



FITTING DIAMETER DISTRIBUTION FOR *Gmelina arborea* (Roxb.) STANDS IN OMO FOREST RESERVE, SOUTH WESTERN NIGERIA

Adedeji E. T. Adekunle V. A. J. Adeagbo D. O.

Federal University of Technology, Department of Forestry and Wood Technology, Akure, Ondo State, Nigeria

*Corresponding Author: adedejiet@futa.edu.ng; +2347062531436

ABSTRACT

Effective forest plantation management and decision making can be achieved by applying probability distribution to predict the stand status in diameter class distribution. The purpose of this study is to develop a suitable diameter distribution model for different age series of *Gmelina arborea* plantation stands in Omo Forest Reserves Ijebu East, Ogun State, south-western Nigeria. The *Gmelina* plantations were established in 2005, 2008, 2010, 2012 and 2014 respectively. A total of 30 temporary sample plots of equal size (25m x 25m) were laid while complete enumeration was carried out in each sampled plot and tree growth variables such as diameter at breast height and total height of all trees were measured. Eight diameter distribution functions that rank best includes Gen.Pareto, Beta, Lognormal (3P), Weibull (3P), Gamma (3P), Exponential (2P), Chi squared and General Extreme Value were fitted and assessed with Kolmogorov Smirnov (K-S), Anderson Darling and Chi-Square. General Pareto Distribution with D value of 0.04382 was ranked the best distribution with the Kolmogorov Smirnov test. The positive skewness recorded is an indication that most of the diameter at breast height are concentrated in the lower diameter class and it is an indication of a healthy and vigour forest.

Keywords: Diameter distribution models; Skewness; Fiting; Omo; Forest Models

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INTRODUCTION

Tropical forests, often referred to as the Earth's green lungs, are among the most biologically diverse and ecologically important ecosystems on the planet. These forests harbor an immense variety of tree species, each adapted to specific ecological niches and contribute significantly to global carbon sequestration, climate regulation and biodiversity conservation (Adeduntan, 2009; Smith, 2023). Understanding the structure and dynamics of tropical tree stands is crucial for sustainable forest management, conservation efforts, and mitigating the impacts of climate change (Keenan, 2015).

According to De Lima (2017), one fundamental aspect of characterizing the structure of tropical forests is through diameter distribution

functions. Diameter distribution functions provide a quantitative representation of the distribution of tree sizes within a given forest area. They describe the proportion of trees in various diameter size classes and help to elucidate the population structure, growth patterns, and recruitment dynamics of tree stands (Adedeji et al., 2020). The diameter distribution of tropical tree stands is influenced by a myriad of ecological factors such as species composition, forest age, disturbance history, soil characteristics and climatic conditions. Different forests may exhibit distinct diameter distributions, reflecting their unique ecological context and historical development (Delima et al., 2017).

There are various mathematical models used to describe diameter distributions in tropical tree stands. Commonly employed models include the Weibull, Lognormal, Gamma, and Negative Exponential distributions. Each of these models has its strengths and limitations and the choice of a particular model depends on the characteristics of the tree stand being studied and the research objectives (Aigbe, 2014; Adedejiet *al.*, 2020). Despite the significance of diameter distribution functions, studying tropical tree stands can be challenging due to the vast and remote nature of many tropical forests, making data collection time-consuming and expensive. Additionally, the diversity of species and growth forms within these forests

adds complexity to the analysis. This study aims to explore the current state of knowledge regarding diameter distribution functions of tropical tree stands in Omo Forest Reserve with a view to contribute to a more comprehensive understanding of these critical ecosystems and their role in the global environment.

MATERIALS AND METHODS

Study Area

This study was conducted at Ogun State Afforestation Project located in Omo Forest Reserve. The Forest Reserve is located in Ijebu East Local Government Areas of Ogun State, Nigeria.

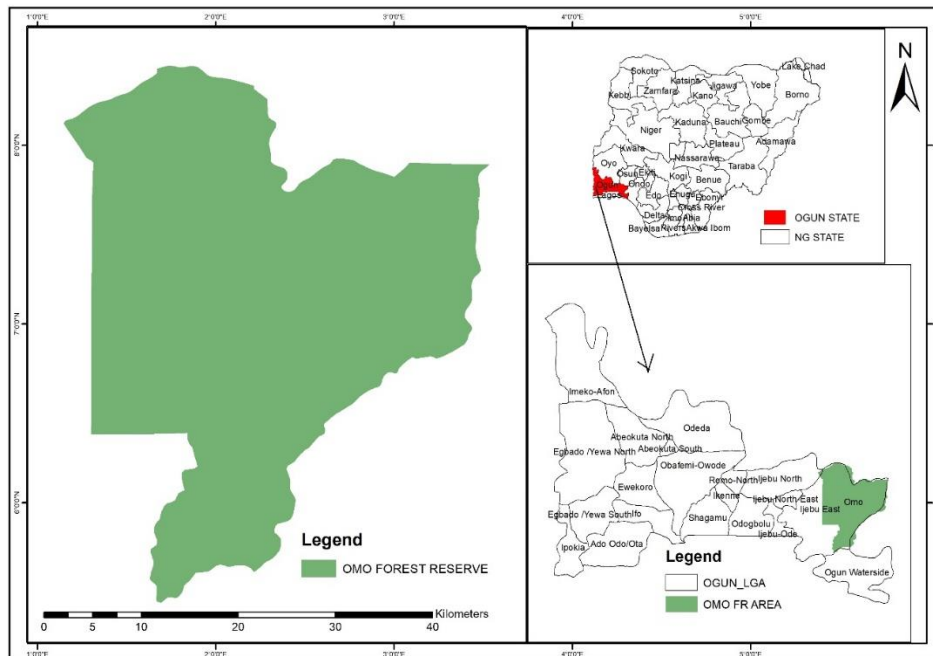


Figure 1: Map of Omo Forest Reserve

Omo Forest Reserve is situated in Ijebu East Local Government Areas of Ogun State, Nigeria, between Latitudes 6°35' and 7°05'N and Longitudes 4°19' and 4°10'E (Chimaet *al.*, 2009; Ogana *et al.*, 2017). It is located in southwest Nigeria's high woodland zone. The region experiences a moist monsoon climate with annual rainfall ranging from 1,500 mm to 1,800 mm, with the most of it falling between March and October. The bimodal distribution of rainfall shows a sharp reduction in August.

Data Collection

Six temporary sample plots were alternatively laid on a line transect of 200 m long. The *Gmelina* plantations were established in 2005,

2008, 2010, 2012 and 2014 respectively. A total of 30 temporary sample plots of equal size (25m x 25m) were laid while complete enumeration was carried out in each sampled plot and tree growth variables such as diameter at breast height and total height of all trees were measured. Eight diameter distribution functions were fitted and assessed with Kolmogorov Smirnov (K-S), Anderson Darling and Chi-Square

Data Analysis

Fitting of Diameter Distribution Model

In fitting the data, various distribution methods for statistic test were tried using Kolmogorov Smirnov, Anderson Darling and Chi-squared

goodness of fit to rank them accordingly. Relatively, eight best ranked distributions were chosen to fit the diameter distribution model. The distribution used for fitting the diameter at breast height data were Gen.Pareto, Beta,

Lognormal (3P), Weibull (3P), Gamma (3P), Exponential (2P), Chisquared and General Extreme Value were fitted and assessed with Kolmogorov Smirnov (K-S), Anderson Darling and Chi-Square.

1. Gen. Pareto Distribution Function

$$\mu \leq x < +\infty \text{ for } k \geq 0; \mu \quad f(x) = \begin{cases} \frac{1}{\sigma} \left(1 + k \frac{(x-\mu)}{\sigma}\right)^{-1-1/k} & k \neq 0 \\ \frac{1}{\sigma} \exp\left(-\frac{(x-\mu)}{\sigma}\right) & k = 0 \end{cases} \dots\dots\dots 1 \leq x < \mu - \sigma/k \text{ for } k < 0$$

2. Beta Distribution Function

The general expression of the beta distribution for a random variable x is given by (Loetsch et al., 1973):

$$f(x) = \frac{y^{\alpha-1} \cdot 1 - y^{\beta-1}}{B. (\alpha, \beta)} \dots\dots\dots (2)$$

3. Lognormal distribution function

This is a continuous distribution and its natural logarithm has a normal distribution.

$$F(x) = \frac{1}{\sqrt{2\pi}\delta x} \exp\left[-\frac{1}{2\delta^2} (\ln x - \mu)^2\right] \dots\dots (3)$$

$\delta > 0, x > 0, 0 < \mu < +\infty$

4. Weibull Distribution function

The probability density function for Three-Parameter Weibull was used in this study. Three-parameter (3P) Weibull distribution (Weibull, 1951) is expressed as:

$$f(x) = \frac{c}{b} \left\{ \left(\frac{x-a}{b}\right)^{c-1} \exp\left[-\left(\frac{x-a}{b}\right)^c\right] \right\} \dots\dots (4)$$

with $x > a, b > 0, c > 0$.

where: x= tree diameter (Dbh), a= location parameter, b= scale parameter, c= shape parameter.

5. The Gamma Function

The model of the gamma PDF has the following expression for a continuous random variable x (Podlaski, 2006; Nelson, 1964):

$$f(x) = \frac{(x-\gamma)^{\alpha-1}}{\beta^\alpha \Gamma(\alpha)} \cdot e^{-\left[\frac{(x-\gamma)}{\beta}\right]} \dots\dots\dots (5)$$

with $x > \gamma, \alpha > 0$ and $\beta > 0$, where α is the shape parameter, β is an inverse scale parameter, γ is the location parameter ($\gamma = 0$ for the two-

parameter gamma distribution) and $\Gamma(\cdot)$ is the gamma function.

6. Exponential Distribution Function (2P)

$$f(x) = \lambda \exp(-\lambda(x-y)) \dots\dots\dots (6)$$

7. Chi squared Distribution Function

$$f(x) = \frac{(x-\gamma)^{v/2-1} \exp(-(x-\gamma)/2)}{2^{v/2} \Gamma(v/2)} \dots\dots\dots (7)$$

$Y \leq x < +\infty$

8. General Extreme Value Distribution Function

$$f(x) = \begin{cases} \frac{1}{\sigma} \exp(-(1+kz)^{-1/k}) (1+kz)^{-1-1/k} & k \neq 0 \\ \frac{1}{\sigma} \exp(-z - \exp(-z)) & k = 0 \end{cases}$$

$$1 + K \frac{(x-\mu)}{\sigma} > 0 \text{ for } k \neq 1$$

$$-\infty < x < \infty \text{ for } k = 1$$

RESULTS

The summary of descriptive statistics and goodness of fit of the tested distribution functions examined in this study was presented in Table 1 and 2. The 2014 stand has the lowest standard error while the highest is observed in the 2005 stand. The skewness value 0.69 is highest in the 2014 stand and lowest 0.09 in the 2005 stands. Moreover, similar trend was observed in the excess kurtosis value 0.13 in the 2014 stand and lowest -0.26 in the 2005 stand. In the pooled data for the entire stands, the coefficient of variation is 0.36, standard error 0.22, skewness 0.78, and excess kurtosis 0.11.

Table 1: Descriptive Statistic for Diameter Distribution of the Age Series in *Gmelina arborea* Stands in Omo Forest Reserve

Statistics	2014 Value	2012 Value	2010 Value	2008 Value	2005 Value
Sample size	194	184	196	140	189
Range	8.6	15	22	17.6	26.6
Mean	12.85	14.93	18.23	20.75	28.29
Variance	3.46	10.46	29.66	13.44	28.42
Std Deviation	1.86	3.23	5.45	3.67	5.33
Std. error	0.13	0.24	0.39	0.30	0.39
Minimum	10	10	10.2	12.5	15.9
maximum	18.6	25	32.2	30.1	42.5
Skewness	0.69	0.67	0.30	-0.37	0.09
Excess Kutosis	0.11	-0.17	-0.98	-0.26	-0.26
Sum	2493.30	2747.4	3574.2	2904.5	5346

Table 2: Descriptive Statistic for Diameter Distribution for the Pooled *Gmelina arborea* Plantation in Omo Forest Reserve

Statistics	Value	Percentile	Value
Sample size	903	Min	10
Range	32.5	5%	10.5
Mean	18.89	10%	11.5
Variance	47.43	25% (Q1)	13.2
Std Deviation	6.89	50%(median)	17.4
Coefficient of Variation	0.36	75%(Q3)	23.4
Std. error	0.22	90%	28.98
Skewness	0.78	95%	32.28
Excess Kutosis	-0.11	Max	42.5

Table 3 shows the value of the summary of the goodness of fit for assessing the distribution functions in the study. The distribution functions were assessed with Kolmogorov Smirnov (K-S), Anderson Darling and Chi-Square. Out of several distributions assessed eight were selected based on their ranking as shown in Table 3. The eight selected distributions were further tested with Kolmogorov Smirnov (K-S), Anderson Darling and Chi-Square, after which Gen. Pareto was adjudged as the best distribution that can adequately fit the diameter in *Gmelina* stands of Omo Forest Reserve, the other two were Beta and Weibull (3P). In Anderson Darling, the

three parameters that gave the best distribution functions with according to their statistical value were Gen. Pareto (1.8435), followed by weibull (3P) (4.4618) and Gamma (3P) (4.9003). The same trend was noticed with the use of Chi-square as the test statistic, where the three parameters that gave the best distribution functions with the Statistical value of 39.162, 39.978 and 46.467 for Gen Pareto, Weibull (3P) and Gamma (3P) respectively. The summary of all parameters of the selected distributions were presented in Table 4. Moreove, the critical values ($\alpha=0.05$) of test statistic for assessing the adjudged best distribution model of Gen. Pareto is shown in Table 5.

Table 3: Summary of Goodness of fit for assessing the Distribution Functions of the *Gmelina arborea* Stand in the Study Area

#	Distribution No	Kolmogorov Smirnov		Anderson Darling		Chi-squared	
		Statistic	Rank	Statistic	Rank	Statistic	Rank
9	Gen. Pareto	0.04382	1	1.8435	1	39.162	1
1	Beta	0.0444	2	13.462	10	N/A	
23	Weibull (3P)	0.04653	3	4.4618	2	39.978	2
7	Gamma (3P)	0.07293	4	4.9003	3	46.467	3
5	Exponential (2P)	0.08246	5	10.878	8	72.356	4
2	Chisquared	0.08429	6	19.307	12	13.16	16
14	Lognormal (3P)	0.08651	7	8.5668	4	73.082	5
8	Gen. Extreme Value	0.08759	8	9.5764	5	97.847	10

Table 4: Summary of Parameters for the selected Diameter Functions for *Gmelina arborea* Plantation

Distribution	a	b	K	V	α_1	α_2	β	γ	λ	σ	μ
Gen. Pareto			-0.35							12.17	9.89
Beta	10.0	44.48			0.98	2.79					
Weibull (3P)					1.22	9.50		9.98			
Gamma (3P)					1.32		6.77	9.97			
Exponential (2P)								10.0	0.11		
Chisquared				18							
Lognormal (3P)								8.32		0.72	2.12
Gen. Extreme Value			0.04							5.34	15.62

Table 5: Critical values ($\alpha=0.05$) of test statistic for assessing the distribution models

Distribution	T-stat	T-crit
Kolmogorov- Smirnov	0.04	0.04
Anderson- Darling	1.84	2.50
Chi-squared	31.96	16.91

Table 6 shows the dbh frequency distribution of 901 trees in the observed diameter class and their evaluation with Gen. Pareto probability distribution 3.3cm dbh class interval. The result of the dbh frequencies shows that there are more trees in the lower dbh class than the upper dbh class. Table 7 shows the the t-test analysis on the

most flexible model. The t= test carried out on the observed and predicted dbh frequencies shows that there is no significant difference ($p > 0.05$) between the observed and predicted frequencies. The correlation coefficient between the observed and predicted dbh frequencies was $R= 0.98$.

Table 6: Observed and Predicted Diameter Distribution According to diameter classes with Gen. Pareto for *Gmelina arborea* Plantation

Dbh Class	Observed	Predicted
10.0 – 13.3	234	238
13.4 – 16.5	180	180
16.6 – 19.8	108	133
19.9 – 23.0	131	117
23.1 – 26.2	108	108
26.3 – 29.5	58	72
29.6 – 32.8	50	45
32.9 – 36.0	18	18
36.1 – 39.3	9	9
39.4 – 42.56	5	3

Table 7: T-Test Analysis for the most Flexible Distribution Model

Distribution	T-stat	T-crit	P-value	Remark
Gen. Pareto	-0.03	2.20	0.87	ns

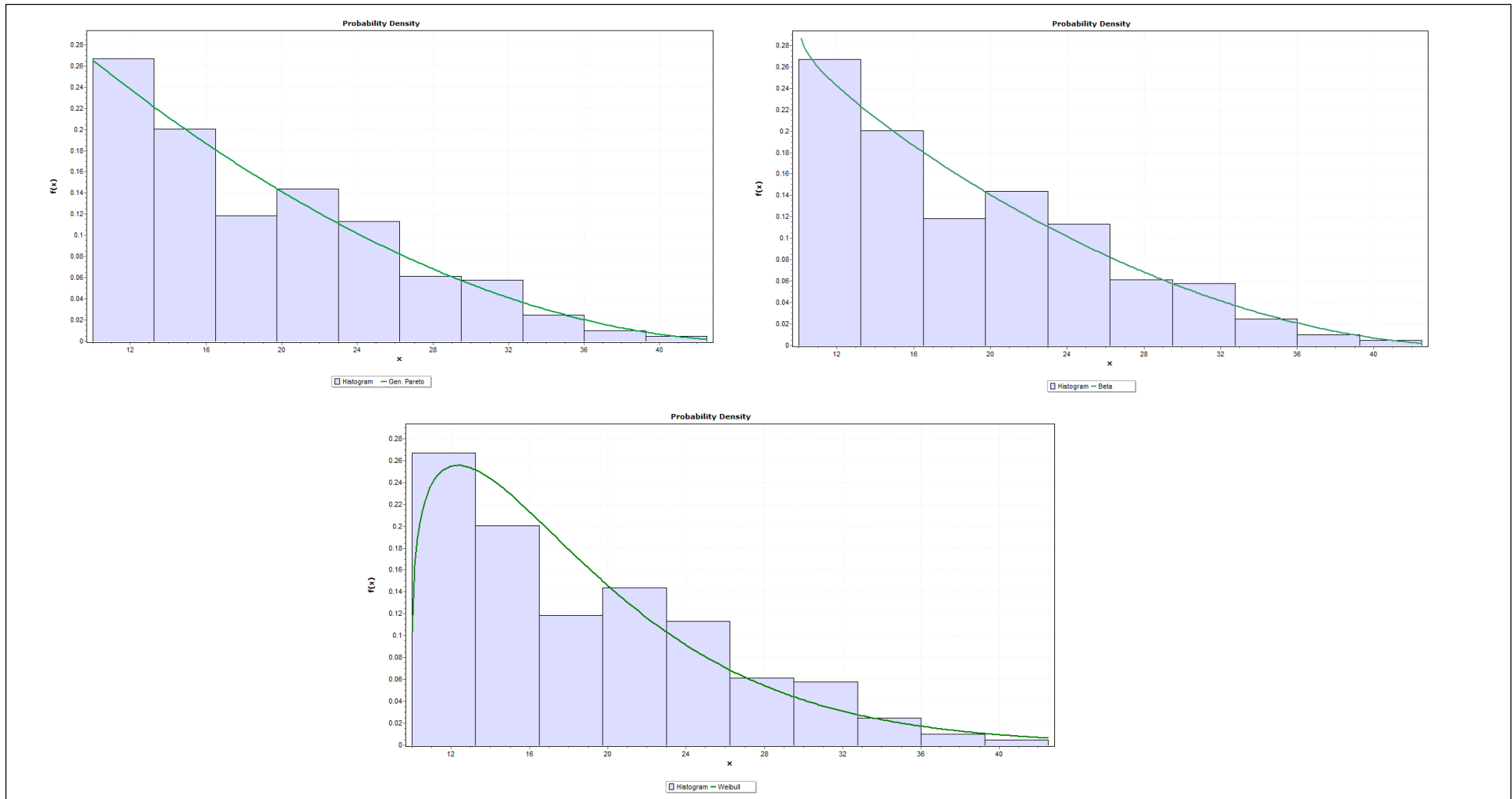


Figure 1: Graphs of observed and estimated probability functions of DBH class selected for *Gmelina arborea* Plantation

DISCUSSION

Diameter Distribution were fitted and assessed with Kolmogorov Smirnov (K-S), Anderson Darling and Chi-Square for the *Gmelina arborea* Stand in Omo Forest Reserve. Eight diameter distribution functions: Gen.Pareto, Beta, Lognormal (3P), Weibull (3P), Gamma (3P), Exponential (2P), Chisquared and General Extreme Value was best after the critical values of test statistic for assessing the distribution models with minimum D-Values were taken. Critical value of the test of statistics is best assessment criteria in describing the diameter distribution of the stands (Aigbe, 2014; Adedeji et al., 2020). The graphs of observed and estimated Probability Distribution Functions were used to validate the output of the distribution models. The descriptive statistics and percentile values of the distribution are presented in Table 1 and Table 2. The results revealed that the mean diameter value was 12.85, 14.93, 18.23, 20.75 and 28.29 cm for 2014, 2012, 2010, 2008, 2005 and 18.89 cm for the pooled data respectively. The positive skewness and peakedness recorded is an indication that most of the values are in the lower diameter class (Gadow, 1983). This is consistent with Adekunle (2002), who reported positive skewness distribution pattern for Ala and Omo Forest Reserves in Nigeria. The implication of this is that the forests are still undergoing regeneration and recruitment, which are vital indicators of forest health and vigour.

The Kolmogorov Sminorv, Anderson Darling and Chi-squared results for the eight distributions for the study area are shown in Table 3. The results revealed that the General Pareto Distribution with D value of 0.04382 was ranked the best distribution with the Kolmogorov Sminorv test, this is followed by Beta (0.0444) and the third is Weibull (3P) with a D- value of 0.04653. They were selected because their tabulated D-value were less than 0.05. This result is in accordance with Mataji et al (2000) and Akindele (2002) have demonstrated the use of Weibull probability distribution functions for predicting diameter distribution in even aged stand.

Furthermore, all the eight best distributions provided a good fit for the diameter distribution but the best three were selected. This is because their statistical values were less than the critical value at 0.05 significant levels this implies that the null hypothesis was accepted for all distribution. The graphs of the observed and estimated probability functions of the DBH distribution functions show that there is no significant difference ($p > 0.05$) between the empirical and theoretical cumulative functions (figure 1). This means there is no difference between the observed and predicted diameter frequencies as also indicated in the student T-test analysis. The correlation coefficient between the observed and predicted dbh frequencies was $R = 0.98$. This finding is in accordance with the report of Aigbe (2013) and Ige et al. (2013) who reported a similar correlation coefficient value for Afi River and Onigambari Forest Reserve.

CONCLUSION

Diameter distribution models play a pivotal role in understanding the structure, dynamics and ecological significance of tree stands in tropical forests. Through these mathematical representations, researchers can gain valuable insights into the abundance, growth, mortality and regeneration patterns of trees, which are fundamental for effective forest management and conservation strategies. The diameter distribution model was estimated effectively using the probability density function graph. Probability distributions were employed to estimate the diameter distribution in this study and statistical approaches were used to develop diameter distribution models. General Pareto Distribution with D value of 0.04382 was ranked the best distribution with the Kolmogorov Sminorv test and positive skewness recorded is an indication that most of the diameter at breast height are concentrated in the lower diameter class.

RECOMMENDATION

The model developed in this study for diameter distribution are recommended for application of scheduling silvicultural treatment.

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