

Error assessment of applying foreign life table in Ghana

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ABSTRACT

The application of foreign life tables in Ghana has been a major concern for actuaries and other stakeholders. The foreign life tables may not reflect the true Ghanaian mortality rates, as mortality experiences vary greatly between countries due to differences in geography and causes of death. However, an empirical assessment of the foreign life tables that reflect similar mortality rates to the Ghanaian population has not been studied. The objective of the study is to assess the foreign life tables that are similar to Ghanaian settings. A cohort dataset of 3,260 subjects was collected from the University of Ghana Hospital which included the subjects' cause of death and gender. The study employed the Hotelling's test and multivariate analysis of variance (MANOVA) to test whether the life tables of UK, USA, and South African populations were similar to the Ghanaian setting. The study revealed that life tables of the UK, USA and South African were significantly different from the Ghanaian mortality experience. Nevertheless, the South African life table was closest to the observed Ghanaian mortality. The study, therefore, recommended that there is a need for Ghana to have her own life table as the foreign life tables are significantly different from the Ghanaian mortality. However, the study recommends that until Ghana successfully constructs her own life table, institutions and agencies such as SSNIT and insurance companies can continue to apply the South African life table with appropriate adjustments.

Key words: Life table, mortality rates, cohort dataset, multivariate analysis of variance

Introduction

Life tables are useful tools for studying the implications of observed mortality rates in a population (Coale, Demeny, & Vaughan, 2013). Life tables provide the most complete description of mortality and serve as a key indicator of the health and wellbeing of any population. They have many applications in various areas of research where birth, death, and illness are studied. There are two forms of the life table - the cohort life table and the period or current life table. A cohort life table records the actual mortality or death till the last member of the group. A period life table is the most common form. It is based on the mortality rates for a particular year, or averages over a few consecutive years (Denton & Spencer, 2011). The period life table draws out the implications for survivorship and life expectancy of the observed age-specific mortality probabilities of a given period. This is done under the assumption that the probabilities remain

constant. Cohort and period life tables may either be complete or abridged. In Complete life tables, the functions or columns of the life table are calculated for each year of life. For the abridged life table, the ages of the year interval are greater than one, taking the initial year as 0 to 1 year (Livingstone *et al.*, 2015).

Life tables are mostly used by life insurance companies in determining the premium for their clients based on the pure risk of death. The required data needed for the construction of a life table are obtained from vital registration and population censuses. In Ghana, these accurate basic data do not exist due to lack of functioning vital registration systems, the incompleteness of coverage and errors in reporting (Mathers *et al.*, 2005). As a result, actuaries and various insurance companies in Sub-Saharan Africa have adopted foreign life tables to study the longevity of their citizens (Katara *et al.*, 2014). The actuaries apply foreign life tables to determine the amount

of premiums to be paid by a person falling in a specific age group. However, this application of the foreign life table in the Ghanaian settings may also have its shortcomings due to mortality experiences varying greatly from place to place because of differences in places of residence and causes of death.

In Ghana, there is little research on life tables. Katara *et al.* (2014); constructed an abridge life table in the Tamale metropolis. It was found that mortality among males was higher than their female counterparts in most age groups. Kpedekpo (1969) also constructed a working life table for males in Ghana and provided information on the expected average number of years of working life. Ghana Statistical Service also constructed empirical life tables in the year 2010. All these tables were computed from census data with many adjustments, estimations, and assumptions. Sometimes the correct ages of individuals were unknown, birth and death records were incomplete, and some individuals were not enumerated on a census day (Murray, *et al.*, 2000). However, assessing the foreign life tables that are similar to Ghanaian mortalities had not been studied. The main objective of the study is to assess the Foreign Life Tables that are similar to the Ghanaian setting.

Materials and Methods

This study used data from the University of Ghana hospital. The data include age and gender from 1979 to 2016 in the hospital. The construction of the life table was done by using Chiang's (1969), widely accepted method. To assess the errors in the application of foreign life tables in the Ghanaian setting, firstly paired differences (errors) were computed by subtracting the estimated life expectancy of Ghana at each age from those the three foreign countries (USA, South Africa, UK) for each sex. This resulted in a random sample from a trivariate normal population for each sex to which the Hotelling's T^2 test was used to assess if the life expectancies of the three foreign countries are significantly different from those of Ghanaian for each sex. The study further used an independent sample t-test to assess which country's life expectancy significantly differs from the Ghanaian setting. Multivariate analysis of variance (MANOVA) was also used to test whether the errors from

the foreign life expectancy were significantly different from each other and also identify the country with the minimum error for each gender.

Life Table Construction

The following represent the Chiang (1969), approached used to construct the life table;

- i. l_j , Number of people alive at age j : This column begins with setting a hypothetical number know as radix l_0 . The value of l_0 is arbitrary but always in a round figure such as 1000, 10000, 100000. The value of l_{j+1} is obtained by subtracting the number of deaths in the previous age group from its corresponding l_j .

$$l_{j+1} = l_j - d_j \quad (1)$$

- ii. \hat{q}_j , Proportion dying ($j, j + 1$): The probability that an individual alive at age will die during the age interval ($j, j + 1$) is calculated directly as

$$\hat{q}_j = \frac{d_j}{l_j} \quad (2)$$

- iii. \hat{p}_j , Proportion dying ($j, j + 1$): The probability that an individual alive at age will survive during the age interval ($j, j + 1$) is calculated directly as

$$\hat{p}_j = \frac{l_{j+1}}{l_j} \quad (3)$$

- iv. a_x , Average fraction of the last year of life for age x . Each of the people who die during the interval ($x, x + 1$) has lived x complete years plus some fraction of the year ($x, x + 1$). Usually, at the beginning of the year it is 0.10 and after 5 years it is taken as 0.5.

- v. L_x The number of years lived in the interval ($x, x + 1$). Each member of the cohort who survives the year ($x, x + 1$) contributes one year to L_x , while each member who dies during the year ($x, x + 1$) contributes to an average fraction a_x . Hence the formula is

$$L_x = l_x - (1 - a_x)d_x \quad (4)$$

- vi. T_x The total number of people lived beyond age x . This is equal to the sum of the number of years lived in each interval. This implies

$$T_x = L_x + L_{x+1} + \dots + L_n \quad (5)$$

- vii. \hat{e}_x Observed expectation of life at age x . This is the average number of years yet to be lived by an individual at exact age x .

$$\hat{e}_x = \frac{T_x}{l_x} \quad (6)$$

Multivariate Analysis of Variance (MANOVA)

Let the paired difference of Ghana mortality settings and the foreign countries be represented as;

UK $x_{10}, x_{11}, \dots, x_{1n}$

USA: $x_{20}, x_{21}, \dots, x_{2n}$

South Africa (SA): $x_{30}, x_{31}, \dots, x_{3n}$

μ_1 = mean paired difference from UK life table

μ_2 = mean paired difference from USA life table

μ_3 = mean paired difference from South Africa (SA) life table

Hypotheses

$$H_0 : \mu_1 = \mu_2 = \mu_3$$

$$H_1 : \mu_1 \neq \mu_2 \neq \mu_3$$

Test Statistic

$$\Lambda = \frac{|W|}{|B+W|} = \det[W(B+W)^{-1}] \quad (8)$$

Where

$$W = \sum_{l=1}^g \sum_{i=1}^{nl} (x_{li} - \bar{x}_l)(x_{li} - \bar{x}_l)' \text{ with a degree of freedom of } g-1$$

$$B = \sum_{l=1}^g nl(\bar{x}_l - \bar{x})(\bar{x}_l - \bar{x})', \text{ with a degree of freedom of}$$

$$\sum_{l=1}^g nl - g$$

g represents the number of groups

Decision Rule

$$\text{Reject } H_0 \text{ if } -\left(n-1-\frac{p+g}{2}\right) \ln \Lambda \geq \chi^2 p(g-1)\alpha$$

Where p is the number of variables; i.e., paired difference for males and females of all the foreign life tables and g refers to the number of groups; i.e., South Africa, USA and UK.

Hotelling's T^2 Test

Let μ_g be the mean paired difference from the Foreign life table, where $g = 1, 2, 3$

Hypotheses

$$H_0 : \mu_g = 0, \quad g = 1, 2, 3$$

$$H_1 : \mu_g \neq 0, \text{ for at least one } g$$

Test Statistic

$$T^2 = (\bar{x} - \mu_0)' \left(\frac{1}{n} S^{-1}\right) (\bar{x} - \mu_0) = n(\bar{x} - \mu_0)' S^{-1} (\bar{x} - \mu_0) \quad (9)$$

Where

$$\bar{x} = \frac{1}{n} \sum x_i \quad S = \frac{1}{n-1} \sum (x_i - \bar{x})(x_i - \bar{x})' \quad \text{and}$$

$$\mu_0 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Decision Rule

Reject H_0 at α the level of significance if

$$T^2 > \frac{(n-1)p}{(n-p)} F_{g, n-g}(\alpha)$$

Empirical Results

This section presents the results of assessing the error of applying foreign life tables in Ghana. The analysis includes results of life expectancy, Hotelling's T^2 , independent sample t-test, MANOVA and ANOVA.

Comparing the Study Life Table with Other Foreign Life Tables

In order to assess the errors of applying the foreign life tables to Ghanaian mortalities, the life expectancy of the complete life table constructed was used to compare three foreign life tables. The foreign life tables were USA, United Kingdom and South Africa. The life expectancies of the countries and those of this study were presented in Figure 1.

Comparing the life expectancy of the various countries in Figure 1, it is clear that the life expectancy at birth for males decreases from a maximum of 79.09 for UK followed by USA (76.53), then South Africa (59.30) with Ghana recording the least life expectancy at birth of 49.51. The same order can be observed for the life expectancy at birth for females of the countries under investigation. Thus, in order of decreasing magnitude, we have UK, USA, South Africa and Ghana with corresponding life expectancy at birth to be 82.88, 81.11, 64.66 and 53.54 respectively. The graph revealed that the life expectancy of USA and UK behaved in almost the same manner and they are far away from the Ghanaian life expectancy. The South Africa's life expectancy is close to that of Ghana.

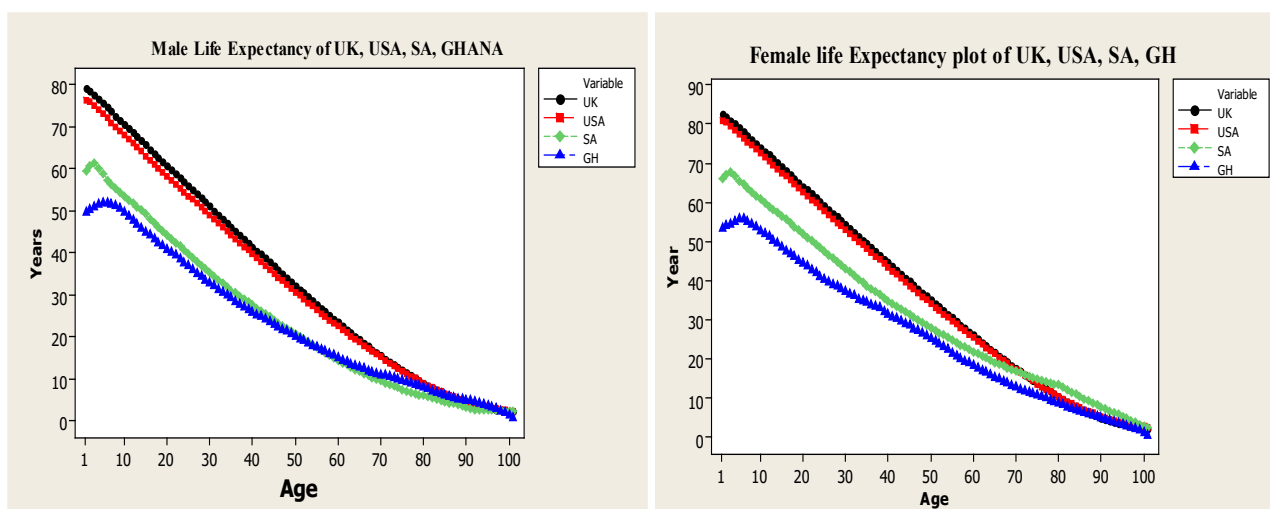


Fig. 1. A graph of Life Expectancy in the various Countries

Hotelling's T^2 test was used to assess if the foreign life tables (South African, USA and UK) were similar to Ghanaian settings. The null hypothesis states that all the foreign life tables (USA, SA, UK) are similar to that of Ghana; i.e., on the average the difference between the age specific life expectancies between each of the foreign countries and Ghana is equal to zero. From Table 1, it can be seen that the Hotelling's T^2 tests for males ($T^2 = 187.12$; p -value = 0.000) and females ($T^2 = 185.60$; p -value = 0.000) are all significant. This

implies that we reject the null hypothesis which states that the foreign life tables (USA, SA, UK) are similar to Ghanaian settings. Therefore, there is sufficient evidence to conclude that at least one of the foreign life tables of both males and females is significantly different from the Ghanaian setting.

Table 1. Hotelling's T^2 Test

Gender	Hotelling's T^2	Critical Value	P-value
Male	187.12	8.26	0.000
Female	185.60	8.26	0.000

To identify which of these foreign life tables is significantly different from the Ghanaian settings, an independent t-test was done for the paired difference of each foreign life table and the Ghanaian setting. The null hypothesis states that on average the foreign life table is similar to the Ghanaian life table, while the alternative states that the foreign life table is different from the Ghanaian life table. From Table 2, it can be seen that all the test statistics were positive and significant for both males and females in all the countries. Therefore, we reject the null hypotheses that on average the foreign life tables are similar to Ghanaian setting. Hence, all the foreign life tables for both males and females are significantly different from the Ghanaian setting.

Table 2. Identifying Which Country Significantly Differs from Ghanaian settings

Gender	Country	Mean	T-value	P-value
Male	USA	10.01	13.29	0.000
	SA	1.78	6.89	0.000
	UK	11.39	13.30	0.000
Female	USA	9.85	13.17	0.000
	SA	2.47	7.82	0.000
	UK	10.68	13.12	0.000

Assessing if the foreign life tables (South African, USA and UK) behave similarly

The MANOVA was used to test whether the paired difference between the foreign life tables (South African, USA and UK) and Ghana mortality setting were significantly different from each other. The null hypothesis states that there is no significant difference between the foreign life tables in terms of gender while the alternative state that at least one of the foreign life tables differs from the rest. From Table 4, it can be seen

that the p-value was less than 0.001, hence we reject the null hypothesis at 5% level of significance and conclude that at least one of the paired differences of male and female foreign life table significantly differ from the rest.

Table 4. Multivariate Analysis of Variance (MANOVA)

Source of Variation	Df	Wilk's Lambda	p-value
Treatment	2	42.867	0.000
Residual	300		
Total	301		

Table 5 represents an Analysis of Variance (ANOVA), which was done to find out which of the variables (paired differences for males on one side and females on the other) are significantly different among the three countries. The ANOVA table shows that the paired difference of males ($F = 63.09$; $p - value = 0.000$) and females ($F = 42.81$; $p - value = 0.000$) among the countries are significantly different. It can also be seen that for both sexes, UK recorded the highest mean difference, followed USA and South Africa. Therefore, the mean paired difference of South African is significantly less than the mean paired difference of USA and UK. Similar results were seen in the case of females where the mean paired difference of South African is significantly less than the mean paired difference of USA and UK. This is because among the three foreign countries the South Africans way of life and causes of death are relatively similar to those of Ghana than those of the foreign countries.

Table 5: Univariate Analysis of Variance (ANOVA)

Gender	Country	Mean	F-value	p-value
Male	USA	10.01	63.09	0.000
	SA	1.78		
	UK	11.39		
Female	USA	9.85	42.81	0.000
	SA	2.47		
	UK	10.68		

Discussion

This paper, constructed a local life table from Ghanaian mortality. The resulting life expectancies at the various ages were compared with corresponding life expectancies from three foreign life tables. The estimated life expectancies at birth for males and females were 49.51 and 53.54 years respectively for Ghana. These results are confirmed by Katara *et al.* (2014) who constructed an abridge life table using deaths experiences and number of inhabitants in Tamale metropolis. The study also shows clearly that foreign life tables (UK, USA, and South African) were significantly different from the Ghanaian setting. According to Mathers *et al.* (2001), the life expectancy of countries differs due to the way of life and health performances. This explains why the foreign life tables significantly differ from the Ghanaian setting. Moreover, the mean difference between the foreign life expectancy and that of the study was positive. This implies that the life expectancies of the foreign countries were higher than the Ghanaian settings. Therefore, premiums of insurance companies which are calculated based on the foreign life tables are underestimated. The study also revealed that the life expectancy of USA and UK behaved similarly while among the three countries South Africa's life table is closest to the Ghanaian mortality.

Conclusion

This paper has shown clearly that applying a foreign life table to Ghanaian mortality has a significant deviation (errors). These errors can have serious implications to the pricing of insurance products, the viability of pension plans and governmental policy decisions. Hence there is a need for Ghana to develop her own life table from local empirical data.

Among the foreign life tables, the South African exhibited the least deviation, therefore, in the short-term, institutions and agencies such as SSNIT and insurance companies can continue to apply the South African life table with appropriate adjustment. However, investments should be made towards the development of an accurate life table based on local experience.

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