Modelling Ghana Stock Exchange Index with Stable Distributions

Gabriel Kallah-Dagadu[1*], Ezekiel N. N. Nortey[1] and Kwabena Doku-Amponsah[1]

Department of Statistics and Actuarial Science, University of Ghana, Lego
*Corresponding author: gkallah-dagadu@ug.edu.gh

ABSTRACT

Major concepts in theoretical and empirical finance developed over the years rest upon the assumption that the return or price distribution for financial data follows a normal distribution, but this assumption is not justified by empirical data. This paper shows that the return distribution of some Ghanaian financial data exhibits excess kurtosis. The paper shows that of the three known methods of estimating the parameters of alpha-stable distributions: Maximum Likelihood estimation, Empirical Characteristic function and Sample Quantile methods, the first method performed better than the other two. The weekly return financial data (GSE all-shares index) is modelled with stable distribution.

Keywords: Stable distribution, GSE all-shares index, Maximum Likelihood, Sample Quantile and Empirical Characteristic function.

Introduction

The theories and models developed in theoretical and empirical finance over the last five decades rest upon the assumption that the return or price distribution of financial assets obeys the normal or Gaussian distribution. But with rare exception, studies have shown that this assumption does not hold and there is ample empirical evidence that many, if not most, financial return series are heavy-tailed and possibly skewed (Rachev *et al.*, 2005).

Stable distributions have been widely used for fitting data in which extreme values are frequent because they accommodate heavy-tailed financial series and therefore produce more reliable measures of tail risk such as value at risk (Garcia *et al.*, 2010).

Asset returns are the cumulative outcome of a vast number of pieces of information and individual decisions arriving almost continuously in time. Hence, in the presence of heavy tails it is natural to assume that they are approximately governed by a stable, non-Gaussian distribution. Stable distributions have been proposed as a model for many types of physical and economic systems. Firstly, they are applied when there are solid theoretical reasons for expecting a non-Gaussian stable model. Secondly, the generalized central limit theorem states that the only possible non-trivial limit of normalized sums of independent identically distributed terms is stable. Thirdly, the argument for modelling with stable distributions is empirical; many large datasets exhibit heavy tails and skewness (Feller, 1971; Uchaikin & Zolotarev, 1999). The strong empirical evidence in favor of these features, combined with the generalized central limit theorem, is used to justify the use of stable models.

Mandelbrot (1963), Fama (1965) and Borak *et al.* (2005) show by empirical evidence that financial asset returns and stock price indices exhibit fat tails; they thus proposed the stable distribution as an alternative for modelling returns. This paper investigated the empirical performance of stable distribution in fitting the behaviour of asset returns (Ghana Stock Exchange all-shares index). It also explored the existing methods for estimating the parameters of stable distribution for fitting stable models.

Materials and Methods

In this paper we consider the Ghana Stock Exchange (GSE) all-shares index spanning a period of ten years (2000-2010). The information was obtained from the GSE. The GSE all-shares index is the market capitalization weighted index of all ordinary shares listed on GSE. The base date for the GSE all-shares index is November 12, 1990 and the base index value was 100. The daily and weekly GSE all-shares is used in this paper.

The Stable Distribution Family

The family of stable distributions is employed due to their stability property and the nature of the financial data. The stable family consists of an α-stable distribution, a Levy stable distribution, a Cauchy distribution and a Gaussian or normal distribution. This family of distributions have a very interesting pattern of shapes, allowing for asymmetry and fat tails, which makes them suitable for the modelling of several phenomena, ranging from engineering to finance (Lombardi, 2007).

Stable distributions are a class of probability laws that have intriguing theoretical and practical properties. The stable family of distributions stems from a more general version of the central limit theorem which replaces the assumption of the finiteness of the variance with a much less restrictive one concerning the regular behaviour of the tails (Gnedenko & Kolmogorov, 1954).

The term stable means that the random variables retain their shape (up to scale and shift) under addition. If $X_1, X_2, \ldots, X_n$ are independent and identically distributed stable random variables with distribution function F, then for every n

$$X_1 + X_2 + \ldots + X_n \overset{d}{=} c_n X + d_n, \quad (2.1)$$

for some constants $c_n > 0$ and d_n. The symbol $\overset{d}{=}$ means equality in distribution, i.e., the right- and left-hand sides have the same distribution. Equation (2.1) is strictly stable if $d_n = 0$, for all n. In terms of financial returns, one could say that the sum of daily returns is up to scale and location equally distributed as the weekly, monthly or yearly returns.

Definition 2.1

A random variable X is α-stable distributed and denoted by $S(\alpha, \beta, \gamma, \delta; 0)$, if it has the characteristic function

$$E\left[\exp(iuX)\right] = \begin{cases} \exp\left(-\gamma^{\alpha}|u|^{\alpha}\left[1 + i\beta\left(\tan\frac{\pi\alpha}{2}\right)(sign\ u)\left(|\gamma u|^{1-\alpha} - 1\right)\right] + i\delta u\right) & \alpha \neq 1, \\ \exp\left(-\gamma^{\alpha}|u|^{\alpha}\left[1 + i\beta\frac{2}{\pi}(sign\ u)\ln|u|\right] + i\delta u\right) & \alpha = 1, \end{cases} \quad (2.2)$$

where $Sign(u) = \begin{cases} 1, & u > 0 \\ 0, & u = 0 \\ -1, & u < 0 \end{cases}.$

Definition 2.2

A random variable X is α-stable distributed, denoted by $S(\alpha,\beta,\gamma,\delta;1)$, if its characteristic function is given as

$$E\left[\exp(iuX)\right]=\begin{cases}\exp\left(-\gamma^{\alpha}|u|^{\alpha}\left[1-i\beta\left(\tan\frac{\pi\alpha}{2}\right)(sign\ u)\right]+i\delta u\right) & \alpha\neq 1,\\ \\ \exp\left(-\gamma^{\alpha}|u|\left[1+i\beta\frac{2}{\pi}(sign\ u)\ln|u|\right]+i\delta u\right) & \alpha=1,\end{cases} \quad (2.3)$$

$$\text{where } Sign(u)=\begin{cases}1, & u>0\\ 0, & u=0\\ -1, & u<0\end{cases}.$$

The parameter α is known as the index of stability (or characteristic exponent or index of the law) and must be in the range $0<\alpha\leq 2$. The parameter β is known as the skewness of the law and ranges $-1\leq\beta\leq 1$. If $\beta = 0$, the distribution is symmetric; if $\beta > 0$ it is skewed towards the right and if $\beta < 0$, it is skewed towards the left. The parameters α and β determine the shape of the distribution. The parameter γ is a scale parameter and it can be any positive number $(\gamma\geq 0)$. The parameter δ is known as the location parameter and falls in the interval, $-\infty<\delta<\infty$. The location parameter shifts the distribution to the right if $\delta>0$ and to the left if $\delta<0$.

The two different definitions of α-stable distribution (2.2 and 2.3), according to Zolotarev (1986) and Nolan (2003), are the two common different parameterizations which this paper considered. Definition 2.1 is used for computations, because it has better numerical behaviour and intuitive meaning. However, the formulation of the characteristic function is more cumbersome and the analytic properties have a less intuitive meaning. The second parameterization has a quite manageable expression of its characteristic function and can straightforwardly produce several interesting analytic results (Zolotarev, 1986), but has a major drawback: it is not continuous with respect to the parameters, having a pole at $\alpha = 1$, but is more commonly used in the literature.

The distribution function can be obtained by either the Fast Fourier Transform (FFT) to the characteristic function (Mittnik *et al.*, 1999) or direct numerical integration of the characteristic function (Nolan, 1997). The FFT based approach is faster for large samples, whereas the direct integration method favours small data sets since it can be computed at any arbitrarily chosen point (Borak *et al.*, 2005).

There are three special cases of the stable distribution, namely Levy stable, Cauchy and Gaussian distributions that have closed-form expression of density functions. The case where $\alpha = 2$ (and $\beta = 0$) and with the reparameterization in scale parameter, where $\hat{\gamma}=\sqrt{2\gamma}$, yields the normal distribution. The case where $\alpha = 1$ and $\beta = 0$ yields the Cauchy distribution with much fatter tails than the normal distribution and the case where $\alpha = \frac{1}{2}$ and $\beta = 1$ yields the Lévy distribution.

Estimating the Parameters of α - Stable Distribution

This study explored three methods of estimating the parameters of $\alpha-$stable distribution using sample information. Lindgren (1993) argues that if the sample is representative of the population, then one can use it to make an estimate that is better than a sheer guess.

The maximum likelihood estimation (MLE) estimates are asymptotically consistent and converge to the true values. It is asymptotically efficient, produces the most precise estimates and is asymptotically unbiased. DuMouchel (1971) developed an approximate MLE method, which was based on grouping the data set into bins and using a combination of means to compute the density to an approximate log-likelihood function which was then numerically maximized.

The sample quantile method was first used by Fama and Roll (1971) to provide very simple estimates for parameters of symmetric $(\beta = 0,\ \delta = 0)$ stable laws with $\alpha > 1$. A decade later McCulloch (1986) generalized this method and provided consistent estimators of all the four stable parameters (with the restriction $\alpha \geq 0.6$). McCulloch (1986) defined the estimators as:

$$\upsilon_\alpha = \frac{x_{0.95} - x_{0.05}}{x_{0.75} - x_{0.25}},\ \upsilon_\beta = \frac{x_{0.95} + x_{0.05} - 2x_{0.50}}{x_{0.95} - x_{0.05}} \text{ and } \upsilon_\gamma = \frac{x_{0.75} - x_{0.25}}{\gamma}, \tag{2.4}$$

where x_f denotes the $f-$th population quantile, the $x_{(i)}$ are ordered in ascending order and the x_f matched, so that $S(\alpha,\ \beta,\ \gamma,\ \delta)(x_f) = f(x;\ \theta)$. The statistics υ_α and υ_β of Equation (2.4) are functions of α and β only. These relationships are inverted and the parameters α and β are viewed as functions of υ_α and υ_β: $\alpha = \psi_1(\upsilon_\alpha,\ \upsilon_\beta)$ and $\beta = \psi_2(\upsilon_\alpha,\ \upsilon_\beta)$.

Using the stable distribution parameters tables provided by McCulloch (1986) yields estimators of $\hat{\alpha}$ and $\hat{\beta}$ through linear interpolation between values read on the theoretical table. The scale parameter, (γ), is also estimated in a similar way and finally, the location parameter, (δ), is estimated using the sample mean $\bar{x}$ (when $\alpha > 1$).

Press (1972) was the first to use the Empirical Characteristic Function method, but Koutrouvelis (1980) presented a much more accurate regression-type method which starts with an initial estimate of the parameters and proceeds iteratively until some pre-specified convergence criterion is satisfied. Koutrouvelis (1980) derived the regression equation for estimating the parameters from the stable characteristic function as

$$\log\left(-\log|\phi(t)|^2\right) = \log\left(2\gamma^\alpha\right) + \alpha \log|t| \tag{2.5}$$

The real and imaginary parts of $\phi(t)$ for $\alpha \neq 1$ are given by equations (2.6) and (2.7):

$$\Re\{\phi(t)\} = \exp\left(-|\gamma t|^\alpha\right)\cos\left[\delta t + |\gamma t|^\alpha \beta sign(t)\tan\frac{\pi\alpha}{2}\right], \tag{2.6}$$

$$\Im\{\phi(t)\} = \exp\left(-|\gamma t|^\alpha\right)\sin\left[\delta t + |\gamma t|^\alpha \beta \text{sign}(t)\tan\frac{\pi\alpha}{2}\right]. \tag{2.7}$$

Apart from considerations of principal values, we have

$$\arctan\left(\frac{\Im\ \{\phi(t)\}}{\Re\ \{\phi(t)\}}\right) = \delta t + \beta\gamma^\alpha \tan\frac{\pi\alpha}{2} sign(t)|t|^\alpha. \tag{2.8}$$

Equation (2.5) depends only on α and γ and these two parameters can be estimated by regressing $y = \log\left(-\log|\phi_n(t)|^2\right)$ on $\omega = \log|t|$ in the model as displayed in Equation (2.9)

$$y_k = m + \alpha\omega_k + \varepsilon_k, \tag{2.9}$$

where ω_k is an appropriate set of real numbers, $m = \log(2\gamma)$, and ε_k denotes an error term. Koutrouvelis (1980) proposed to use $\omega_k = \frac{\pi k}{25}$, $k = 1, 2, ..., N$; with N (the sample size) ranging between 9 and 134 for different values of α and sample sizes for the model, Equation (2.9). Once $\hat{\alpha}$ and $\hat{\gamma}$ have been obtained and α and γ fixed at these values, estimates of β and δ are now obtained using Equation (2.8). Next, the regressions are repeated with $\hat{\alpha}$, $\hat{\beta}$, $\hat{\gamma}$ and $\hat{\delta}$ as the initial parameters. The iterations continue until a prespecified convergence criterion is satisfied. Koutrouvelis suggested the use of Fama and Roll's (1971) formula to estimate γ and the 25% truncated mean ($\bar{X}$) for initial estimates of γ and δ , respectively.

Koutrouvelis' (1980) method was slower in estimating the parameters and therefore Kogon and Williams (1998) eliminated that iteration procedure and simplified the regression method using McCulloch's (1986) method with the continuous representation of the characteristic function instead of the classical one. They used a fixed set of only ten (10) equally spaced frequency points ω_k and in terms of computational speed, their method was favourably faster than the original regression method (Borak *et al*., 2010).

Two goodness of fit tests (Kolmogorov-Smirnov (K-S) and chi-square tests) are used to examine the distribution of the employed dataset. The root mean square error (RMSE), mean absolute error (MAE) and Akaike information criterion (AIC) are used with the K-S and chi-square tests to determine the best method for estimating the parameters of stable distribution. The R statistical software, together with STABLE program developed by Nolan (2005), was used for the analysis.

Empirical Results

The study considered weekly GSE all-shares index of 571 observations spanning from 2000-2010. It was observed that the daily GSE all-shares index show little or no volatility, hence the use of weekly observations. The weekly logarithm return $\left(X(t)\right)$ of the data is computed using the Equation (3.1).

$$X(t) = \log(Y_{t+1}) - \log(Y_t), \qquad (3.1)$$

where Y_t is the weekly all-shares index at week t and Y_{t+1} is the successive weekly all-shares index. The time series plot of GSE all-shares index (Fig. 1) displays a constant growth between 2000 and 2002, which increased till mid-2004 and then started decreasing till the end of the third-quarter of 2005. The process began again and attained its maximum index in mid-2008. The rise and fall in the GSE all-shares index can be attributed to high inflation rates during the general elections in 2004 and 2008 and pressure on the economy.

The histogram plot illustrates that the log-returns are highly picked and more heavy-tailed than the normality which conformed to the studies which found that asset returns are heavy-tailed. Fig. 2 graphically demonstrates that the logarithm returns of the GSE all-shares index are not normally distributed. The density plots of Stable, Gaussian and Cauchy distributions demonstrate that the Stable distribution produces a better fit to the logarithm returns of GSE all-shares index than the Gaussian and Cauchy distributions (Fig. 3). This displays graphically that the weekly logarithm returns of GSE all-shares index come from leptokurtic distribution and are heavy-tailed, in line with Mandelbrot's (1963) proposal that the returns can be modelled by stable distributions with four parameters.

The Stable fit diagnostic tests for log-returns of the weekly GSE all-shares index comprise a density plot of the data, a Q-Q plot, a P-P plot and a Z-Z plot (Fig. 4). The Q-Q plot illustrates a good fit of the quantiles of the weekly GSE all-shares log-returns to the quantiles of the theoretical α-stable fit, but with few outliers at the tails. Also, the P-P plot displays a perfect fit of the probabilities of the weekly GSE all-shares log-returns to the theoretical probabilities of the α-stable distribution. Finally, the Z-Z plot also demonstrates a good fit to the data, where the inverse cumulative distribution of α-stable fit of the data is plotted against the theoretical inverse of the normal cumulative distribution.

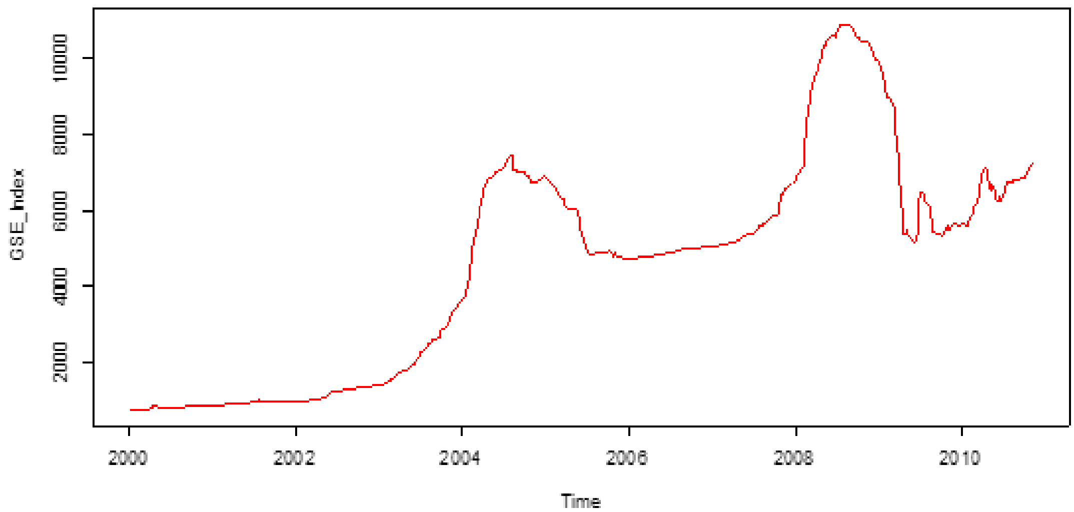

Fig. 1: Time series plot of the weekly GSE All-Share index evolution from 02/01/00 to 31/12/10.

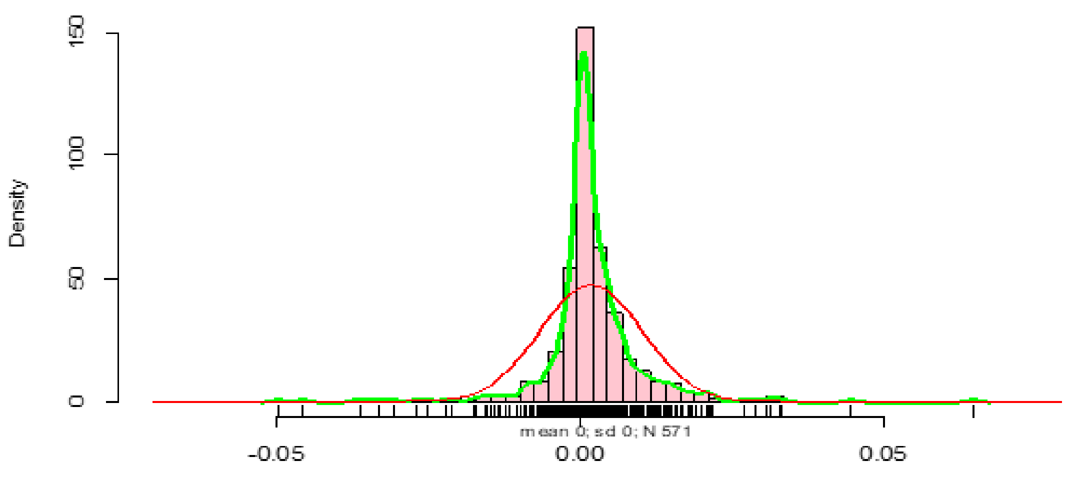

Fig. 2: A histogram plot of the weekly logarithm returns with a fitted line to the data and Normal distribution fit.

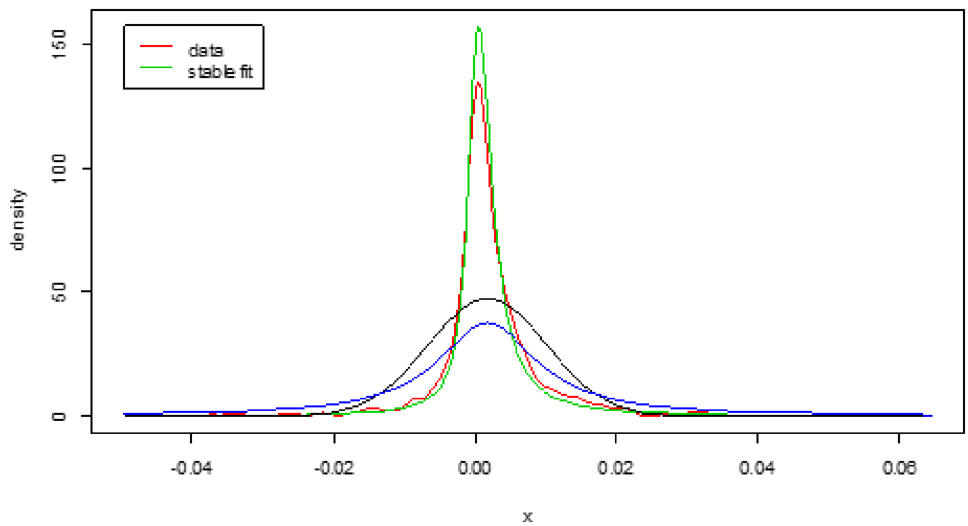

Fig. 3: The density plots of log-returns of GSE All-Shares index.
Data = red line, Stable fit = green line, Normal fit = black line and Cauchy fit = blue line

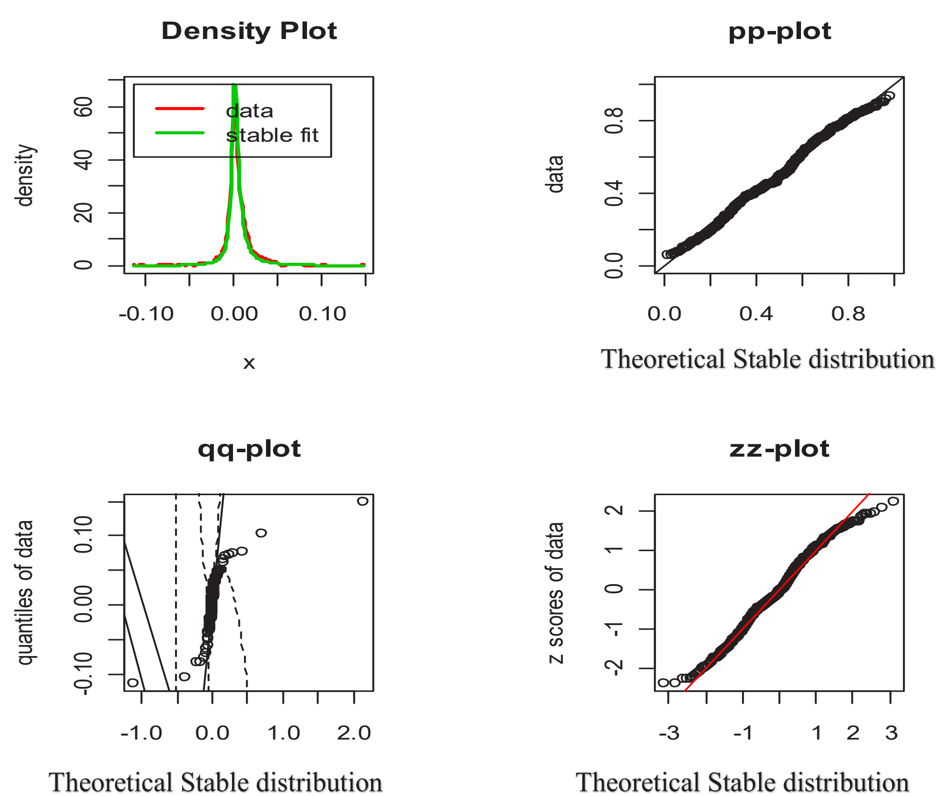

Fig. 4: The diagnostic tests of the weekly logarithm returns of GSE all-shares index.

Table 1 displays estimates of the four parameters of the stable fit with a 95% confidence interval bound of the estimates displayed in parentheses. The three methods produce different estimates of the four parameters (alpha, beta, gamma and delta). The Kolmogorov-Smirnov and Chi-Square goodness of fit tests show that maximum likelihood estimates of the stable distribution fit better to the logarithm returns of GSE all-shares index than the other two methods (Table 2). The root mean square error (RMSE), mean absolute error (MAE) and Akaike information criterion (AIC) test results also show that the maximum likelihood estimation method produces more accurate estimates than empirical characteristic function and sample quantile methods.

Table 3 demonstrates the goodness of fit for the logarithm returns of GSE all-shares index to the three distributions under consideration. The four tests for normality (Kolmogorov-Smirnov, Chi-square, Anderson-Darling and Shapiro-Wilk) rejected logarithm returns of GSE all-shares index being normally distributed even at 1% significance level. Since Anderson-Darling and Shapiro-Wilk tests cannot be used to test goodness of fit for the Cauchy and the α-stable distributions, only the K-S and chi-square tests were employed. The K-S test and chi-square test both rejected the claim that logarithm returns of GSE All-Shares index is Cauchy distributed.

The K-S and chi-square goodness of fit tests show that logarithm returns of GSE all-shares index are α-stable distributed at 5% or 10% significance level. These results conformed to those obtained by Alfonso *et al.* (2011) and Xu *et al.* (2011) which indicate that asset returns are α-stable distributed.

Table 1: Estimates of α-stable distribution with 95% confidence intervals

Methods	Alpha	Beta	Gamma	Delta
MLE	1.005 (0.104)	0.31 (0.135)	0.002 (0.0002)	0.001 (0.0002)
SQ	1.124 (0.115)	0.196 (0.155)	0.0023 (0.0002)	0.0007 (0.0003)
ECF	1.174 (0.116)	0.399 (0.155)	0.0023 (0.0002)	0.0009 (0.0003)

The 95% confidence interval bound in the parentheses (MLE: Maximum likelihood estimation; SQ: Sample Quantile; ECF: Empirical characteristic function)

Table 2: Goodness of fit tests for the estimation methods of Stable distribution

Methods	K-S test	Chi-square test	RMSE	MAE	AIC
MLE	0.048 (0.138)	3.28 (>0.20)	0.065473	0.022489	-2881.02
SQ	0.084 (0.0006)	21.36 (<0.005)	0.185857	0.037198	-2880.98
ECF	0.059 (0.035)	6.75 (>0.10)	0.078326	0.022075	-2880.98

The p-values of K-S and Chi-square test are in parentheses

Table 3: Goodness of fit tests of the stable distributions

Distributions	K-S	Chi-square	Anderson-Darling	Shapiro-Wilk
Gaussian fit	0.18 (0.00)	408.26 (<0.005)	36.78 (<0.005)	0.78 (0.00)
Cauchy fit	0.486 (0.00)	32432 (0.017)		
Stable fit	0.048 (0.138)	3.28 (>0.20)		

The p-values of the tests are in parentheses

Discussion

In this paper, we have presented the three common techniques for the estimation of the parameters of models or distributions. We show that the daily logarithm returns distribution of Ghana Stock Exchange all-shares index possesses heavy tails and can be described by a leptokurtic distribution from a general stable family of distributions. However, the Central Limit Theorem states that the sum of a large number of independent and identically distributed random variables has a limiting distribution after appropriate shifting and scaling and belongs to a stable class (McCulloch, 1996). Therefore, the diagnostics show that the weekly returns of the all-shares index of the Ghana stock exchange are well described by an alpha-stable model, which agreed with the studies of McCulloch (1996) and Nolan (2005). Though the stable models (alpha stable) do not give the perfect fit to the data, we have shown that they can give a much better fit than the normal models (distributions) do. Maximum likelihood estimation has been shown to be more powerful than the other two methods (sample quantile and empirical characteristic function) in estimating the parameters of the alpha stable model. Since not all financial data returns can be modelled with stable distributions, it is important to perform diagnostic analysis before fitting stable models to the data. Furthermore, in cases where the return distribution is non-Gaussian and leptokurtic, the sample quantile estimation method tends to outperform maximum likelihood estimation, because the former tries to match the shape of the empirical distribution and ignores the top and bottom 5% of the data (Nolan, 2005).

Conclusion

In this paper we have shown that the weekly returns of GSE all-shares index are well described with an alpha stable distribution and the daily returns possess heavy tails that come from a leptokurtic distribution. The maximum likelihood estimation method is found to produce better estimates for an alpha stable model than the other two estimation methods presented. We conclude that in managing risk and pricing returns of assets, economists and financial analysts that use GSE dataset may consider stable distributions as the basis of their analysis, since the assumption of normality is not supported. Furthermore, for a large dataset where the central limit theorem can be applied, the results may be misleading, since empirically the returns are heavy-tailed, asymmetry and leptokurtic, compared with normality. This may yield more successful risk management strategies, particularly in the forecast of weekly returns of the GSE all-shares index.

Acknowledgement

This paper is based of an MPhil thesis which was supported financially by Miss Jessica Reath, Mr. Sammy Appiah and Philipine Mordzinu. Thank you all for your support and encouragement.

References

Alfonso, L., Mansilla, R. and Terrero-Escalante, A. C. (2011). On the scaling of the distribution of daily price fluctuations in Mexican financial market index. *Physica A: Statistical Mechanics and its Applications*, Vol. 391 (10): 2990-2996.

Borak, S., Misiorek, A. and Weron, R. (2010). Models for Heavy-tailed Asset Returns. *SFB 649 Economic Risk Berlin,* 2010-049.

Borak, S., Härdle, W., Wolfgang, K. & Weron, R. (2005). Stable distributions. *SFB 649 Economic Risk Berlin,* 8.

DuMouchel, W. H. (1971). *Stable Distributions in Statistical Inference*, Ph.D. Thesis, Department of Statistics, Yale University.

DuMouchel, W. H. (1973). On the Asymptotic Normality of the Maximum–Likelihood Estimate when Sampling from a Stable Distribution. *Annual Statistics, **1**(5)*, 948–957.

Fama, E. F. (1965). The behaviour of stock prices. *Journal of Business, 38*, 34-105.

Fama, E. F. and Roll, R. (1971). Parameter Estimates for Symmetric Stable Distributions. *Journal of the American Statistical Association, 66*, 331–338.

Feller, W. (1971). *An Introduction to Probability Theory and its Application. Vol. 2* (2nd Ed.), New York: John Wiley & Sons, Inc.

Garcia, R., Renault, E. and Veredas, D. (2010). Estimation of stable distributions by indirect inference. *Journal of Econometrics*, CORE DP 2006/112.

Gnedenko, B. V. and Kolmogorov, A. N. (1954). Limit Distributions for Sums of Independent Variables. Moscow: Addison-Wesley.

Kogon, S. M. & Williams, D. B. (1998). Characteristic function based estimation of stable parameters. *A Practical Guide to Heavy Tailed Data*, pp. 311–338.

Koutrouvelis, I. A. (1980). Regression type estimation of the parameters of stable laws. *JASA, 75*, 918–928.

Lindgren, B.W. (1993). *Statistical Theory, Fourth Edition*. London: CRC Press LLC.

Lombardi, M. J. (2007). Bayesian inference for α -stable distributions: A random walk MCMC approach. *Computational Statistics and Data Analysis,* 51(5): 2688–2700.

Mandelbrot, B. B. (1963). The variation of certain speculative prices. *Journal of Business,* 36 (4): 394-419

McCulloch, J. H. (1986). Simple consistent estimators of stable distribution parameters. *Communications in Statistics, Simulation and Computation, 15*, 1109–1136.

McCulloch, J.H. (1996). Financial applications of stable distributions. *Statistical Methods in Finance*, 14, 393-425.

Mittnik, S., Rachev, S. T., Doganoglu, T. and Chenyao, D. (1999). Maximum Likelihood Estimation of Stable Paretian Models. *Mathematical and* Computer *Modelling*, 29, 275–293.

Nolan, J. P. (1997). Numerical calculation of stable densities and distribution functions. *Communication in Statistics and Stochastic Models, 13*, 759–774.

Nolan, J. P. (2003). Modelling financial data with Stable Distributions. In Rachev, S.T. (Eds.), *Handbook of Heavy Tailed Distributions in Finance* (pp: 107-129).

Nolan, J. P. (2005). STABLE program. http://academic2.american.edu/~jpnolan/stable/stable.exel.

Press, S. (1972). Estimation in univariate and multivariate stable distributions. *Journal of the American Statistical Association, 67*, 842–846.

Rachev, T. S., Menn, C. and Fabozzi, F. J. (2005). *Fat-tailed and Skewed Asset Return Distributions: Implications for Risk Management, Portfolio Selection and Option Pricing.* Hoboken, New Jersey: John Wiley & Sons, Inc.

Uchaikin, V.V. and Zolotarev, V.M. (1999). *Chance and Stability: Stable Distributions and their Applications*. Zeist: VSPBV.

Xu, W., Wu, C., Dong, Y. and Xiao, W. (2011). Modelling Chinese stock returns with stable distribution. *Mathematical and Computer Modelling, 54(1-2),* pp: 610-617.

Zolotarev, V. M. (1986). One-Dimensional Stable distributions. In American Mathematics Society Translation of Mathematics Monographs, Vol. 65, *American Mathematics Society,* Providence, RI.