

MHD Flow of Rarefied Gas

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ABSTRACT

The problem of combined heat and mass transfer in natural convection flow of an incompressible, rarefield, viscoelastic fluid along infinite vertical porous plate with constant heat source under transverse magnetic field is considered. The perturbation technique is used to solve the differential equations. The formula for stress, rate of heat and mass transfer are obtained in the slip flow regime

NOMENCLATURE

x', y'	co-ordinate system
u', v'	velocities in x_1 and y_1 directions
B_0	external magnetic field
c', c', c'	species concentration, in the boundary layer, at the plate and away from the plate
C_p	Specific heat at constant pressure
D	chemical Molecular diffusivity
g	acceleration due to gravity
h_1	velocity slip parameter
h_2	temp. Jump coefficient
h_3	Concentration jump co-efficient
k	thermal conductivity
K_0	elastic constant
M	magnetic field parameter
p'	pressure
Pr	Prandtl Number
q'	rate of heat transfer

q_1	rate of mass transfer
S_c	Schmidt number
t'	time
T', T'_w, T'_∞	temp. of the fluid in the boundary layer, at the plate and away from the plate.
T_m	mean shear stress (non-dimensional)
V_0	suction velocity
ω	frequency
β	coefficient of volume expansion due to thermal diffusion
β^*	coefficient of volume expansion due to mass diffusion
G	a content.
ρ'	density of the fluid
σ	electrical conductivity
η_0	viscosity of the fluid
ν	kinematic viscosity
α	heat source parameter (dimensionless form).

INTRODUCTION

Because of the recent advent of supersonic air planes, rockets and missiles which can operate at high altitudes, considerable research has been under taken on fluid flow and heat transfer at high Mach numbers and in some cases with rarefied gases. Some interesting findings have been made, but these fields are still in the process of exploration and clarification.

The phenomenon of viscous incompressible gases when their density is slightly reduced due to low absolute pressure or an increase in temperature is called rare-fraction of the medium and hence there is accordingly same departure from continuum gas dynamics. No slip boundary conditions fail to describe such a flow and slip flow boundary conditions are suggested (Schaat

and Chambre, 1961). The first effects of gas rarefaction has been observed as a velocity slip and temperature jump at the plate and the flow regime is then called slip flow.

Inman (1965) investigated that in the case of an electrically conducting, visco-incompressible rarefied gas flowing between two stationary non-conducting walls, the velocity profiles, skin friction and rate of mass flow are affected by gas rarefaction. He conjectured that velocity gradient of the upper wall was unaffected by rarefaction of the medium. Some important contributions in this aspect have been given by street (1960) and Redely (1964).

The vertical natural convection flow resulting from these combined buoyancy mechanism have been studied in the past. In a series of papers Oldroyd (1958) proposed and studied, a set of constitutive equations for elastico-viscous fluids. Revlin and Ericksen (1956) formulated the nature of the boundary layer flow of visco-elastic fluids.

The survey of literature reveals that the combined effects of buoyancy forces from the thermal and mass diffusion on free convective heat and mass transfer for visco-elastic fluid in slip flow regime in presence of constant heat source have not been studied.

In the study, attention is directed to free convection flow of an incompressible rarefied visco-elastic fluid past an infinite vertical porous plate with con-

stant heat source in presence of uniform transverse magnetic field under the combined buoyancy force effects of thermal and mass diffusion.

The equations and boundary conditions used are limited to processes which occur at low concentration difference since the boundary conditions at the surface are assumed unaffected by interfacial velocities. Species thermal diffusion and species diffusion of thermal energy, some time important in gases are also neglected in the analysis.

MATHEMATICAL FORMULATION AND SOLUTION

The partial differential equation that represents the free convective flow of an incompressible, rarefied, visco-elastic fluid causes by the combined buoyancy effect of thermal and chemical species concentration disparity in the presence of uniform transverse magnetic field and constant heat source (absorption type) are written below in terms of fluid velocity u' , v' along x' -axis and y' -axis. All the fluid properties are considered constant except that the influence of density variation with temperature and concentration are considered only in the body force temperature. The equations are simplified by usual Boussinesq approximation and by the assumption that concentration of single diffusing species are small compared with other concentrations.

CONTINUITY EQUATION

$$\frac{\partial v'}{\partial y'} = 0 \quad \dots(1)$$

Momentum equations

$$\begin{aligned} \rho' \left(\frac{\partial u'}{\partial t'} + v' \frac{\partial u'}{\partial y'} \right) &= \rho' g \beta (T - T'_{\infty}) + \rho' g \beta^* (c' - c'_{\infty}) \\ &+ \eta_0 \frac{\partial^2 u'}{\partial y'^2} - 6B_0^2 u' - Ko \left[\frac{\partial^2 u'}{\partial y'^2} + v_0 \frac{\partial^3 u'}{\partial y'^3} \right. \\ &\quad \left. - 3 \frac{\partial u'}{\partial y'} \frac{\partial^2 v'}{\partial y'^2} - \frac{\partial v'}{\partial y'} \frac{\partial^2 u'}{\partial y'^2} \right] \quad \dots(2) \end{aligned}$$

$$\rho' \left(\frac{\partial v'}{\partial t'} + v' \frac{\partial v'}{\partial y'} \right) = - \frac{\partial p'}{\partial y'} + 2\eta_0 \frac{\partial^2 v'}{\partial y'^2} - 2Ko \left[\frac{\partial^3 v'}{\partial y'^3} \right]$$

$$-3 \frac{\partial v'}{\partial y'} \frac{\partial^2 u'}{\partial y'^2} + \frac{\partial^3 v'}{\partial y'^3} \quad \dots(3)$$

Energy equation

$$\rho' c_p' \left(\frac{\partial T'}{\partial t'} + v' \frac{\partial T'}{\partial y'} \right) = K' \frac{\partial^2 T'}{\partial y'^2} + \Phi' \quad \dots(4)$$

Mass diffusion equation

$$\left(\frac{\partial c'}{\partial t'} + v' \frac{\partial c'}{\partial y'} \right) = D \frac{\partial^2 c'}{\partial y'^2} \quad \dots(5)$$

The boundary conditions are

$$u' = L_1 \frac{\partial u'}{\partial y'}, T'_w = L_2 \frac{\partial T'}{\partial y'}, c' - c'_w = L_3 \frac{\partial c'}{\partial y'} \text{ at } y = 0 \quad \dots(6)$$

$$u' \rightarrow 0, T' \rightarrow T'_{\infty}, c' \rightarrow c'_{\infty} \text{ as } y \rightarrow \infty$$

Continuity equation (1) integrates to

$$v' = v_0 \quad \dots(7)$$

In the present investigation, we consider heat generation of the type $Q' = Q_0(T'_{\infty} - T')$, Vajravdu and Sastri (1978).

Now we introduce the following non-dimensional quantities

$$\eta = \frac{y'}{\nu}, t = \frac{v_0^2 t'}{4\nu^2}, u = \frac{u'}{v_0}, K = K + \frac{v_0^2}{\nu^2}$$

$$\theta_1 = \frac{T' - T'_{\infty}}{T'_w - T'_{\infty}}, \theta_2 = \frac{c' - c'_{\infty}}{c'_w - c'_{\infty}}, Pr = \frac{\eta_0 c_p'}{K}$$

$$Sc = \frac{\nu}{D}, G_1 = \frac{\nu g (T'_w - T'_{\infty})}{v_0^3}, G_2 = \frac{\nu g \beta^* (c'_w - c'_{\infty})}{v_0^3}$$

$$M = \frac{6 B_0^2 \nu}{\rho v_0^2}, \alpha = \frac{Q \nu^2}{v_0^2 K} \quad \dots(8)$$

Equation(1) to (5), in light of equation (7) and (8) are modified to

$$\frac{\partial^2 u}{\partial \eta^2} + \frac{\partial u}{\partial \eta} - \frac{1}{4} \frac{\partial u}{\partial t} = K \left(\frac{1}{4} \frac{\partial^3 u}{\partial \eta^2 \partial t} - \frac{\partial^3 u}{\partial \eta^3} \right) - Mu \quad \dots(9)$$

$$= -G_1 \theta_1 - G_2 \theta_2 \quad \dots(10)$$

$$\frac{\partial^2 \theta_1}{\partial \eta^2} + Pr \frac{\partial \theta_1}{\partial \eta} - \frac{Pr}{4} \frac{\partial \theta_1}{\partial t} - \alpha \theta_1 = 0$$

$$\frac{\partial^2 \theta_2}{\partial \eta^2} + Sc \frac{\partial \theta_2}{\partial \eta} - \frac{Sc}{4} \frac{\partial \theta_2}{\partial t} = 0 \quad \dots(11)$$

where $\nu = \eta_0/\rho'$, $K^* = \frac{K_0}{\rho}$

The modified boundary conditions are

$$\begin{aligned} u &= h_1 \frac{\partial u}{\partial n}, \theta_1 = 1 + h_2 \frac{\partial \theta_1}{\partial n}, \theta_2 = 1 + h_3 \frac{\partial \theta_2}{\partial n} \text{ at } \eta = 0 \\ u &\rightarrow 0, \theta_1 \rightarrow 0, \theta_2 \rightarrow 0 \text{ as } \eta \rightarrow \infty \end{aligned} \quad (12)$$

where

$$h_1 = \frac{L_1 v_1}{\nu}, \quad h_2 = \frac{L_2 v_0}{\nu}, \quad h_3 = \frac{L_3 v_0}{\nu}$$

In order to solve the coupled non-linear simultaneous equation (9) to (11), we assume that in the neighbourhood of the plate

$$\theta_1 = [1 - f_1(\eta)] + \epsilon e^{i\omega t} [1 - f_2(\eta)]$$

$$\theta_2 = [1 - g_1(\eta)] + \epsilon e^{i\omega t} [1 - g_2(\eta)]$$

$$u = u_0(\eta) + \epsilon e^{i\omega t} u_1(\eta) \quad (13)$$

Equation (9) to (11) in connection with equation (13) have the form neglecting the term containing K and its higher order, and equating the like power of K, we get

$$K u_0'''' + u_0'' + u_0' - M^2 u_0 = G_1 \theta_1 - G_2 \theta_2 \quad (14)$$

$$K u_1'' + (1 - K \frac{i\omega}{4}) u_1' + u_1' - (\frac{i\omega}{4} + M) u_1 = 0 \quad (15)$$

$$f_1'' + Pr f_1' - f_1 \alpha = -\alpha \quad (16)$$

$$f_2'' + Pr f_2' - (\frac{Pr i\omega}{4} + \alpha) f_2 = -(\frac{Pr i\omega}{4} + \alpha) \quad (17)$$

where prime denotes the differential with respect to η .

Solving equation (16) and (17)

$$f_1 = 1 - \frac{1}{h_2 H_2 + 1} \exp(-H_2 \eta), \quad f_2 = 1 \quad (18)$$

$$g_1 = 1 - \frac{1}{1 + h_3 S_c} \exp(-S_c \eta), \quad g_2 = 1 \quad (19)$$

Thus

$$\theta_1 = \frac{1}{1 + h_2 H_2} \exp(-H_2 \eta) \quad (20)$$

$$\theta_2 = \frac{1}{1 + h_3 S_c} \exp(-S_c \eta) \quad (21)$$

To solve equation (14) and (15) we again expand u_0 and u_1 in power of K, i.e.

$$u_0 = u_{01}(\eta) + K u_{02} + O(K^2) \quad (22)$$

$$u_1 = u_{11}(\eta) + K u_{12} + O(K^2) \quad \dots(23)$$

Substituting (22) and (23) in (14) and (15), equating the coefficient different power of K and neglecting the term containing the power K² and its higher order and using (20) and (21), we get the following equations

$$u_{o1}'' + u_{o1}' - M u_{o1} = - \frac{G_1}{1 + H_2 h_2} \exp(-H_2 \eta) - \frac{G_2}{1 + h_3 S_c} \exp(-S_c \eta) \quad \dots(24)$$

$$u_{o2}'' + u_{o2}' - M u_{o2} = - \mu''_{o1} \quad \dots(25)$$

$$u_{11}'' + u_{11}' - \left(\frac{i\omega}{4} + M\right) u_{11} = 0 \quad \dots(26)$$

$$u_{12}'' + u_{12}' - \left(\frac{i\omega}{4} + M\right) u_{12} = - u_{11}''' + i\omega u_{11}'' \quad \dots(27)$$

and the corresponding boundary conditions are

$$u_{o1} = h_1 u_{o1}', u_{o2} = h_1 u_{o2}', u_{11} = h_1 u_{11}', u_{12} = h_1 u_{12}' \quad \dots(28)$$

at $\eta = 0$

$$u_{o1} \rightarrow 0, u_{o2} \rightarrow 0, u_{11} \rightarrow 0, u_{12} \rightarrow 0 \text{ as } \eta \rightarrow \infty$$

Solving equation (24) to (27) under the boundary condition (28) and substituting it in (23) and (24) we get

$$u_o = B_3 \exp(-H_1 \eta) - B_1 \exp(-H_2 \eta) - B_2 \exp(-S_c \eta) + K [B_7 \exp(-H_1 \eta) + B_4 \eta - \exp(-H_1 \eta) - B_5(-H_2 \eta) + B_6 \exp(-S_c \eta)] \quad \dots(29)$$

and $u_1 = 0$

Now the shear stress at the plate in the slip flow regime (in non-dimensional form is

$$T_m = \left(\frac{\partial u}{\partial \eta}\right)_{\eta=0} = -H_1 (B_3 + K B_7) + K B_4 + H_2 (B_1 + K B_5) + S_c (B_2 + K B_6) \quad \dots(30)$$

The rate of heat transfer at the plate in non-dimensional form is

$$q = \left(\frac{\partial \theta_1}{\partial \eta}\right)_{\eta=0} = \frac{-H_2}{1 + H_2 h_2} \quad \dots(31)$$

The rate of mass diffusion in non-dimensional form is

$$q_1 = \left(\frac{\partial \theta_2}{\partial \eta}\right)_{\eta=0} = - \frac{S_c}{1 + S_c h_3} \quad \dots(32)$$

RESULTS AND DISCUSSION

To study the effect of constant heat source on velocity of newtonian and non-newtonian rarefied gas, some calculations are carried out. Fig.1 and Fig.2 show the velocity profiles of newtonian and non-newtonian rarefied gas against y with respect to constant heat source of absorption type. From Fig.1 and Fig.2 we achieve an important conclusion that there exists an inverse relation between α and u where α is a heat source parameter. There is minute change in velocity profiles of Newtonian and non-newtonian rarefied gas for same value of α .

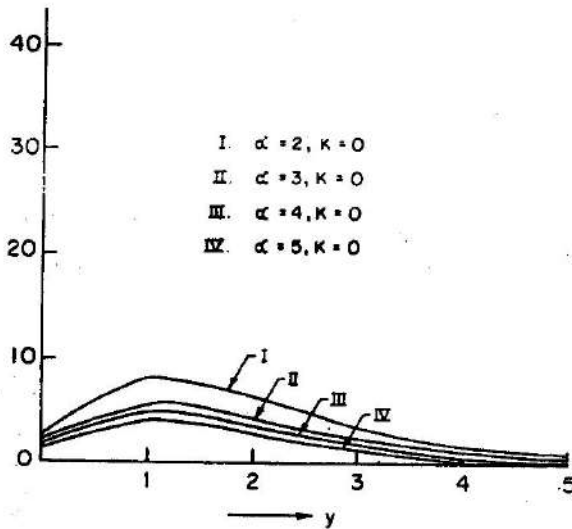


Figure 1: Newtonian rarefied gas

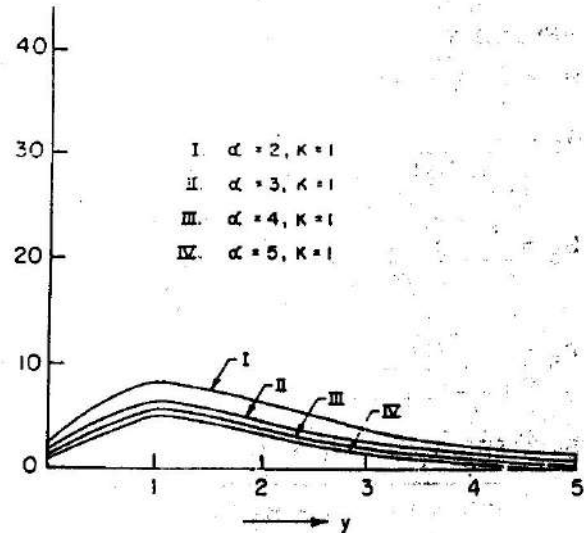


Figure 2: Non-Newtonian rarefied gas

Fig.3 and Fig.4 describe the nature of velocity profiles in case of newtonian - non rarefied and Non-newtonian - non rarefied gas against y in presence of constant heat source. From Fig. 3 and Fig.4 it is observed that there is an also inverse relation between α and u .

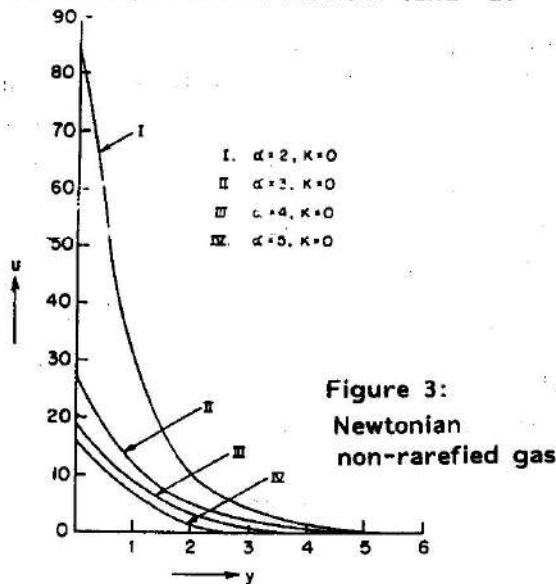


Figure 3: Newtonian non-rarefied gas

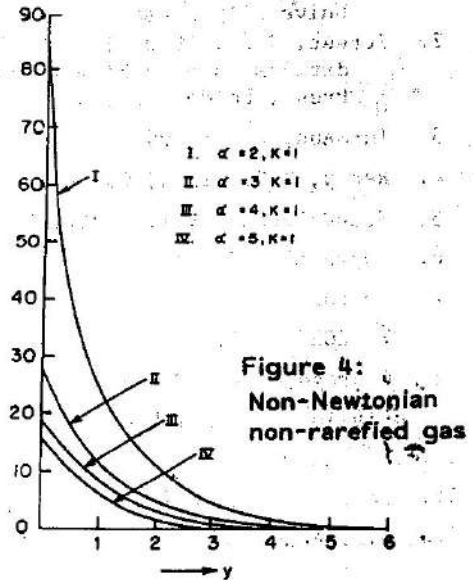


Figure 4: Non-Newtonian non-rarefied gas

CONCLUSION

In the case of newtonian rarefied gas and non-newtonian rarefied gas the velocity profiles first increases then decreases but in the case of Newtonian non-rarefied and non-Newtonian rarefied gas velocity profiles decreases continuously.

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APPENDIX

$$H_1 = \frac{1}{2} [1 + (4M + 1)^{\frac{1}{2}}], \quad H_2 = \frac{1}{2} Pr + \sqrt{Pr^2 + 4\alpha}$$

$$B_1 = \frac{C_1}{(1+H_2 h_2)(H_2^2 - H_2 - 1)}, \quad B_2 = \frac{G_2}{(1 + S_c h_3)(S_c^2 - S_c - M)}$$

$$B_3 = \frac{B_1(h_1 H_2 + 1) + B_2(1 + S_c h_1)}{1 + H_1 h_1}, \quad B_4 = \frac{B_3 H_1^3}{1 - 2H_1}$$

$$B_5 = \frac{H_2^3 B_1}{(H_2^2 - H_2 - M)}, \quad B_6 = \frac{S_c^3 B_2}{(S_c^2 - S_c - M)}$$

$$B_7 = \frac{(H_2 h_1 + 1)H_5 + B_6(1 + S_c h_1) + B_4 h_1}{1 + H_1 h_1}$$

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