FRACTURE MECHANICS

5
LOAD-CARRYING CAPABILITY EVALUATION BY THE METHOD OF ENVELOPE EQUATIONS

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ABSTRACT

Envelope equations have been developed to provide a simple method for evaluating the load-carrying capability of process components such as found in nuclear and petro-chemical plants. The use of the equations eliminates the, otherwise, elaborate and time-consuming analyses required to determine whether or not a given set of loads could be successfully applied to a specific structure. In particular, the equations are suitable for the evaluation of the different service code criteria such as those of the ASME.

The method provides conservative upper bounds for the loads that a given structure can carry under a specified service condition. Thus, the equations could provide useful guidelines at the preliminary design stage to be backed later by more elaborate calculations for ultimate verification. Sample calculations are also presented to verify the validity of the equations.

Keywords: load-carrying capability, envelope equations, process components, ASME service code criteria

Nomenclature

F = force
M = moment
N = total number of applied loads
S = allowable stress intensity
Z = generalized maximum load (force or moment)

Greek Symbols

ξ = unit load (force or moment
η = dimensionless load, defined by eq. (5)
λ = dimensionless force
μ = dimensionless moment
σ = actual stress intensity
ϕ = surface

Subscripts

c = critical
d = discharge nozzle
s = suction nozzle
T = temperature
P = pressure

INTRODUCTION

In the design and analysis of nuclear power and petrochemical plant components, it is necessary to determine whether certain critical components such as pumps, valves or vessels would meet a specified stress criterion under the action of a given set of loads. The corollary to this problem is the determination of the minimum thickness required by such a component to meet a specified stress criterion under the simultaneous action of a given set of loads.

Oftentimes, the problem cannot be satisfactorily resolved by the superposition of the results of classical engineering formulas because of structural complexity or the combination of the loads involved or both. In such instances, one's best recourse is the use of finite element computer programs. The usual procedure is to determine 'unit load' stresses corresponding to each of the loads (forces and/or moments) in the given set. The results are stored on magnetic tapes or discs for later retrieval. The value of the unit load is chosen such that it leads to appreciable stress in each element of the structure. Values of the order of 10^3 to 10^6 are frequently chosen with appropriate dimensions such as Newton (N), pound force (lb_f), m-N or in-lb_f.

The next step would be to determine the load factors for each set of loads that are to be evaluated. These are obtained by dividing each load in the set by the magnitude of the unit load. The final results are obtained with the help of a post-processor program which recalls the unit-load stress, scales them appropriately by the corresponding set of scale factors and ultimately combines the stresses for the number of loads in the given set.

Experience shows that these types of post-processor programs are rather expensive to run and thus become uneconomical if several sets of loads are to be considered. It is for such cases that the envelope equations developed below provide an inexpensive and quick means of estimating the adequacy or otherwise of a structure to carry a given set of loads, especially at the preliminary design stage.

A concept similar to the envelope equation was applied by Erdogan and Sih [1] to two-dimensional failure modes due to mechanical fracture. The concept was later extended to three-dimensional crack modes by Tuba and Wilson [2]. The efforts resulted in the development of an expression for 'surface of critical stress intensities' for the three-dimensional failure mode which degenerates into a curve of critical stress intensities for the two-dimensional mode.

In the development of the envelope equations, however, there is no restriction on either the number of loads or the location of application.

Derivation of Envelope Equations

Consider a structure, such as a pump or vessel, with a set of N-loads acting simultaneously on it. These loads are in the form of forces and moments which are applied through nozzles, support lugs, anchor points and other similar locations. The following two assumptions are fundamental to the formulation of the envelope equations:

(i) equilibrium of the structure is preserved upon the application of the external loads; and

(ii) each load δ_i is assumed to act independently of the other (N - 1) loads.

Now consider a non-metrized, N-dimensional, real space, VN, in which

$$\left| \;_i - Z_i \right| < \;_i \;;\; (i = 1,2..., N) \qquad (1)$$

such that δ_i, Z_i V_N and $_i$ is a small quantity.

Since the space V_N is real, the inequalities in eq. (1) can be written in the form

$$Z_i \leq \delta_i \leq Z_i, \;(1 = 1,2 .., N) \qquad (2)$$

where Z_1 and Z_1 are minimum and maximum forces in the space, respectively. Equation (2) defines the range of variables $\S_i$. The N-dimensional space is further assumed to bear a one-to-one correspondence with the N-loads acting on the structure.

The effects of thermal and pressure stresses could also be taken into account if they act simultaneously with the N-loads. In such a case, the allowable stress intensity limit, S_m, for the N-loads is given by

$$S_m = S_m - (S_T + S_P) \qquad . \quad (3)$$

Now let the stress intensity at a point of interest due to δ_i be σ_i. Therefore, the maximum independent value of the ith load, Z_i that the structure can withstand and still meet the given criterion is

$$Z_i = \frac{S_m \;\delta_i}{\sigma_i} \qquad (4)$$

where $\sigma_i \leq S_m$

Eq. (4) is based on the proposition of linear relationship between load and stress. The limiting loads designated by the equation represent the pure-mode critical loads. This equation can be further rearranged in dimensionless form as

$$\eta_i = \frac{\delta_i}{Z_i} = \frac{\sigma_i}{S_m} \leq 1 \qquad (5)$$

The surface described by the limits of eq. (5) represents a critical surface ϕ_C A degenerate form of ϕ_C for the three-dimensional space is illustrated in Fig.1.

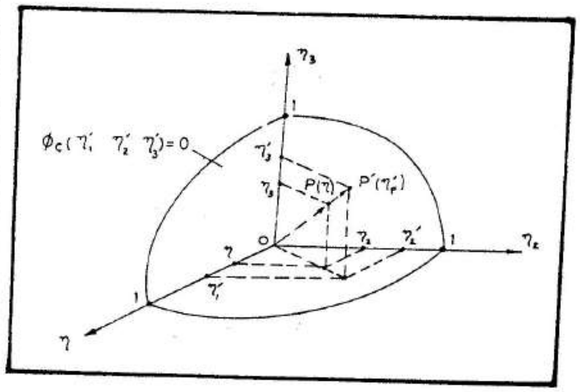

Fig. 1: Illustration of Surface of Critical stress intensity

In general, the stress intensity at a point of interest could reach the critical value as a result of a combination of N simultaneous loads acting on the structure; and would correspond to mixed-mode critical loads. Such a point, p' would lie on the critical surface. It is obvious that

$$\eta_p = \frac{\delta_p}{Z_p} = 1. \qquad (6)$$

The components of $_p$ can now be designated as fractions of the pure-mode critical values (Fig.1.1), i.e.,

$$(\;p\;)_i = \eta_i \qquad (7)$$

where $o \le \eta_i \le 1$ (8)

The surface of critical loads can be formally represented as

$$Q_c(\eta_1, \eta_2, \eta_3, \ldots \eta_N) = O \qquad (9)$$

For any non-critical stress intensity loading, $\eta_p < \eta_p$, and $\eta_i < \eta_i$.

Any functional relation can be chosen to represent Qc subject to numerical experimentation. For example, a polynomial could be chosen such that

$$\sum_{i=1}^{N} (\eta'_i)^{n_i} = 1 \qquad (10)$$

where, in general, $n_i \neq r_{i+1}$.

Simplified forms of the Envelope Equation

Eq. (10) could be simplified in several ways; one of which is to make the exponent n_i a constant n independent of load S_i. the particular value assigned to n could result from experience and/or numerical experimentation; and could depend on the nature of the structure being considered.

However, the simplest value that could be assumed for such experimentation is 1 and this would make eq. (10) take a linear, namely

$$\sum_{i=1}^{N} \eta_i = \eta_1 + \eta_2 + \eta_3 + \ldots \eta_N = 1 \qquad (11)$$

A less conservative integer value would be 2; and would result in a parabolic form of the equation, viz.,

$$\sum_{i=1}^{N} (\eta_i)^2 = (\eta_1)^2 + (\eta_2)^2 + (\eta_3)^2 + \ldots (\eta_N)^2 = 1 \qquad (12)$$

As stated above, other values could be assigned to n. For the present, only the forms expressed by eqs. (11) and (12) are examined.

Sample Application of the Envelope Equations

The foregoing results are now applied to a double volute (7 x 7 x 10m) recirculating coolant pump which would be analyzed as a class 1 nuclear component in accordance with the requirements of [3]. The aim is to determine the ability of the pump design to withstand specified service condition primary loads. The loads, which are in forms of forces and moments, act through the suction and discharge nozzles; and are specified in relation to a global coordinate system as shown in Fig.3. Eq. (10) can now be

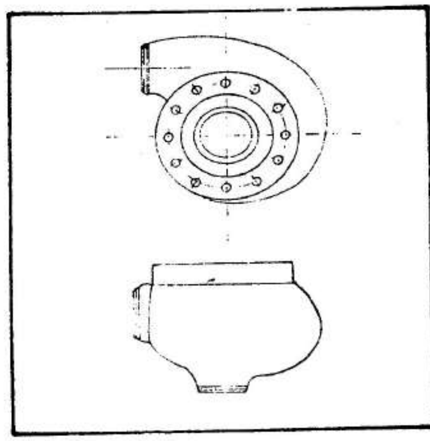

Fig.2: Top and Side View of Pump Casing

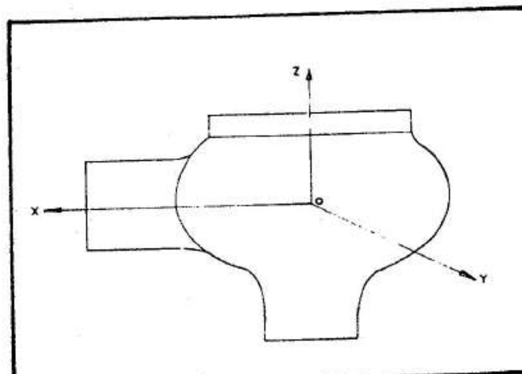

Fig.3: Model For Analysis Showing Coordinate System

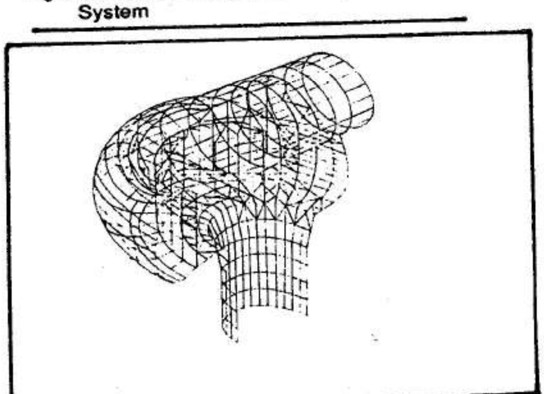

Fig.4:Isometric View of the Finite - Element Model

particularized for the present configuration as follows:

$$\sum_{i=1}^{3} \sum_{j=1}^{2} (\lambda_{ij}^{n} + \mu_{ij}^{n}) \le 1 \qquad (13)$$

where

$i = 1, 2, 3$ (the coordinates)

$$j = \begin{cases} 1, \text{ for suction nozzle} \\ 2, \text{ for discharge nozzle} \end{cases}$$

and

$$n = \begin{cases} 1, \text{ for linear envelope} \\ 2, \text{ for parabolic envelope} \end{cases}$$

In this instance, eq. (5) takes the form

$$ij = \frac{\S_{ij}}{Z_{ij}} \qquad \frac{\bar{\sigma}_{ij}}{S_m} \tag{14}$$

which yields

$$Z_{ij} = \frac{\S_{ij}}{\bar{\sigma}_{ij}} S_m \tag{15}$$

Model for Analysis

The pump casing is expected to be mostly affected by the primary loads of Table 2 since the closure region is relatively more rigid as a result of its structural form and support. These roads are not arbitrary; they, indeed, formed the original design bases of the pump. Fig.2 illustrates the full configuration of the casing while Fig. 3 shows the model selected for analysis.

Table 1*
Expected nozzle loads at diferent loading conditions

Load Designation	Loading Condition		
	Design, Normal & Emergency	Upset	Faulted
F_{11}	3.56	11.12	18.68
F_{21}	48.93	147.68	246.43
F_{31}	-51.15	-96.08	-141.01
M11	74.28	103.60	103.90
M21	85.35	193.32	193.31
M31	-11.91	-90.39	-168.88
F12	32.03	243.32	454.61
F22	-61.83	-154.80	-247.32
F32	84.96	154.35	224.19
M12	54.47	138.42	222.38
M22	-94.85	-120.58	-146.30
M32	62.83	294.35	525.87

* Forces are in kN; Moments are in kN-m

The selected model includes the discharge nozzle, the crotch region and one-half of the suction region. Continuity with the other half is simulated by the appropriate boundary conditions. The pump casing is constructed of high alloy steel, ASME SA-351 CF8M and the mechanical properties used are those specified in [3]

A finite element form of the selected model was constructed for computer code analyses. Three dimensional triangular and quadrilateral shell elements were used to describe the model boundaries. The finite element model was

Table 2
Summary of ASME limit criteria for nozzle loads

Loading Condition	Stress Limit Criteria
Design	$P_m \leq 1.0S_m$ *
	$P_L + P_b \leq 1.5S_m$
Normal and Upset	$(P_L + P_b + P_e + Q) \leq 3.0S_m$
	$P_m \leq (1.2\,S_m \text{ or } S_y)$
Emergency	$P_L \leq (1.8S_m \text{ or } 1.5S_y)$ *
	$P_L + P_b \leq (1.8S_m \text{ or } 1.5S_y)$ *
	$P_m \leq 2.4S_m \text{ or } 0.7S_y$ **
Faulted	$P_L \leq 1.5\,(2.4S_m \text{ or } 0.7S_u)$ **
	$P_L + P_b \leq 1.5\,(2.4S_m \text{ or } 0.7S_u)$ **

* Use greater of limits specified
** Use lesser of limits specified

subjected to primary unit loads in the forms expected during service conditions. The results obtained include, inter alia, stresses at three points across (inside, middle and outside) each element.

Envelope Equations for ASME Service Conditions

The primary task in the present exercise is to identify the most critical region(s) of the given structure. While experience could be helpful in identifying such regions, the results obtained from the unit load calculations would effectively identify the most highly stressed regions. In order to determine the limiting loads as expressed by eq. (15), it is necessary to know the stress intensity S_m whose value depends on the applicable criterion. Table 2 presents the relevant criteria for the different ASME loading conditions. The values in this table are to be used in conjunction with eq. (3) to obtain the applicable S_m.

Except for the Normal and Upset conditions which have one stress limit criterion, other conditions have two or more criteria (Table 2). Thus, it is conceivable that different regions (elements) may have to be considered for the different criteria under a given loading condition. The most conservative set of loads emanating from the different criteria is assumed to govern the loading condition and would lead to a set of limiting primary loads for each loading condition. The application of the foregoing to the pump casing under consideration leads to the limiting loads of Table 3.

Under an actual service condition, the particular set of applied loads would be expected to be less than the limiting set of loads. Table 3

Table 3
Limiting independent nozzle loads* that casing can withstand at different loading conditions

Load Designation	Design	Normal	Loading Upset	Condition Emergency	Faulted
F'_{11}	1,073.80	956.81	205.93	864.29	277,861.62
F'_{21}	1,024.43	457.28	4,922.67	868.74	69,304.16
F'_{31}	1,091.59	986.61	3,697.57	881.64	220,373.24
M'_{11}	309.10	233.61	51,898.57	246.94	18,275.64
M'_{21}	256.24	264.75	9,673.14	202.54	57,606.42
M'_{31}	296.05	302.95	45,213.68	234.45	19,744.36
F'_{12}	966.15	990.17	5,954.63	828.70	117,725.70
F'_{22}	717.94	458.17	1,143.74	484.41	21,231.80
F'_{32}	851.83	447.94	1,189.32	684.58	36,926.01
M'_{12}	277.42	265.10	69,712.98	214.42	16,777.97
M'_{22}	237.90	238.85	40,263.33	203.60	31,828.80
M'_{33}	306.70	268.66	294,573.68	229.82	11,657.23

* Forces are in kN, Moments are in kN-m

presents the postulated set of loads that the pump design is expected to carry. A comparison of the two tables shows that every load in Table 1 is less than the calculated limiting load of Table 3. Thus, when any set of loads in Table 1 acts on the pump, depending on the loading condition, the region (element) which produced the corresponding limiting loads of Table 2 would possess a stress intensity which is less than the critical value. The state of such an element is represented by point

In the present example, the component η_{ij} is given by

$$\eta_{ij} = Z_{ij} / Z_{ij} \qquad (16)$$

where Z_{ij} and Z_{ij} are obtained from Tables 1 and 3, respectively. Eq.(16) can now be used to evaluate expression (13) for the linear ($n = 1$) and parabolic ($n = 2$) forms. When using the data of Table 1 in eq. (16), the absolute value of

Table 4
Verification of the Linear form of the Envelope Equation (eq.11)

λ_{ij} or μ_{ij}	Loading Condition				
	Design	Normal	Upset	Emergency	Faulted
λ_{11}	0.0033	0.0037	0.0540	0.0041	-
λ_{21}	0.0478	0.1070	0.0300	0.0563	0.0036
λ_{31}	0.0469	0.0518	0.0260	0.0580	0.0006
μ_{11}	0.2403	0.3180	0.0020	0.3008	0.0057
μ_{21}	0.3331	0.3224	0.0200	0.4214	0.0034
μ_{31}	0.0402	0.0393	0.0020	0.0508	0.0066
λ_{12}	0.0332	0.0323	0.0409	0.0387	0.0039
λ_{22}	0.0861	0.1349	0.1354	0.1276	0.0116
λ_{32}	0.0997	0.1897	0.1298	0.1241	0.0061
μ_{12}	0.1963	0.2055	0.0020	0.2540	0.0132
μ_{22}	0.3967	0.3971	0.0030	0.4659	0.0046
μ_{32}	0.2049	0.2339	0.0010	0.2734	0.0451
$\Sigma\lambda_{ij} + \Sigma\mu_{ij}$	1.7305	2.0356	0.4461	2.1751	0.1064

Table 5
Verification of the Parabolic Form of the Envelope Equation (eq.12)

λ^2_{ij} or μ^2_{ij}	Loading Condition				
	Design	Normal	Upset	Emergency	Faulted
λ^2_{11}	-	-	0.0029	-	-
λ^2_{21}	0.0023	0.0114	0.0009	0.0032	-
λ^2_{31}	0.0022	0.0027	0.0007	0.0034	-
μ^2_{11}	0.0577	0.1011	-	0.0905	-
μ^2_{21}	0.1109	0.1039	0.0004	0.1776	-
μ^2_{31}	0.0016	0.0015	-	0.0026	0.0001
λ^2_{12}	0.0011	0.0010	0.0017	0.0015	-
λ^2_{22}	0.0074	0.0182	0.0183	0.0163	0.000L
λ^2_{32}	0.0099	0.0360	0.0168	0.0154	-
μ^2_{12}	0.0386	0.0422	-	0.0645	0.0002
μ^2_{22}	0.1590	0.1577	-	0.2170	-
μ^2_{32}	0.0420	0.0547	-	0.0747	0.0020
$\Sigma\lambda^2_{ij} + \Sigma\mu^2_{ij}$	0.4327	0.5304	0.0417	0.6667	0.0024

CONCLUSION

A comparison of the two sets of results show that some of the elements which satisfy the constraints of the parabolic criterion fail to meet the linear requirements, whereas such elements have been found to satisfy the relevant ASME requirements of Table 2. Thus, the results show that the linear form of the envelope equation is more conservative than the parabolic form.

In the specific case under consideration, the parabolic form of the equation shows that the pump design meets the ASME structural requirements of [3]. Hence, it can be concluded that the parabolic form of the envelope equation developed can provide preliminary design guidelines for structures like pump casing.

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