Predicting Lecturers Promotional Mobility Using Markov Chain Model

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ABSTRACT

Stochastic mobility models are probabilistic descriptions of how movements take place from one class to another. The main objective of the study was to forecast the number of members of University of Ghana (UG) academic staff in various categories or states of a system. In the literature, a stochastic mobility model for an open system has been developed. This work adopts this preexisting stochastic model to forecast promotion patterns for UG academic staff over specified periods. This is done through the generation of a probability transition matrix for academic staff promotions (open system) from 2001 to 2014. The findings of the study indicate that the total expected size of the university increased steadily over the period under consideration. A member of academic staff recruited to the position of a lecturer who wishes to rise through the ranks and to retire as a professor is likely to spend 27 years in the service (10.3 years as lecturer, another 7.5 years as senior lecturer, 5.6 years as associate professor and 3.6 years as full professor). A member of academic staff of UG recruited into the entry point as a lecturer has a 78.78% chance of becoming a senior lecturer, a 45.56% chance of becoming an associate professor and a 26.51% chance of becoming a full professor.

Keywords: Stochastic mobility model; Probability transition matrix; Markov chain; Open system; Closed system

Introduction

Human societies are often stratified into classes based on demographic variables such as age, sex, income, occupation, social status and place of residence. Members of such societies move from one class to another in what often seems to be a haphazard manner. In a free society a person has some degree of choice about changing his/her job or residence. The essential ingredient of any stochastic model of mobility is thus a probabilistic description of how movement takes place from one class to another. The underlying assumption for the simplest model is that the chance of moving depends only on the present class and not on the past. If movement can be regarded as taking place at discrete points in time, then the appropriate model becomes a simple Markov chain.

In a social system in which our interest is the changing internal structure, it is expedient to assume closeness. A closed social system is one which either no member moves in or out of, or any losses are replaced immediately by identical recruitments. The assumption of a closed system is reasonable for the applications to social class and labour mobility (Gani, 1963).

Nevertheless, there are many systems in which gains and losses are an important feature of the process. One example of a situation in which such a model is appropriate is provided by an educational system. Such a model was first used by Gani (1963) in projecting enrolment and degrees awarded in Australian universities. This model can also be applied promotions in an organization. The model has been applied to the student population of the University of California by Marshell (1973) and by Oliver and his co-workers in several unpublished reports. Musiga *et al.* (2011) modeled the bachelor's degree system using the Markov Chain approach in which the proportions of students who graduate and drop out of the system are separately grouped into double absorbing states. Adeleke *et al.* (2014) used the model to study the pattern of students' enrolment and their academic performance in the Department of Mathematical Sciences (Mathematics Option) at Ekiti State University, Ado – Ekiti, Nigeria. Both Markov's model and the capacity models of Menges and Elstermann (1971) incorporate a Markovian component. Armitage *et al.* (1969) have discussed the applicability of the model in educational planning and Armitage *et al.* (1970) dealt with it in relation to a model of the English Secondary School system. Kamat (1968b ;1968c) proposed a special case of the model suitable for describing the progress of a cohort through the educational system.

The academic staff of the University of Ghana (UG) is categorized into 4 ranks, which are arranged in increasing order of seniority as follows:

(1) Lecturer (*Denoting the first state*)
(2) Senior Lecturer (*Denoting the second state*)
(3) Associate Professor (*Denoting the third state*)
(4) Professor (*Denoting the fourth state*)

Aside these four grades (states), there is an Assistant Lecturer position in the university teacher's ranking. This position is excluded from this paper due to its peculiar terms of engagement. One is appointed to this position for a period (mostly 3 years) within which one is expected to obtain a PhD degree which is the minimum qualification for a lecturer; otherwise one is disengaged. In addition, movement from an Assistant Lecturer position to a Lecturer position is not considered a promotion per the university's terms of reference. Thus, the number of Assistant Lecturers who progress to Lecturer in a particular year are included in the total number of new entrants into Lecturer positions for that year.

Using the model first proposed by Gani (1963), our main interest here was in the number of UG academic staff in various categories or states of the system. These stocks change over time as a result of the operation of transition probabilities of flow between states. The main emphasis was on the stochastic behavior of the stock number and, in particular, on their means and variances.

Method

The number of new lecturers that enter the j^{th} state (status) in year T is denoted by $n_{0j}(T)$, where j = 1, 2, 3, 4. The number of lecturers leaving the university from the i^{th} state in the year T is denoted by $n_{i5}(T)$, where i = 1, 2, 3, 4. The n_{ij}'s $(i = 0, 1, 2, 3, 4$ and $j = 1, 2, 3, 4, 5)$ can be conveniently set in standard matrix form as shown in Table 1 below.

Table 1: Values of n_{ij} for the year T

		j				
		1	2	3	4	5
i	1	$n_{11}(T)$	$n_{12}(T)$	$n_{13}(T)$	$n_{14}(T)$	$n_{15}(T)$
	2	$n_{21}(T)$	$n_{22}(T)$	$n_{23}(T)$	$n_{24}(T)$	$n_{25}(T)$
	3	$n_{31}(T)$	$n_{32}(T)$	$n_{33}(T)$	$n_{34}(T)$	$n_{35}(T)$
	4	$n_{41}(T)$	$n_{42}(T)$	$n_{43}(T)$	$n_{44}(T)$	$n_{45}(T)$
	0	$n_{01}(T)$	$n_{02}(T)$	$n_{03}(T)$	$n_{04}(T)$	

Let $n_j(T) = \sum_{i=1}^{4} n_{ij}(T)$ denote the number of university teachers in state (status) j at year T (where T = 0, 1, 2, … and j = 1, 2, 3, 4). The initial state sizes, $n_j(0) = \sum_{i=1}^{4} n_{ij}(T)$ (j = 1, 2, 3, 4), are assumed to be given and we define

$$N(T) = \sum_{j=1}^{4} n_j(T) \tag{1}$$

For $T > 0$ the state (status) sizes are random variables. The expected number, $E\left[n_j(T)\right]$, in state j year T is denoted by $\bar{n}_j(T)$. The number of new entrants into the system in year T is written as $R(T)$, whilst the expected new entrants in year T is denoted by $\bar{R}(T)$.

Let p_{ij} $(j, i = 1, 2, 3, 4)$ be the probability that an individual in state i in a particular year will be found in state j in the following year. Let $\boldsymbol{P}$ denote the matrix with elements $\{p_{ij}\}$

$$\boldsymbol{P} = \begin{pmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \\ p_{41} & p_{42} & p_{43} & p_{44} \end{pmatrix}. \tag{2}$$

Since transitions out of the university are possible (open system), $\sum_{j=1}^{4} p_{ij} \le 1.$ The probability that an individual in state i in a particular year will be out of the university the following year is given by $p_{i5} = 1 - \sum_{j=1}^{4} p_{ij}$. The proportion of new entrants that enter the j^{th} state is denoted by p_{0j}, where $\sum_{j=1}^{4} p_{0j} = 1$, with expectation $R(T)\,p_{0j}$ entering the j^{th} state (j = 1, 2, 3, 4). Let $\boldsymbol{Q}$ be the transpose of the transition matrix $\boldsymbol{P}$ (which is sub-stochastic). The one-step transition probabilities, p_{ij} (i = 0, 1, 2, 3, 4 and j = 1, 2, 3, 4, 5), that specify the process can be represented by a matrix $\boldsymbol{M}$ as follows:

Final state

$$\boldsymbol{M} = \text{Initial state}\quad \begin{array}{c} \\ i=1 \\ i=2 \\ i=3 \\ i=4 \\ i=0 \end{array} \begin{array}{c} \begin{array}{ccccc} j=1 & j=2 & j=3 & j=4 & j=5 \end{array} \\ \left(\begin{array}{cccc|c} p_{11} & p_{12} & p_{13} & p_{14} & p_{15} \\ p_{21} & p_{22} & p_{23} & p_{24} & p_{25} \\ p_{31} & p_{32} & p_{33} & p_{34} & p_{35} \\ p_{41} & p_{42} & p_{43} & p_{44} & p_{45} \\ \hline p_{01} & p_{02} & p_{03} & p_{04} & \end{array}\right) \end{array}, \qquad (3)$$

The expected values $\bar{n}_j(T+1)$ satisfy the recurrent relations

$$\bar{n}_j(T+1) = \sum_{i=1}^{4} p_{ij}\bar{n}_i(T) + R(T+1)p_{0j},$$

which can be expressed as

$$\bar{\boldsymbol{n}}(T+1) = \boldsymbol{Q}\bar{\boldsymbol{n}}(T) + R(T+1)\boldsymbol{p}_0, \qquad (4)$$

where $\bar{\boldsymbol{n}}(T) = \left(\bar{n}_1(T),\ \bar{n}_2(T),\ \bar{n}_3(T),\ \bar{n}_4(T)\right)'$ and $\boldsymbol{p}_0 = \left(p_{01}, p_{02}, p_{03}, p_{04}\right)'$. Substituting $\bar{\boldsymbol{n}}(T) = \boldsymbol{Q}\bar{\boldsymbol{n}}(T-1) + R(T)\boldsymbol{p}_0$

into (4), we have

$$\begin{aligned} \bar{\boldsymbol{n}}(T+1) &= \boldsymbol{Q}\left[\boldsymbol{Q}\bar{\boldsymbol{n}}(T-1) + R(T)\boldsymbol{p}_0\right] + R(T+1)\boldsymbol{p}_0 \qquad (5) \\ &= \boldsymbol{Q}^2\bar{\boldsymbol{n}}(T-1) + R(T)\boldsymbol{Q}\boldsymbol{p}_0 + R(T+1)\boldsymbol{p}_0 \end{aligned}$$

Proceeding in this manner, we can write

$$\bar{\boldsymbol{n}}(T+1) = \boldsymbol{Q}^{T+1}\bar{\boldsymbol{n}}(0) + \sum_{t=0}^{T} R(T+1-t)\boldsymbol{Q}^t\boldsymbol{p}_0$$

$$\bar{\boldsymbol{n}}(T) = \boldsymbol{Q}^{T}\bar{\boldsymbol{n}}(0) + \sum_{t=0}^{T-1} R(T-t)\boldsymbol{Q}^t\boldsymbol{p}_0 \qquad (6)$$

If $R(T)$ has a suitable mathematical form it may be possible to sum the matrix series appearing in (6) and so obtain the solution in closed form. This is the case if $R(T)$ is constant for all T or more generally if $R(T) = R.x^T$ (R > 0, x > 0, T ³ 1). In this case (6) becomes;

$$\begin{aligned} \bar{\boldsymbol{n}}(T) &= \boldsymbol{Q}^T\bar{\boldsymbol{n}}(0) + \sum_{t=0}^{T-1} Rx^{T-t}\boldsymbol{Q}^t\boldsymbol{p}_0 \\ &= \boldsymbol{Q}^T\bar{\boldsymbol{n}}(0) + Rx^T\left(\sum_{t=0}^{T-1} x^{-t}\boldsymbol{Q}^t\right)\boldsymbol{p}_0 \qquad (7) \end{aligned}$$

The sum $\sum_{t=0}^{T-1} x^{-t}\boldsymbol{Q}^t$ is a geometric series of first term $x^{-0}\boldsymbol{Q}^0 = \boldsymbol{J}$ $\;x^{-0}\boldsymbol{Q}^0 = \boldsymbol{Q}^0 = \boldsymbol{J}$(the unit matrix) with the common ratio $x^{-1}\boldsymbol{Q}$. The sum of the first T terms of the sequence is given by

$$\sum_{t=0}^{T-1} x^{-t}\,\boldsymbol{Q}^t = (\boldsymbol{J} - x^{-1}\boldsymbol{Q})^{-1}(\boldsymbol{J} - x^{-T}\boldsymbol{Q}^T) = x(x.\boldsymbol{J} - x^{-1}\boldsymbol{Q})^{-1}(\boldsymbol{J} - x^{-T}\boldsymbol{Q}^T)$$

$$Rx^T\left(\sum_{t=0}^{T-1} x^{-t}\,\boldsymbol{Q}^t\right)\boldsymbol{p}_0 = Rx(x.\boldsymbol{J} - \boldsymbol{Q})^{-1}(x^T\boldsymbol{J} - \boldsymbol{Q}^T)\boldsymbol{p}_0$$

Hence

$$\bar{\boldsymbol{n}}(\boldsymbol{T}) = \boldsymbol{Q}^T\bar{\boldsymbol{n}}(0) + Rx(x.\boldsymbol{J} - \boldsymbol{Q})^{-1}(x^T\boldsymbol{J} - \boldsymbol{Q}^T)\boldsymbol{p}_0 \qquad (8)$$

Consider the random variables $X_{ii}^{(r)}$, defined as

$$X_{ij}^{(r)} = \begin{cases} 1, & \text{if an entrant to grade } i \text{ is in grade } j \text{ after } r \text{ units} \\ 0, & \text{otherwise} \end{cases}$$

where i, j = 1, 2, 3, 4 and r = 0,1, 2, … . If a person is recruited into the university in state i, the total time spent by such an individual in state j is

$$X_{ij} = \sum_{r=0}^{\infty} X_{ij}^{(r)} \quad (i, j = 1, 2, 3, 4)$$

(Note that $X_{ij}^{(0)} = 0$ if j ¹ i and = 1 otherwise). The expected length of time he will spend in state j is

$$E\left(X_{ij}\right) = \sum_{r=0}^{\infty} E\left(X_{ij}^{(r)}\right) \qquad (9)$$

It is well known from the general theory of Markov chains that (Stone, 1972)

$$P\left(X_{ij}^{(r)} = 1\right) = p_{ij}^{(r)},$$

where $p_{ij}^{(r)}$ is the $(i, j)^{\text{th}}$ element of $\boldsymbol{P}^r$. Hence $E\left(X_{ij}^{(r)}\right) = p_{ij}^{(r)}$ and therefore

$$E\left(X_{ij}\right) = \sum_{r=0}^{\infty} p_{ij}^{(r)} \qquad (10)$$

If we introduce the matrix $\boldsymbol{X} = \left\{X_{ij}\right\}$, then (10) yields

$$E(\boldsymbol{X}) = \sum_{r=0}^{\infty} \boldsymbol{P}^{(r)} = \sum_{r=0}^{\infty} \boldsymbol{P}^{r} = \boldsymbol{I} + \boldsymbol{P} + \boldsymbol{P}^2 + \ldots = (\boldsymbol{I} - \boldsymbol{P})^{-1}. \quad (11)$$

The expected stay of an entrant into grade i in the whole system is

$$E(X_i) = \sum_{j=1}^{k} E(X_{ij}) = d_i \quad (12)$$

Let π_{ij} denote the probability that an entrant into state i spends some time in state j before leaving. If μ_{ij} is the $(i, j)^{th}$ element of $(\boldsymbol{I} - \boldsymbol{P})^{-1}$ then

$$\mu_{ij} = \pi_{ij}\mu_{jj} + (1 - \pi_{ij}) \times 0,$$

$$\text{or } \pi_{ij} = \frac{\mu_{ij}}{\mu_{jj}}, \quad (i, j = 1, 2, 3, 4) \quad (13)$$

The diagonal elements of $\left\{\pi_{ij}\right\}$ must obviously be unity; the off-diagonal elements give the chance of reaching the grade corresponding to the column, given that we enter that corresponding to the row.

Results

Table 2 shows the mobility of academic staff at the UG from 2009 to 2014. The individual elements of the j^{th} column for each year shows how the total number of lecturers in the j^{th} state is divided among the various states (status) from which they are moving.

Table 2: Values of n_{ij} from 2009 to 2014

2009

		J				
		1	2	3	4	5
i	1	317	19			11
	2		193	9		13
	3			110	3	8
	4				59	5
	0	48	9	0	1	

2010

		j				
		1	2	3	4	5
i	1	374	22			3
	2		205	13		8
	3			115	10	1
	4				63	3
	0	39	4	0	3	

2011

		J				
		1	2	3	4	5
i	1	335	31			5
	2		184	9		8
	3			120	2	6
	4				67	6
	0	73	8	2	3	

2012

		j				
		1	2	3	4	5
i	1	343	26			5
	2		184	13		7
	3			125	7	2
	4				68	4
	0	60	6	4	0	

2013

		J				
		1	2	3	4	5
i	1	306	32			9
	2		172	21		6
	3			136	10	4
	4				72	3
	0	88	6	2	0	

2014

		j				
		1	2	3	4	5
i	1	307	37			12
	2		158	9		12
	3			151	7	7
	4				66	10
	0	75	3	1	6	
	Total	382	198	161	79	820

The values of n_{ij} (i = 0, 1, 2, 3, 4 and j = 1, 2, 3, 4, 5) for the year 2014 are taken to be the initial values at T = 0. In the year 2014, it can be seen that out of the total number of academic staff who were Lecturers in the previous year, 37 were promoted to Senior Lecturers $(n_{12}(0) = 37)$, 12 left the university $(n_{15}(0) = 12)$, whilst 307 maintained the position of Lecturer $(n_{11}(0) = 307)$. The total number of academic staff recruited in 2014 was 85, of which 75 where Lecturers, 3 Senior Lecturers, 1 Associate Professor and 6 Professors. The initial number of teachers $n_j(0)$ at the j^{th} state is given by

$$n_j(0) = n_{1j}(0) + n_{2j}(0) + n_{3j}(0) + n_{4j}(0) + n_{0j}(0) \quad (14)$$

Thus, $n_1(0) = 382$, $n_2(0) = 198$, $n_3(0) = 161$ and $n_4(0) = 79$, which are the total numbers of Lecturers, Senior Lecturers, Associate Professors and Professors, respectively, in the year 2014. Thus, the total number of academic staff in the year 2014 is given by

$$N(0) = n_1(0) + n_2(0) + n_3(0) + n_4(0) = 820.$$

The one-step transition probabilities, p_{ij} $(i = 0, 1, 2, 3, 4 \text{ and } j = 1, 2, 3, 4, 5)$, that specify the process can be represented by a matrix $\boldsymbol{M}$ as follows:

$$\boldsymbol{M} = \begin{array}{c} \\ i=1 \\ i=2 \\ i=3 \\ i=4 \\ i=0 \end{array} \begin{pmatrix} j=1 & j=2 & j=3 & j=4 & j=5 \\ 0.9034 & 0.0761 & 0 & 0 & 0.0205 \\ 0 & 0.8954 & 0.0605 & 0 & 0.0441 \\ 0 & 0 & 0.9187 & 0.0473 & 0.0340 \\ 0 & 0 & 0 & 0.9272 & 0.0728 \\ 0.8685 & 0.0816 & 0.0204 & 0.0295 & \end{pmatrix}. \quad (15)$$

(Rows: Initial state; columns: Final state.)

Note: $\sum_{j=1}^{4} p_{0j} = 1$ and $\sum_{j=1}^{5} p_{ij} = 1.$ The sub-stochastic transition matrix $\boldsymbol{P}$ is given by

$$\boldsymbol{P} = \begin{array}{c} \\ i=1 \\ i=2 \\ i=3 \\ i=4 \end{array} \begin{pmatrix} j=1 & j=2 & j=3 & j=4 \\ 0.9034 & 0.0761 & 0 & 0 \\ 0 & 0.8954 & 0.0605 & 0 \\ 0 & 0 & 0.9187 & 0.0473 \\ 0 & 0 & 0 & 0.9272 \end{pmatrix} \quad (16)$$

In the UG, advancement of academic staff through the hierarchy (i.e. from lecturer through to the position of professor) is mainly one step (level) for any period as in most management hierarchies. This explains why in the transition matrix $\boldsymbol{P}$ of Equation (16), the values of the p_{ij}'s are zero $(p_{ij} = 0)$ for $j = i + 1$. Since transitions within a hierarchy are to a higher grade only, $p_{ij} = 0$, for $j < i$. The transition matrix $\boldsymbol{P}$ shows much bigger values for the diagonal elements. This is because very few academic staff get promoted to the next rank, while the majority remain at the same rank. This reflects the kind of conditions usually found in a typical management hierarchy. In the promotion of students in an educational system, where very few repeat a class, the diagonal elements would tend to be much smaller.

The individual elements in the rows of $(\boldsymbol{I} - \boldsymbol{P})^{-1}$ show how the total expected length of service of an entrant is divided among the various states, where $\boldsymbol{I}$ is a 4 ´ 4 identity matrix.

$$(\boldsymbol{I} - \boldsymbol{P})^{-1} = \begin{pmatrix} j=1 & j=2 & j=3 & j=4 & \textbf{Row total} \\ 10.3520 & 7.5314 & 5.6045 & 3.6414 & 27.1293 \\ 0 & 9.5602 & 7.1143 & 4.6223 & 21.2969 \\ 0 & 0 & 12.3001 & 7.9917 & 20.2918 \\ 0 & 0 & 0 & 13.7363 & 13.7363 \end{pmatrix} \quad (17)$$

For instance, a member of academic staff of UG is expected to spend 10.3 years in the first state (as a lecturer). After transition to the next state (as a Senior Lecturer) he is expected to spend 7.5 years in that state, 5.6 years in the third state as an Associate Professor and finally 3.6 years in the fourth state as a Professor. If an academic is recruited to join the second grade (state 2) as a Senior Lecturer, the pattern then changes. This is evidenced in the second row of equation (17). The average time he spends in state 2 is 9.6 years. He is then expected to spend 7.1 years and 4.6 years in the third and the fourth states respectively. This reflects the fact that the individual was not recruited through the first grade. A recruit who enters at state 2 is expected to spend 4.6 years in state 4.

The expected length of service of an entrant into the 1st state (Lecturer) = 27.1 years

The expected length of service of a recruit into the 2nd state (Senior Lecturer) = 21.3 years

The expected length of service of a recruit into the 3rd state (Associate Professor) = 20.3 years

The expected length of service of a recruit into the 4th state (Professor) = 13.7 years.

Thus to obtain the matrix π of probabilities $\{\pi_{ij}\}$ we must divide the elements in each column of $(\boldsymbol{I}-\boldsymbol{P})^{-1}$ by the diagonal elements of that column. Therefore

$$\boldsymbol{\pi} = \begin{pmatrix} 1 & 0.7878 & 0.4556 & 0.2651 \\ 0 & 1 & 0.5784 & 0.3365 \\ 0 & 0 & 1 & 0.5818 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \qquad (18)$$

The diagonal elements of equation (18) must obviously be unity. This means, there is a certain probability that an academic staff will remain in his grade (status). A member of academic staff of UG recruited to enter the first state as a lecturer has a 78.78% chance of becoming a senior lecturer, a 45.56% chance of becoming an associate professor and a 26.51% chance of becoming a full professor.

It can be seen from Table 2 that the average number of teachers recruited each year into the University of Ghana is $R = 74$. Let $\boldsymbol{Q}$ be the transpose of the transition probability matrix $\boldsymbol{P}$ (which is sub-stochastic) so that

$$\boldsymbol{Q} = \begin{pmatrix} 0.9034 & 0 & 0 & 0 \\ 0.0761 & 0.8954 & 0 & 0 \\ 0 & 0.0605 & 0.9187 & 0 \\ 0 & 0 & 0.0473 & 0.9272 \end{pmatrix} \qquad (19)$$

From Table 2

$$\overline{\boldsymbol{n}}(0) = \begin{pmatrix} 382 & 198 & 161 & 79 \end{pmatrix}' \text{ and}$$

$$\boldsymbol{p}_0 = \begin{pmatrix} 0.8685 & 0.0816 & 0.0204 & 0.0295 \end{pmatrix}'$$

The expected total number of academic staff after T years is $\overline{n}(T)$ and is given by (Bartholomew, 1973)

$$\overline{\boldsymbol{n}}(T) = \boldsymbol{Q}^T \overline{\boldsymbol{n}}(0) + R(\boldsymbol{I}-\boldsymbol{Q})^{-1}(\boldsymbol{I}-\boldsymbol{Q}^T)\boldsymbol{p}_0 \qquad (20)$$

We find that

$$(\boldsymbol{I}-\boldsymbol{Q})^{-1} = \begin{pmatrix} 10.3520 & 0 & 0 & 0 \\ 7.5314 & 9.5602 & 0 & 0 \\ 5.6045 & 7.1143 & 12.3001 & 0 \\ 3.6414 & 4.6224 & 7.9917 & 13.7363 \end{pmatrix}$$

Thus, from the model, the distribution of the total number of university teachers after 1 year (i.e. 2015) is given by the vector

$$\overline{\boldsymbol{n}}(1) = \boldsymbol{Q}\overline{\boldsymbol{n}}(0) + R(\boldsymbol{I}-\boldsymbol{Q})^{-1}(\boldsymbol{I}-\boldsymbol{Q})\boldsymbol{p}_0$$

which gives $\overline{\boldsymbol{n}}(1) = (409, 212, 161, 83)'$. The set of entries in the vector $\overline{\boldsymbol{n}}(1)$ gives the estimated number of academic staff in the various ranks in 2015. Based on Equation (20), the estimated grade sizes from 2015 to 2025 are computed and the results are given in Table 3. The expected size of the academic staff at UG in each category shows a steady increase over the period.

Table 3: Expected number of lecturers from 2015– 2025

Rank	Years										
	2015	2016	2017	2018	2019	2020	2021	2022	2023	2024	2025
Lecturer	409	434	456	477	495	511	526	540	552	563	573
Senior Lecturer	212	257	285	310	331	251	369	385	400	412	425
Associate Professor	161	211	236	256	273	289	304	316	328	338	347
Professor	83	120	141	158	172	186	198	209	219	227	236
Total	865	1022	1118	1201	1271	1237	1397	1450	1499	1540	1581

Table 4: The expected mobility in 2017

		j				
		1	2	3	4	5
i	1	413	24			10
	2		280	16		14
	3			238	8	9
	4				148	11
	0	64	6	2	2	

The expected mobility for 2017 is as given in Table 4. The individual elements of the j^{th} column of Table 4 show how the total expected number of academic staff in the j^{th} state is divided among the various states where they are moving from. The number of new entrants into the first and second states as lecturers and senior lecturers for 2017 were 64 and 6, respectively, whilst 2 members of staff were expected to be recruited to enter into each of the remaining higher grades (i.e. associate professor and professor). The number of teachers expected to be promoted from the first grade to the second (i.e. from the position of lecturer to senior lecturer) was 24 whilst 16 were expected to be promoted from senior lecturer to associate professor. The total number of teachers expected to leave the university in 2017 was 44. The values in the 5^{th} column show how these 44 teachers who were expected to leave the university in 2017 were divided among the various states from which they were resigning. For example, 10 teachers were expected to exit from the position of lecturer whilst in the same year 9 associate professors were expected to leave. The diagonal elements in Table 4 give the number of teachers in state *i* in the year 2016 who were expected to remain in the same state in 2017, where $i = 1, 2, 3, 4$. For instance, the expected number of senior lecturers in 2017 who were still senior lecturers in 2017 was found to be 280.

Table 5: Total staff strength as at 2017

Rank	Total
Lecturer	472
Senior Lecturer	321
Associate Professor	130
Professor	81
Grand Total	1004

Ghana adopted a national policy which controlled public service recruitments in 2015 and 2016. Most public institutions were not allowed to recruit staff. This affected the University academic staff enrolment. The total academic staff strength of UG as at the end of the 2016/2017 academic year is given in Table 5. The study therefore cannot use the 2017 actual staff strength to validate the predicted stock of the university academic staff in Table 3.

Figure 1.0 is a graph of the expected and actual number of academic staff based on rank for 2017.

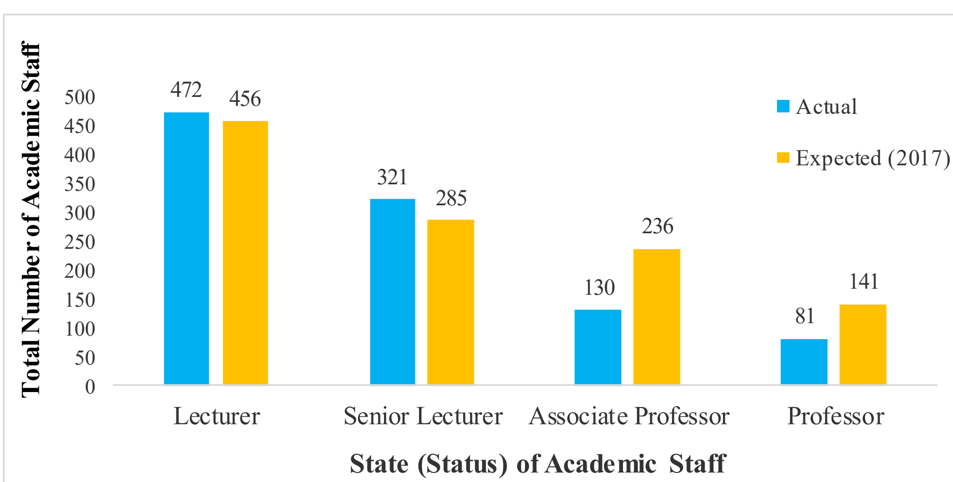

Fig. 1.0: Expected and Actual Distributions of Academic Staff based on rank for 2017.

Conclusion and Recommendation

The study predicted the expected number of academic staff of University of Ghana at specified periods and the respective transitions based on promotions and exits. The study found that a member of academic staff of UG recruited to enter the first state as a lecturer has a 78.78% chance of becoming a senior lecturer, a 45.56% chance of becoming an associate professor and a 26.51% chance of becoming a full professor. The total expected size of the university teachers increased steadily through the period under consideration (2001 to 2015). The expected number of professors decreased gradually over the period (2001 to 2015) while the number of lecturers and senior lecturers grew steadily within the same period.

It is recommended that the academic staff of the university of Ghana increase efforts toward promotion.

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