MAP COORDINATE REFERENCING AND THE USE OF GPS DATASETS IN GHANA

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ABSTRACT
The concepts of coordinate systems required to identify points in space and represent them on maps uses mathematical methods of assigning numbers, called coordinates to each point in space. There are different coordinate systems the commonest being the system of latitudes, longitudes and ellipsoidal heights. The situation is complicated because the latitudes and longitudes of the same point differ slightly depending on the geodetic coordinate system of a country, the result being that different systems of latitude and longitude in use for the same point can disagree in coordinates by more than 200 metres. The GPS for instance uses a WGS84 system and gives latitude and longitude values which can not be integrated directly into the mapping system of a country without suitable mathematical conversions. Similarly, it is not possible to just measure distances from a graticule map without appropriate projection conversions. In order to use data obtained from GPS measurements correctly and effectively in Ghana, we need to use appropriate transformation parameters that relate our Ghana National Survey Mapping coordinate system to that used for the Global Positioning System (GPS). Datum transformation parameters define functional relationship between two reference frames. This paper looked at geodetic coordinate systems and transformations between the WGS84 and the coordinate systems used in Ghana (the Ghana war office system and also the Clarke1880 system) using the Bursa-Wolf model. Particular attention was given to the derivation of ellipsoidal heights through the use of the Abridged Molodensky formulas.

Keywords: *Coordinate Systems, ellipsoidal heights, Global positioning System, WGS84, Transformation parameters*

INTRODUCTION

The concepts of coordinate systems are required in order to identify points in space and to represent them on maps. Position recognition is done by using a mathematical method of assigning numbers, called coordinates to each point in space. There are different coordinate systems the commonest being the system of latitudes, longitudes and ellipsoidal heights.

René Descartes (1596-1650) introduced a system of coordinates based on orthogonal axes often referred to as a Cartesian system. This system charts a point based upon its location along intersecting axes, denoted as X , Y and Z axes. On a global scale, position can be more accurately located using spherical coordinates, in terms of latitude, longitude, and altitude.

These concepts are used to convey information on maps, so to use map data correctly and effectively, we need to understand:

- The basic concepts of the terrestrial coordinate systems[1];
- The coordinate systems used for our Ghana National Mapping.
- The coordinate systems used for the Global Positioning System (GPS); and how these two relate to each other.

Knowledge of these concepts would help answer the following questions;

- Why do coordinate systems use ellipsoids and why are there so many different ellipsoids?
- Is it possible to convert coordinates from one ellipsoid to another?
- Is an ellipsoid the same as a datum?
- What is the difference between height above mean sea level and height above an ellipsoid?
- How is the National Survey Grid defined and how are grid references converted to latitude and longitude coordinates?
- Why should it be difficult to relate the National Grid coordinates to GPS coordinates?
- Exactly what is WGS84 and how does it relate to map coordinates in Ghana?
- Is it possible to relate GPS heights to mean sea level (orthometric) heights?
- In order to answer these questions, it must be understood that:

[1]*A terrestrial coordinate system is a coordinate system designed for describing the positions of objects on the land surface of the Earth.*

A point on the ground has no one unique latitude and longitude because the latitude and longitude of the same point differ slightly depending on the geodetic coordinate system of each country with the result that, different systems of latitude and longitude in use can disagree on the coordinates of a point by more than 200 metres. Fig.1. below shows three points which have the same latitude and longitude, in three different coordinate systems- Ghana Grid called war office, WGS84 and '*Clarke1880*'. Each one of these coordinate systems is used in Ghana but the differences between them are a result of the different ellipsoids used for their definition.

A horizontal plane is not necessarily a level surface because the Earth is round and so any gravitationally level surface must curve as the Earth curves. A level surface is everywhere at right angles to the direction of gravity. This gravity direction is generally towards the centre of the Earth, but varies in direction and magnitude from place to place in a complex way due to the irregular distribution of mass and the variable density of the Earth so all level surfaces are actually bumpy and complex.

The shape of the Earth

The simplest mathematical approximation of the shape and the size of the Earth, on which the coordinate systems are based are *ellipsoids* and *Geoids*. The geometric shape which most closely approximates the shape of the earth is an *ellipsoid*. Because the ellipsoid shape doesn't fit the Earth perfectly, there are lots of different ellipsoids in use, some of which are designed to best fit the whole Earth, and some to best fit just one region. So the various ellipsoids used in different regions differ in size, shape or orientation. (Jackson, 1980; Smith, 1997)

For height measurements, a reference 'zero height' surface is required. This height reference surface must be a level surface and for it to be used worldwide, must be a closed shape like the shape of an ellipsoid. The level surface which is closest to the average surface of all the world's

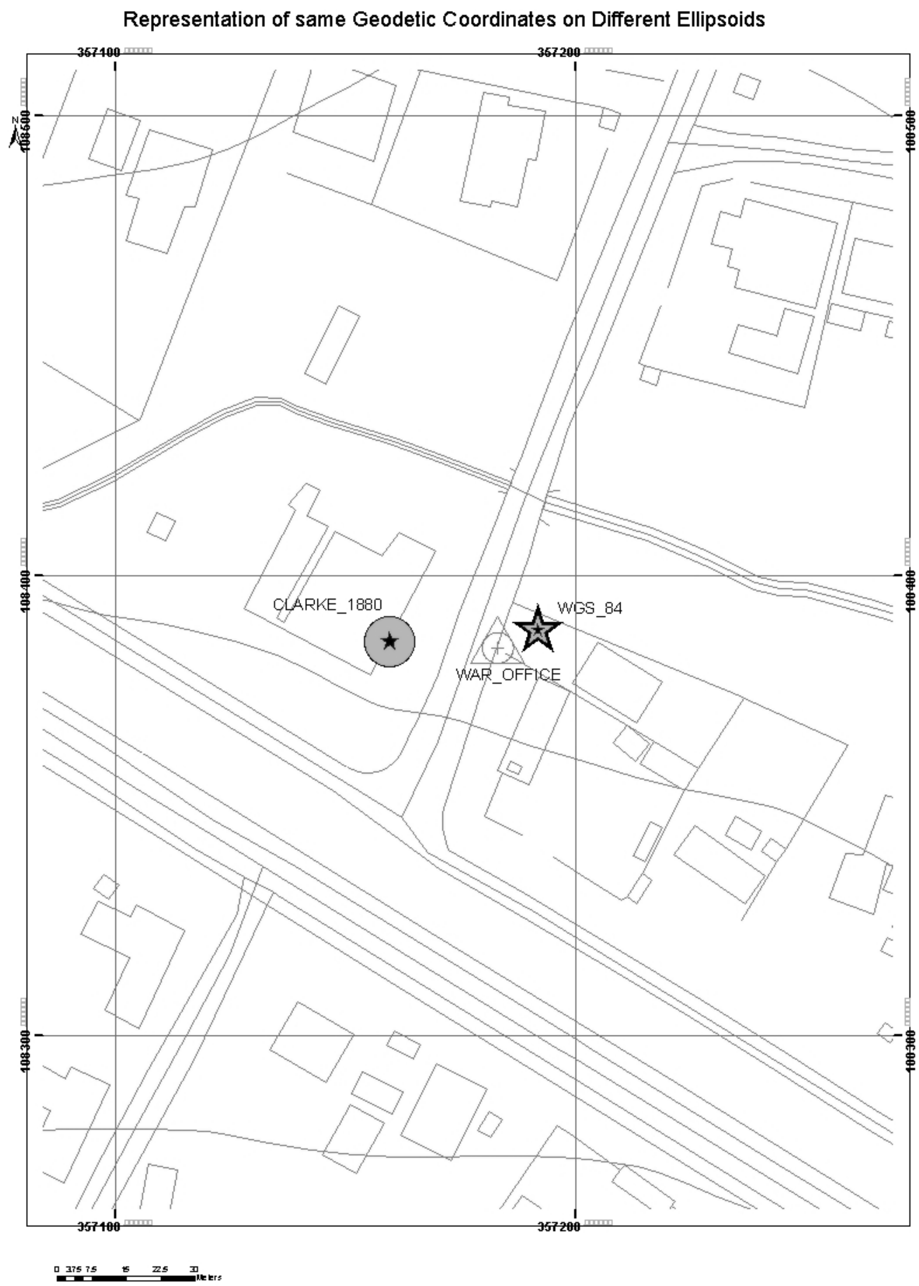

Fig. 1: Representation of same latitude and longitudes on different ellipsoids

oceans is the *Geoid and is chosen as the reference level surface* because it has the property that every point on it has exactly the same height, throughout the world. Height measurements on maps are however stated to be *height above mean sea level* determined by tide-gauge observations in the local mapping region rather than the average of global ocean levels. These local reference surfaces can be considered to be parallel to the Geoid but offset from it, because of local oceanic currents, effects of tides, and variations in water temperature and purity. (Deakin, 1996).

Positioning

To describe with certainty where we are on that Earth, or where any feature is in a simple numerical way, a coordinate system is defined within which each position or topographic feature has an unambiguous set of numbers. Position then means a set of coordinates in a clearly defined coordinate system.

The two types mainly used are:

Latitude, longitude and ellipsoid height: - Latitudes and longitudes define points on the surface of a stated ellipsoid which approximately fits the globe. Since real points on the ground are actually above (or possibly below) the ellipsoid surface, a third coordinate, called *ellipsoid height* is added.

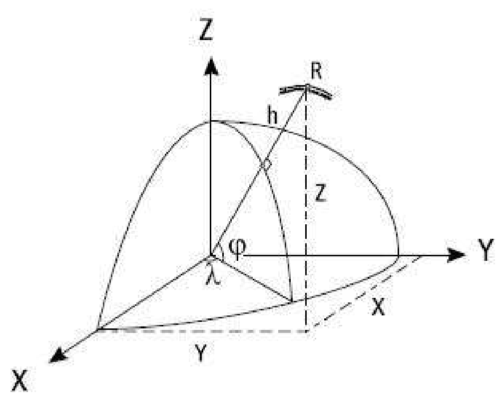

Fig. 2: Ellipsoid surface showing latitude, longitude and ellipsoidal height

Φ = latitude
λ= longitude
h = height above ellipsoid.

Cartesian coordinates:- Rectangular Cartesian coordinates describe positions in three dimensions, using three perpendicular axes X, Y and Z. These can be used as an alternative to latitude, longitude and ellipsoid height system. The origin of the Cartesian system is at the centre of the ellipsoid. Any position uniquely described by latitude, longitude and ellipsoid height can also be described by a unique 3-D Cartesian coordinates, and vice versa.

The forward transformation from geodetic coordinates **(***Ø*,*λ*, h**)** to an earth centre Cartesian coordinates (X,Y,Z) is given in Heiskanen and Moritz (1967) as:

$$X = (v + h) \cos Ø \cos \lambda \qquad (1)$$

$$Y = (v + h) \cos Ø \sin \lambda \qquad (2)$$

$$Z = (v (1 - e^2) + h) \sin Ø \qquad (3)$$

Where the prime vertical radius of curvature (v) is:

$$\upsilon = \frac{a}{\left(1 - e^2 \sin^2 \phi\right)^{1/2}} \qquad (4)$$

With 'a' the semi-major axis of the ellipsoid, 'e' the first eccentricity of the ellipsoid.

The non-iterative reverse transformation from Cartesian coordinates (X,Y,Z) to geodetic coordinates (φ, λ, h) is also given by Bowring (1985) as:

$$\varphi = \arctan\left[\frac{Z + \varepsilon^2 b \sin^3 u}{\mathrm{P} - e^2 a \cos^3 u}\right] \qquad (5)$$

$$\lambda = \arctan\left[\frac{Y}{X}\right] \qquad (6)$$

$$h = P\cos\varphi + Z\sin\varphi - a\sqrt{1 - e^2 Sin^2\varphi} \qquad (7)$$

where $u = \arctan\left[\frac{a\mathrm{Z}}{bP}\right]$

$$P = \sqrt{X^2 + Y^2} \quad \varepsilon = \frac{e^2}{1 - e^2} \qquad (8)$$

With 'u' as the parametric latitude, 'b' the semi-minor axis of ellipsoid, and 'ε' the second eccentricity.

Geoid height and ellipsoidal height:- The distance above a reference ellipsoid does not necessarily indicate height in the real sense of the word because the ellipsoid surface is not a level surface. To ensure that the relative heights of points correctly indicate the height gradient between them, we must measure height as the distance between the ground and the Geoid and not the ellipsoid. This measurement is called the 'orthometric height'. Survey contours, spot heights and bench mark heights on maps are orthometric heights. The relationship between ellipsoid height *h* and Geoid height *H is:*

$$h = H + N \qquad (9)$$

where *N* is the Geoid-ellipsoid separation. (Hofmann-Wellenhof *et al* ,1997).

Because the Geoid is a complex surface, *N* varies in a complex way but if we can develop a model for N, we would obtain a geoid model that could permit a determination of orthometric heights from ellipsoid heights.(Leick, 1995).

Grid coordinates (Eastings and northings)

Eastings and *Northings*, are used to locate positions on a map, which is a two-dimensional plane surface depicting features on the curved surface of the Earth. Map coordinates use a 2-D Cartesian system in which the two axes are known as northings and eastings. They are computed from the ellipsoidal latitude and longitude by a standard formula known as a *map projection.* (Snyder, 1987; Bugayevskiy *et al.*,1995).

The map projection used in Ghana for mapping is the Transverse Mercator projection.(Survey Records, 1936). This result from projecting the globe onto a cylinder tangent to a central meridian resulting in a mesh of longitude and latitude lines superimposed on its plane surface when cut opened. This network of latitudes and meridians formed is called a graticule. Such a graticule map is not suitable for measurement of distances between points directly since the distance between same meridians for example is not constant in different latitudes. Mathematical projection formulae are available to convert the graticule maps to rectangular grids. The formulae for the Transverse Mercator projection are given in Pearson, 1990 as:

Transverse Mercator mapping from Ellipsoid to the plane

$$\frac{E}{k_O v} = \lambda\cos\varphi + \frac{\lambda^3\cos^3\varphi}{6}\left(1 - t^2 + \eta^3\right) + \frac{\lambda^5\cos^5\varphi}{120}\left(5 - 18t^2 + t^4 + 14\eta^2 - 58t^2\eta^2\right) \qquad (10)$$

$$\frac{N}{k_O v} = \frac{S}{v} + \frac{\lambda^2}{2}\sin\varphi\cos\varphi + \frac{\lambda^2}{24}\cos^3\varphi\left(5 - t^2 + 9\eta^2 + \eta^4\right) + \frac{\eta^6}{720}Sin\varphi\cos^5\varphi\left(61 - 58t^2 + t^4 + 270\eta^2 - 330t^2\eta^2\right) \qquad (11)$$

Transverse Mercator Inverse Mapping from Plane to Ellipsoid

$$\varphi = \varphi f - \tfrac{1}{2}\left(1 + \eta^2\right)\left[\frac{E}{k_o v}\right]^2 + t/24\left(5 + 5t^2 + 6\eta^2 - 6\eta^2 t^2 - 3\eta^4 - 9t^2\eta^4\right)\left[\frac{E}{k_o v}\right] - t/720$$
$$\left(61 + 90t^2 + 45t^4 + 107\eta^2 - 162t^2\eta^2 - 45t^4\eta^2\right)\left[\frac{E}{k_O v}\right]^6 \qquad (12)$$

$$\lambda = \sec\ \varphi f\left(\left[\frac{E}{k_0 v}\right] - \frac{1}{6}\left[\frac{E}{k_0 v}\right]^3\right)\left(1 + 2t^2 + \eta^2\right) + \frac{1}{120}\left[\frac{E}{k_0 v}\right]^5(5 + 28t^2 + 24t^2 + 6\eta^2 + 8t^2\eta^2)) \qquad (13)$$

where

$t = \tan \varphi f$, φf Is the footpoint latitude.

$$\eta^2 = \frac{e^2}{1-e^2}\cos^2\varphi, \quad S = \frac{a}{1+n}\left(a_0\varphi - a_2\sin 2\varphi + a_4\sin 4\varphi - a_6\sin 6\varphi + a_8\sin 8\varphi\right)$$

$$a_0 = 1 + \frac{n^2}{4} + \frac{n^4}{64},\ a_2 = \frac{3}{2}\left(n - \frac{n^3}{8}\right),\ a_4 = \frac{15}{16}\left(n^2 - \frac{n^4}{4}\right),\ a_6 = \frac{35}{48}n^4\ \ a_8 = \frac{315}{512}n^4,\ \ n = \frac{f}{(2-f)}$$

Different National Mapping Departments put these formulae in suitable forms and develop tables for ease of the computations.

Datums

For each coordinate system used, an origin is defined with respect to which it is stated. This definition is called a datum[2] and consists of the origin point of the X,Y,Z, axis (three parameters), the orientation of the axes (three parameters), the ellipsoid used and its shape and size (two parameters) (a, f). The complete datum definition then consists of eight parameters- the 3-D location of the origin (three parameters), the 3-D orientation of the axes (three parameters), the size of the ellipsoid (one parameter) and the shape of the ellipsoid (one parameter). (Harvey, 1986; Steed, 1990).

Terrestrial Reference Frameworks (TRF)

For a coordinate system based on a defined datum to be used over a large area, an infrastructure of points to which users can have access and whose coordinates are known is set up. Such a framework is a geodetic Reference framework. These reference points are either on the ground or on satellites orbiting the Earth and all positioning methods rely on line-of-sight from an observing instrument to such reference points of known coordinates.

Putting some of the reference points on orbiting satellites makes them visible to a large area of the Earth's surface at any one time to be used for global positioning (Leick, 1995).

GPS COORDINATE SYSTEMS

The datum used for GPS positioning is called WGS84 (World Geodetic System, 1984). It consists of a three-dimensional Cartesian coordinate system and an associated ellipsoid. WGS84 positions can be described as XYZ Cartesian coordinates or latitude, longitude and ellipsoid height coordinates. The WGS84 definition (D.M.A., 1987; 1991) includes the following items:

- The WGS84 Cartesian axes and ellipsoid are geocentric; that is, their origin is the centre of mass of the whole Earth including oceans and atmosphere.
- The orientation of the axes, and hence the orientation of the ellipsoid equator and prime meridian of zero longitude coincided with the equator and prime meridian of the Bureau Internationale de l'Heure at the moment in time 1984.0 -that is, midnight on New Year's Eve 1983.
- Since 1984.0, the orientation of the axes and ellipsoid has changed but the average motion of the crustal plates relative to the ellipsoid is zero.
- The shape and size of the ellipsoid is defined by the semi-major axis length a = 6378137.0 metres, and the reciprocal of flattening 298.257223563. This ellipsoid is the same shape and size as the GRS80 ellipsoid designed to best-fit the Geoid of the Earth as a whole.

[2]*This term is often misused. The term datum refers only to the arbitrarily chosen elements of a coordinate system necessary to define the origin of coordinates - not to any control network based on this.*

COORDINATE REFERENCING IN GHANA

The Ghana National framework consists of monuments planted at points whose coordinates are well known and have assigned coordinates to become control stations in the framework net. National geodetic controls in Ghana dated back to June 1904, with observations for latitude taken by Captain P. G. Guggisberg from a pillar established in the house of the then Secretary of Native affairs. This pillar was subsequently connected by traverse to GCS 547 in Accra and the longitude of GCS 547 determined by telegraphic signal exchange with Cape Town in November 1904 to establish both latitude and longitude values for GCS547. Pillar GCS 547 was connected to GCS 121 at Legon by triangulation and thence the latitude and longitude of GCS 121 computed. The value obtained was accepted to be used for further computations in Ghana.

To provide a proper network of framework points as the basis for cadastral surveys throughout the whole country, the nature of the country necessitated the judicious blend of triangulation and traversing. Triangulation was done in the southern areas where there were high mountains and hills that afforded long sights whereas traversing was done in the Northern territories and other low lying regions. To control bearings, supplementary observations were also done at various times, for latitude and longitude at triangulation points to obtain a series of comparison data. Most of the triangulation was completed between 1926 and 1929. (Survey Records, 1936)

Heights in Ghana are based on the mean sea level determined in Accra by means of tidal observations from 9th April 1922 to 30th April 1923. In order to transfer the heights to other established monuments, a double line of spirit levels was run to connect pillar GCS 121 and this gave a reduced level of H as 483.80 feet to GCS 121. Trigonometric heights were consequently involved in the framework computation with the observed vertical heights weighted in inverse proportion to the respective lengths of the lines to obtain orthometric heights for the trigonometric points (Survey Records, 1936).

Because of the limitations in computational aids at the time of the original triangulation, the triangulation net was adjusted in smallish figures so that homogeneity or lack of homogeneity for the whole network could not be guaranteed. The primary traverses similarly were very good individually but could be out of sympathy collectively.

Surveying computations are much simpler if plane projected coordinates of Easterns and Northerns are used in place of the spherical coordinates φ and λ. For deducing such coordinates, Ghana uses a national horizontal datum that uses an ellipsoid which was suggested by the British war office and has subsequently come to be known as the “war office” ellipsoid. The parameters of this ellipsoid are a=6378299.99 m., b=6356751.69 m. and inverse flattening 296 with a feet to meter conversion factor of 0.304799706846. For the computation of planimetric coordinates, the Transverse Mercator projection is used for Ghana with the whole country placed at the same origin of the intersection of longitude 1^{o} W and latitude 4^{o} 40’ N. The values assigned this origin are 0.000ft. for northerns and 900,000.000ft. for easterns and a scale factor along central meridian of 0.99975. The *National Grid* is therefore a horizontal coordinate system, consisting of a geodetic datum that uses the war office ellipsoid called Accra Datum. The realization of this TRF is established using trigonometric pillars and a Transverse Mercator projection for deriving easting and northing coordinates. This is the coordinate system used to indicate positions of features on all Ghana Survey Maps. (Survey Records, 1936).

Re-adjustment of the Ghana framework and projection on Clarke 1880

In 1977, the triangulation and traverse framework was supplemented with the inclusion of secondary triangulations and additional traverses and re-adjusted together using variation of Geographical coordinates. Observed astronomical azimuths were also included in the new adjustment with the existing latitude and longitude value for Legon GCS 121 accepted as error-free for the re-computation. Also the coordinates were computed on the original Transverse Mercator projection using the same origin being the intersection of latitude 4°40'N and longitude 1°W with grid values of 0.00 for Northern and 900000 feet for Eastern but the projection was done using the Clarke 1880 ellipsoid so that Ghana can be brought in line with the rest of the African nations who already use this ellipsoid. The re-computation was completed in August 1978 (Survey Records, 1978) and has come to be known as the "Leigon datum". Even though the re-computation and adjustment of coordinates on Clarke 1880 was completed in August 1978, the results have never been put into the public domain by the Ghana Survey Department. Instead, the Department had continued to maintain its coordinates and publish maps in the original War office ellipsoid. The positional shift between the two different ellipsoidal projected coordinates is on the average of 25 feet (Table 1.). An un-informed usage of both without the provision of conversion formulae between the two series and a clear annotation difference between plans in both frames would invariably result in the creation of two parallel map series at large scale with overlaps which may not necessarily exist on the ground.

GEODETIC TRANSFORMATIONS BETWEEN COORDINATE SYSTEMS

A geodetic transformation is a mathematical operation which takes the coordinates of a point in one coordinate system and computes the coordinates of the same point in a second coordinate system. Many types of mathematical models are used to accomplish this task of deducing a best *estimate* of the transformation between the two TRFs by using points which have known coordinates in both systems.

The ways in which two ellipsoidal datums can differ are in:

- position of the origin of coordinates;
- orientation of the coordinate axes;
- Size and shape of the reference ellipsoid.

Table 1:. Sample comparison of clark1880 and war office TM coordinates

	CLARK 1880 COORDINATES		WAR OFFICE CO-ORDINATES		
Points	**X**	**Y**	**X**	**Y**	**Position change**
GCS101	234385.64	981191.49	234410.91	981195.24	25.5467297
GCS102	222439.57	996467.52	222464.16	996471.72	24.9461039
GCS107	271196.46	1079443.06	271221.57	1079447	25.4219
GCS110	315972.99	1126449.56	315997.57	1126452.9	24.7992036
GCS112	335947.62	1144337.64	335971.73	1144340.3	24.2584934
GCS113	353177.93	1160808.82	353201.36	1160835.60	35.5827669
GCS116	322784.00	1171268.00	322808.15	1171270.1	24.2446303
GCS121	356061.16	1192115.80	356084.33	1192117.9	23.2658763
GCS141	649826.78	971839.51	649852.77	971841.16	26.0423232
CFP162	490814.13	858919.39	490834.70	858922.00	20.7349222
CFP180	502120.47	795977.15	502139.98	795978.88	19.5865515
CFP217	461968.30	998067.45	461992.40	998070.31	24.2691079
CFP184	653805.59	647799.76	653823.60	647795.79	18.4423697
CFP213	529101.25	991064.37	529124.19	991066.89	23.0779982
CFP153	449952.65	1285361.85	449974.09	1285363.7	21.515427
CFP201	265160.44	1004563.19	265185.99	1004567.4	25.8977876
CFP152	549970.11	1279089.30	549992.55	1279092.3	22.6383237

The abridged Molodensky datum transformation formulae, given in equations 14-16 (Molodensky *et al*., 1962) provides a simple one-stage procedure for converting the geodetic latitude and longitude of the points in one datum directly to the second datum. It also provides a suitable model from which the shift parameters could be derived.

$\Delta Ø''1/\rho\sin1\rho''[-\Delta X\sin Ø_1\cos\lambda_{-1}-\Delta Y\sin Ø$
$\sin\lambda_1+\Delta\cos Ø_1+(a_1\Delta f+f_1\Delta a)\sin 2Ø_1]$ (14)
$\Delta\lambda''=1/v\cos Ø_1 \mathrm{Sin}1''[-\Delta X\sin\lambda_1+\Delta Y\cos]$ (15)
$\Delta h = \Delta X \cos Ø_1\cos\lambda_1+\Delta Y \cos Ø_1\sin Ø_1 +$
$\Delta Z \sin Ø_1 + (a_1\Delta f + f_1\Delta a)\sin^2 Ø_1 - \Delta a$ (16)

Where

$Ø_1,\lambda_1,h_1$ = geodetic coordinate in first datum
$Ø_2,\lambda_2,h_2$ = corresponding coordinate in second datum
a_1,f_1 = ellipsoidal parameter (first datum)
Δa, Δf = the difference between the ellipsoid Parameters
v = the radius of curvature in the prime vertical.

$$\upsilon = \frac{a}{\left(1-e^2\sin^2\phi\right)^{1/2}}$$

ρ is the radius of curvature in the meridian

$$\rho = \frac{a\left(1-e^2\right)}{\left(1-e^2\sin^2\phi\right)^{3/2}}$$

e is the eccentricity of the ellipsoid

$$e^2 = 2f - f^2$$

MATERIALS AND METHODS FOR DERIVING DATUM TRANSFORMATION PARAMETERS

The longitudes ($\lambda_{WGS\ 84}$), latitudes ($Ø\text{-}_{WGS\ 94)}$) and ellipsoidal heights (h_{WGS84}) are obtained from GPS observations at the trigonometric stations whereas the corresponding longitude and latitude values for the same stations in the war office and also the Clarke ellipsoids ($\lambda_{warOffice}$), ($Ø_{waroffice}$), (λ_{clerke}), ($Ø_{clarke}$) are available from Survey records. The

Table 2: Obtained Geodetic coordinates for test stations (war office)

	WGS84		WAR OFFICE	
POINTS	LAT	LONG	LAT	LONG
CFP 155	5° 56’ 20.49553	0° 07’ 19.27312	5° 56’ 10.4880	0° 07’ 20.2460
GCS 124	5° 49’ 09.95049	0° 08’ 40.1798	5° 48’ 59.8086	0° 08’ 41.1823
GCS 125	5° 45’ 58.96239	0° 03’ 54.58319	5° 45’ 48.9299	0° 03’ 55.6264
GCS 101	5° 18’ 56.68755	0° 46’ 35.10575	5° 18’ 46.4840	0° 46’ 36.0480
GCS 102	5° 16’ 57.90578	0° 44’ 03.84429	5° 16’ 47.8500	0° 44’ 04.8400

Table 3: Obtained Geodetic coordinates for test stations (Clarke 1880)

	WGS84		CLARKE 1880	
POINTS	LAT	LONG	LAT	LONG
CFP 155	5° 56’ 20.49553	0° 07’ 19.27312	5° 56’ 10.5671	0° 07’ 20.2456
GCS 124	5° 49’ 09.95049	0° 08’ 40.1798	5° 48’ 59.8597	0° 08’ 41.1797
GCS 125	5° 45’ 58.96239	0° 03’ 54.58319	5° 45’ 48.9666	0° 03’ 55.6223
GCS 101	5° 18’ 56.68755	0° 46’ 35.10575	5° 18’ 46.3848	0° 46’ 36.0797
GCS 102	5° 16’ 57.90578	0° 44’ 03.84429	5° 16’ 47.7497	0° 44’ 04.8751

values used in this study are shown in Tables 2 and 3.

These values were substituted in the equations 14. and 15, and then using least squares the equations were solved for ΔX, ΔY and ΔZ. The transformation shift parameters obtained from this solution for WGS84 and the war office and also for WGS84 and CLARKE 1880 ellipsoid are shown in Tables 4 and 5.

These (ΔX, ΔY, ΔZ) are substituted in equation 16 to obtain Δh for the various triangulation points from which the ellipsoidal heights could be determined.

Table 4: Transformation shift parameters from WGS 84 and War office

Parameters	Values	Units
ΔX	-241.871	Meters
ΔY	32.560	Meters
ΔZ	318.803	Meters
Δf	$-5.4707\text{x}10^{-5}$	Radians
Δa	-112	Meters
a	6378137	Meters
f	1/298.2572	Radians

Table 5: Transformation shift parameters from WGS 84 and Clarke 1880

Parameters	Values	Units
ΔX	-129.955	Meters
ΔY	32.2067	Meters
ΔZ	366.423	Meters
Δf	$-5.4707\text{x}10^{-5}$	Radians
Δa	-112	Meters
a	6378137	Meters
f	1/298.2572	Radians

For the purpose of converting the orthometric heights (H) to their corresponding ellipsoidal heights (h) on the war office and also the Clarke 1880 spheroid, the relationship between orthometric and ellipsoidal heights (h = H+ N) is used. Using the GPS heights h_{WGS84} and orthometric heights H in the relationship, Geoid heights (N_{WGS84}) are obtained on WGS84. The war office geoid heights (N_{war}) were computed from:

$$N_{war} = N_{WGS84} - \Delta h \qquad (17)$$

The ellipsoidal heights above the war office spheroid is then computed from

$$h_{war} = H + N_{war} \qquad (18)$$

From the above equations, h_{war} on the local spheroid can be computed form:

$$h_{war} = H + N_{WGS84} - \Delta h \qquad (19)$$

The results obtained for ellipsoidal heights for both the war office and the Clarke 1880 spheroids together with the corresponding geoidal heights are shown in Table 6.

HELMERT DATUM TRANSFORMATION FOR GHANA ELLIPSOIDS

The geodetic coordinates **(φ, λ**, h) for the local ellipsoids and for WGS84 are used to deduce Helmert transformation parameters between the WGS84 and war office or the Clarke 1880. The seven parameter similarity transformation model relating coordinates of points in the X_B, Y_B, Z_B network to coordinates in the X_A, Y_A, Z_A network is:

Table 6: Derived Ellipsoidal and Geoidal heights

Point	h_{wgs84}	N_{wgs84}	h_{ciarke}	N_{ciarke}	$h_{waroffice}$	$N_{waroffice}$	H
GCS 124	344.161	36.0082	327.965	19.8117	391.2246	83.0718	308.158
GCS125	109.213	36.061	93.257	20.1050	156.5181	83.3661	73.152
CPF155	536.26	35.9003	519.411	19.0511	582.2669	82.3072	500.3596
GCS102	92.959	32.6390	79.986	20.3416	143.2335	82.9136	60.31992
GCS101	162.189	33.4719	149.059	19.6663	212.3068	83.5198	128.7170

$$\begin{bmatrix} X_B \\ Y_B \\ Z_B \end{bmatrix} = S \times R \begin{bmatrix} X_A \\ Y_A \\ Z_A \end{bmatrix} + \begin{bmatrix} \Delta_x \\ \Delta_Y \\ \Delta_Z \end{bmatrix}$$

where ***S*** is the scale factor and ***R*** is a 3x3 orthogonal rotation matrix. This model is adequate for relating two networks which are homogeneous and is called the *Bursa-Wolf model*. (Burford 1985; Bursa, 1962; Wolf, 1963)

Rotation Matrices

The rotation matrices about the X-, Y-, and Z-axes are:

$$R_Z(K) = \begin{pmatrix} CosK & SinK & 0 \\ -SinK & CosK & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R_Y(\theta) = \begin{pmatrix} Cos\,\theta & 0 & -Sin\,\theta \\ 0 & 1 & 0 \\ Sin\,\theta & 0 & Cos\,\theta \end{pmatrix}$$

$$R_X(\omega) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & Cos\omega & Sin\omega \\ 0 & -Sin\omega & Cos\omega \end{pmatrix}$$

Yielding a combined rotation matrix R given by:

$$R = \begin{pmatrix} CosKCos\theta & CosKSin\theta Sin\omega + SinKCos\omega & SinKSin\omega - CosKSin\theta Cos\omega \\ -SinKCos\theta & CosKCos\omega - SinKSin\theta Sin\omega & SinKSin\theta Cos\omega + CosKSin\omega \\ Sin\theta & -Cos\theta Sin\omega & Cos\theta Cos\omega \end{pmatrix}$$

For small rotations this matrix may be approximated by:

$$R \approx \begin{pmatrix} 1 & K & -\theta \\ -K & 1 & \omega \\ \theta & -\omega & 1 \end{pmatrix}$$

where ω, θ, and κ , are the rotation angles in radians about the X-, Y-, and Z-axes respectively. (Harvey, 1986: Steed, 1990.)

RESULTS AND DISCUSSION

The results from the Bulsa-Wolf model solution are shown in Tables 6 and 7.

The determinations made in this study compare with previous ones by NIMA (National Imaging and Mapping Agency) of the United States using eight satellite points.

Table 6: Seven Parameters for transforming WGS84 to War Office

Parameter	Values	Units
Δx	-241.6774716	Meters
Δy	23.77915512	Meters
Δz	335.1341164	Meters
Rx	1.05473E-05/2.18’’	Radians/Seconds
Ry	2.49197E-06/0.51’’	Radians/Seconds
Rz	2.38859E-06/0.49’’	Radians/Seconds
Scale	0.999999712	

Table 7: Seven Parameters for transforming WGS84 to Clarke 1880

Parameter	Values	Units
Δx	-128.9147286	Meters
Δy	25.94999122	Meters
Δz	377.9779344	Meters
Rx	7.58278E-06/1.56’’	Radians /Seconds
Ry	1.77552E-06/0.37’’	Radians/ Seconds
Rz	1.7008E-06/0.35’’	Radians/ Seconds
Scale	0.999999655	

Table 8: Variances obtained for Clarke 1880

Points	**X(m^2)**	**Y(m^2)**	**Z(m^2)**
CFP155	-0.040637118	-0.314055826	0.029044657
GCS124	-0.014182827	0.149373892	-0.005816225
GCS125	0.01248615	0.173752154	-0.080647133
GCS 101	0.004896733	-0.10313153	0.130331349
GCS 102	0.037559421	0.094274004	-0.072762682

Values determined for Clarke 1880 to WGS84 (see Table 5 pg. 15) are:

Δλ= –130±2, ΔY=29±3, and ΔZ=364=36±2*m* (D.M.A, 1991)
ΔX=–129.995, ΔY=32.2067, and ΔZ=366.423 (from present study)

Values determined for war office to WGS84 (see Table 4. pg. 14) are:

ΔX= –199, ΔY=32, and ΔZ=322 (D.M.A, 1987)
ΔX= –241.871, ΔY=32.560, and ΔZ=318.803 (from present study).

A comparison shows that values obtained for the Clarke 1880 were more consistent with the previous determinations than those for the war office spheroid. This could probably be due to the non-homogeneous nature of the framework based on war office. The geoidal heights obtained from the determined ellipsoidal heights show that the

Clarke 1880 ellipsoid fits the Ghana geoid better than the war office ellipsoid and even the GRS80. The variances obtained for the Clarke 1880 met a 99% confidence level Chi-squared testing whereas the war office parameters satisfy the chi-squared test at 95% confidence level.

CONCLUSION

The coordinate systems in use in Ghana were discussed and three and seven parameter transformations obtained between the WGS84 datum and the Datums in use in Ghana using the Bursa-Wolf Helmert transformation model. The values obtained for the Accra datum transformation could be improved if homogeneously adjusted coordinates were used for the transformations.

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