I_{doi}]^T$$

$$S'_V = \text{Block diag}(S'_{V1}, S'_{V2}, \text{——}, S'_{Vn})$$

$$S'_{Vi} = [E'_{qoi}, 0]^T$$

$$\Delta D_\delta = [\Delta\delta_1, \Delta\delta_2, \ldots\ldots, \Delta\delta_n]^T$$

$$\Delta V'_M = [0, \Delta E'_{q1}, 0, \Delta E'_{q2}, \ldots, 0, \Delta E'_{qn}]^T$$

D) Current Components In Expanded Form

$$\Delta I_{di} = \{I_{qoi} + y_{2i-1,2i-1} \times [(x'_{di} - x_{qi})I_{doi} - E'_{qoi}]$$
$$+ y_{2i-1,2i} \times (x_{qi} - x'_{di})I_{qoi}\}\,\Delta\delta_i$$
$$+ \sum_{\substack{j=1 \\ j \neq i}}^{n} \{ y_{2i-1,2j-1} \times [(x'_{dj} - x_{qj})I_{doj} - E'_{qoj}]$$
$$+ y_{2i-1,2j} \times (x_{qj} - x'_{dj})I_{qoj}\}\,\Delta\delta_j$$
$$+ \sum_{j=1}^{n} y_{2i-1,2j} \times \Delta E'_{q}{}^{j}$$

$$\Delta I_{qi} = \{-I_{doi} + y_{2i,2i-1} \times [(x'_{di} - x_{qi})I_{doi} - E'_{qoi}]$$
$$+ y_{2i,2i} \times (x_{qi} - x'_{di})I_{qoi}\}\,\Delta\delta_i$$
$$+ \sum_{\substack{j=1 \\ j \neq i}}^{n} \{ y_{2i,2j-1} \times [(x'_{dj} - x_{qj})I_{doj} - E'_{qoj}]$$
$$+ y_{2i,2j} \times (x_{qi} - x'_{dj})I_{qoj}\}\,\Delta\delta_j$$
$$+ \sum_{j=1}^{n} y_{2i,2j} \times \Delta E'_{qj}$$

E) Derivation Of Current Component Equation

The equation which describes the constraint imposed by interconnecting network in linearized form is

$$\Delta I_N = Y_N \Delta V_N$$

where

$$\Delta I_N = [\Delta I_{D1}, \Delta I_{Q1}, \Delta I_{D2}, \Delta I_{Q2}, \ldots\ldots, \Delta I_{Dn}, \Delta I_{Qn}]^T$$

$$\Delta V_N = [\Delta V_{D1}, \Delta V_{Q1}, \Delta V_{D2}, \Delta V_{Q2}, \ldots\ldots, \Delta V_{Dn}, \Delta V_{Qn}]^T$$

The relations between d-q and D-Q components of voltages and currents are

$$I_{Mi} = T_i^T I_{Ni}$$

$$V_{Mi} = T_i^T V_{Ni}$$

where

$$I_{Mi} = [I_{di}, I_{qi}]^T$$

$$V_{Mi} = [V_{di}, V_{qi}]^T$$

$$T_i = \begin{bmatrix} \cos\delta_i & -\sin\delta_i \\ \sin\delta_i & \cos\delta_i \end{bmatrix}$$

For small variations about the operating point, we obtain

$$\Delta I_{Mi} = S_{Ii}\Delta\delta_i + T_{Oi}^T \Delta I_{Ni} \qquad \text{(B.6)}$$

$$\Delta V_{Mi} = S_{Vi}\Delta\delta_i + T_{Oi}^T \Delta V_{Ni} \qquad \text{(B.7)}$$

The matrix S_{Vi} is defined in the same manner as S_{Ii} in (B.4). In compounded form, (B.1), (B.2) (B.6), (B.7) become

$$\Delta V_M = -Z_A \Delta I_M + \Delta V'_M \qquad \text{(B.8)}$$

$$\Delta V_M = S_V \Delta D_\delta + T_O^T \Delta V_N \qquad \text{(B.9)}$$

$$\Delta I_M = S_I \Delta D_\delta + T_O^T \Delta I_N \qquad \text{(B.10)}$$

where

$$T_O = \text{Block diag}(T_{O1}, T_{O2}, \ldots\ldots, T_{On})$$

Substituting the solution of ΔV_M of (B.8) into (B.9) and solving for $\Delta V'_M$ from the resulting equation, (B.5) and (B.10) gives

$$\Delta V'_M = S_V \Delta D_\delta + [Z_A + T_O^T Y_N^{-1} T_O]\,\Delta I_M$$
$$- [T_O^T Y_N^{-1} T_O]\, S_I \Delta D_\delta \qquad \text{(B.11)}$$

The stator equations (B.1) and (B.2) though linearized indicate that

$$S_{Vi} = -Z_{Ai} S_{Ii} + C_i S_{Ii} + S'_{Vi}$$

which when compounded gives

$$S_V = -Z_A S_I + C S_I + S'_V$$

Substituting this into (B.11), we obtain after simplification

$$\Delta I_M = [S_I - Y(C S_I + S'_V)]\,\Delta D_\delta + Y \Delta V'_M$$