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NONLINEAR HEIGHT-DIAMETER MODELS FOR *Gmelina arborea* (Roxb.) IN ACHUSA, MAKURDI LOCAL GOVERNMENT AREA OF BENUE STATE, NIGERIA

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ABSTRACT

Gmelina arborea belongs to the family Lamiaceae and is a rapid growing species that is often grown to produce timber, paper and pulp as well as fodder. The height of a tree is a very vital information to monitor biomass and carbon stocks. Due to the increasing demand and high consumption for wood and wood-based forest products has incessantly been on a spike, and it could multiply because of an increase in the growing population. There is therefore the need to develop height-diameter models for the study area. This study was carried out at Achusa, in Makurdi Local Government Area of Benue State. The simple random sampling technique was used for the establishment of ten (10) sample plot size of 20m x 20m in the woodland. Six (6) non-linear height-diameter relationship model forms were used and evaluated statistically. The results of the fitted height-diameter models showed that Chapman-Richards and Weibull model were the best function for estimating tree height of Gmelina arborea for the training dataset. They had the smallest RMSE 3.223, a Bias of -0.005, R^2value of 0.734, AIC of 510.314 and 510.321 respectively. For the validation dataset the Logistics model was the best with the smallest RMSE 2.906 and Bias of 0.013. Chapman-Richards and Weibull model were the best models in predicting tree height based on diameter at breast height values within the study area. The research recommends that variables such as soil fertility, density, spacing, crown area, age and silvicultural practices can be incorporated in height-diameter models so as to improve the accuracy and reliability of the models.

Keywords: *Gmelina arborea*, non-linear models, height-diameter, Chapman-Richards, Weibull

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INTRODUCTION

Gmelina arborea is a rapid growing species of the family Lamiaceae that varies from a shrub to medium to big size deciduous tree (Singh *et al*., 2021). It can grow up to 35 m in total height and a diameter of 3 m (Iwuoha *et al*., 2021). The IUCN catergorised it as a species that falls under the Least Concern category (de Kok, 2019). *Gmelina arborea* is not threatened (Iwuoha and Seim, 2023). *Gmelina arborea* originates from Asia. However, it has been widely planted in tropical areas of West Indies, Australia, America, Africa and numerous islands in Pacific Ocean (PROTA, 2016). The artists of Central India use the close- grained wood of *Gmelina arborea* as the standard material for carving (Krishen, 2013). The species has diverse use in the form of plantation and timber tree, landscape ornamental because of its large leaves and medicinal plants and utilized in light construction, industrial wood, it is a preference for paper and pulp, fodder production and it has a wide application in agroforestry and woodlot systems because it is used as a shade producer (Singh *et al*., 2021).

The demand in the medicinal industry is high (500–1000 metric tonnes annually) (Warrier *et al*., 2019). The root bark is the main part of interest, the tree is destroyed before harvesting leading to the removal of the entire tree (Warrier *et al*., 2021). There is a continuous reduction in the site extent and quality of the habitat (de Kok, 2019). The plants are used as a raw material to extract semi-synthetic compounds for the perfume, cosmetic as well as food sectors and it is an important material of therapeutic agents (Shakya, 2016). The height of a tree is a very vital information in order to understand the autecological characteristics (e.g., size, ontogenetic stage, fitness, trophic position) and important to monitor biomass and carbon stocks (Sullivan *et al*., 2018). In the discourse of global climate change and national biodiversity conservation, the forest has gotten an increase notice because of its pivotal role in carbon stock reservoir and productivity (Anderson *et al*., 2023). Direct estimation of forest biomass in wide locations in a state is unrealistic and requires applying refined statistical methods (Von Gadow *et al*., 2021).

Forest management on the bases of sustainable development principles needs correct and practical data (Baumgartner, 2019). Tree height and diameter at breast height are used to describe vertical and horizontal structures of a forest stand as well as in decision making in forest management (Baral *et al*., 2020; Subedi *et al*., 2021). Simple height-diameter models, having just diameter as a predictor variable, is more eligible for a location that has uniform stand characteristics despite the fact that there is a strong correlation between height and diameter, the relationship can vary by stand characteristics and tree species in large locations that have heterogeneous stand structures (Seki, 2022). Bhandari *et al*. 2021 noted that height-diameter allometry of Jarrah species and growth of Oriental beech (Yucesan *et al*., 2015) were influenced by thinning.

Nonlinear regression models have various merits, which include creating biologically logical estimations, interpretability and parsimony, that is small is required in order to know the relationships between the variables (Mehtätalo *et al*., 2015). Nonlinear height–diameter models are more accurate compared to linear models in showing height–diameter relationship of individual trees (Han *et al*., 2021; Bolat *et al*., 2022). Tree heights not measured can be estimated by using a befitting height–diameter model (Koulelis *et al*., 2018). Nonlinear models are highly recommended to forest managers, because it uses one factor as input and gives great results despite the environmental effects (Štefančík *et al*., 2018).

Due to the increasing demand and high consumption of wood and wood-based forest products has incessantly been on a spike, and it could multiply due to increasing growing population. There is need to develop height-diameter models for the study area. Numerous research has been conducted to develop height-diameter models, but many of these models are usually species and location specific (Eby *et al*., 2017; Ogana *et al*., 2020), therefore, there is need to develop height-diameter models for this tree species in the study area.

Diameter and height variables are the bedrock of forest inventory, measuring of tree height is time consuming, more difficult and costly than diameter measurement (Baral *et al*., 2022), particularly in mountainous locations (Seki and Sakici, 2022a). The objective of the study was to develop models for predicting height of *Gmelina aborea* in Achusa, Makurdi Local Government Area of Benue State.

MATERIALS AND METHODS

Study Area

This study was carried out at Achusa, Nyima, in Makurdi Local Government Area of Benue State, which is located within the Southern Guinea Savanna zone on latitude 7°44′0″N and longitude 8°32′0″E (Figure 1). The area is characterized by two distinct seasons; wet and dry season. The wet season is between April to October, and dry season between November to March.

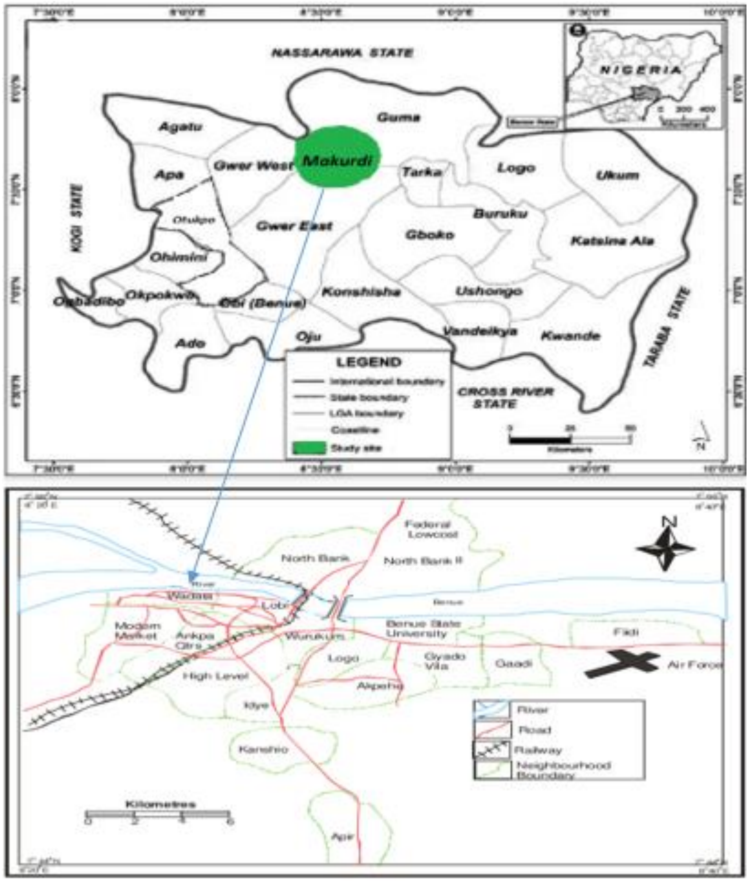


Figure 1: Map of Makurdi Local Government Area

Sampling and Data Collection

The simple random sampling technique was used for the establishment of ten (10) sample plots size of 20m x 20m (0.04ha) in the woodland. The total height of trees was measured using a Spiegel Relaskop and Diameter at breast height of all the trees in the sample plot were measured using diameter tape.

Data Analysis

Six (6) non-linear height-diameter relationship model forms were used. The data set was divided into two, 80% of the data set (training set) were used to fit the six height-diameter models, while the remaining 20% was used for model validation (Aghimien *et al*., 2016).

Power law (Stage, 1975 and Arabatzis and Burkhart, 1992):

$H = 1.3 + aD^b$ ……………………(1)

Gompertz (Winsor, 1932):

$H = 1.3 + a\exp^{-\exp(b-cD)}$……(2)

Logistics (Zeide, 1993):

$H = 1.3 + \frac{a}{1 + b\exp^{-cD}}$ …… (3)

Naslund (Naslund, 1936):

$H = 1.3 + \left(\frac{D}{a+b.D}\right)^2$ …….. (4)

Chapman-Richards (Richards, 1959 and Chapman, 1961):

$H = 1.3 + a(1 - \exp(-bD))^c$ ….. (5)

Weibull (Yang *et al*., 1978 and Zeide, 1993):

$H = 1.3 + a(1 - \exp(-bD^c))$ ……… (6)

Where:

H = Height, D = Diameter at Breast Height (DBH), a, b and c = Parameters exp = Exponential

Model Evaluation Criteria

The six non-linear height-diameter models were evaluated statistically and graphically (Dag *et al*., 2018; Mangiafico *et al*., 2018). This is in line with suggested applications in forestry modeling (Mensah *et al*., 2017). This was carried out statistical using the following:

i **Coefficient of Determination (R^2)**

It measured the proportion of variation in the predicted variable that is explicated by the activity of the predictor variable. The Coefficient of Determination figure ranges from 0 to 1 and is also expressed in percentage through multiplication of the figure by 100.
A good model R^2 value should be high i.e>50% in order to be accepted (Thomas, 1977).

$$\boldsymbol{R^2} = 1 - \frac{SSE}{SST} \text{ ….. (7)}$$

ii **Regression Mean Square Error (RMSE)**

It is also known as the standard deviation or residual of the error variance of the estimate. It gives information concerning a brief performance of an equation via a term-by-term view of the actual difference of measured and the predicted value, calculates the spread of data and is an excellent key of precision. The smaller the value, the better the performance of the model.

$$\boldsymbol{RMSE} = \sqrt{\frac{\sum(\boldsymbol{h_i} - \boldsymbol{h})^2}{\boldsymbol{n}}} \text{ ….. (8)}$$

Note: h_i = observed height
h = predicted height
n = number of observations

iii **Akaike's information criterion (AIC)**

It is used in evaluating the simplicity and accuracy of the model. It is a single number score which is applied to know the best model among other models. It estimates equations relatively that means Akaike's information criterion scores are just important when compared with other AIC scores for just the same data set, a low AIC score is better (Burnham and Anderson, 2004)

AIC = n × ln(RMSE) + 2p …… (9)

Where n = total number of data used for fitting the model; p = number of model parameters

iv **Mean prediction bias (MPB)**

It is the error in relation to prediction for the ith tree and shows the deviation of the model with respect to the measured value. The purpose of MPB is for reliability of the model. A value of 0 indicates no bias.

$$MPB = \frac{1}{n}\sum_{i=1}^{n}(h_i - h) = \sum_{i=1}^{n}\frac{(h_i - h)}{n} \text{ ...(10)}$$

Note: h_i = observed height
h = predicted height
n = number of observations

RESULTS

Descriptive statistics for two variables: Diameter at Breast Height (DBH) and Height, measured in centimeters and meters, respectively is displayed in Table 1. The data was spilt into two subsets: a training set and a validation set, and the combined values for both subsets are also reported.
The mean DBH is slightly higher in the training set compared to the validation set (35.99 cm vs. 32.08cm). The mean Height follows a similar pattern, with a higher value in the training set (19.2m vs. 17.46 m) and a combined mean of 18.84m

Table 1: Summary statistics of measured growth variables

Descriptive Statistics	Diameter at Breast Height (cm)			Height (m)		
	Training	Validation	All	Training	Validation	All
Mean	35.99	32.08	35.19	19.2	17.46	18.84
Minimum	17.1	17	17	7.2	8	7.2
Maximum	80	58	80	37.4	29.3	37.4
Standard Deviation	11.77	10	11.5	6.29	5.52	11.5
Sample Size	97	25	122	97	25	122

Figure 1 is a scatter plot that displays the relationship between tree height and diameter at breast height. Each point on the graph represents a tree in the dataset. The x-axis represents DBH and the y-axis represents height. From the scatter plot, we can observe that there is a positive correlation between tree height and diameter. As the diameter of the tree increases, so does its height. This is indicated by the upward trend in the scatter plot. However, we can also see that there is a significant amount of variability in the data. Some trees with smaller diameters have relatively large heights, while some trees with larger diameters have relatively small heights. This indicates that other factors besides diameter may also affect tree height.

Table 2 shows the parameters estimates of six different models developed for the study. The models aim to predict some outcome variable that is tree height Y based on an input variable D. The input variable D is the diameter at breast height (DBH), which is a commonly used measure in forestry. Model 1 is the power model, which has a form of Y=a D^b $H = 1.3 + aD^b$. The parameter estimates for model 1 a = 0.676 and b = 0.916. These estimates indicate that as input variable D increases, the outcome variable Y increases at an increasing rate.

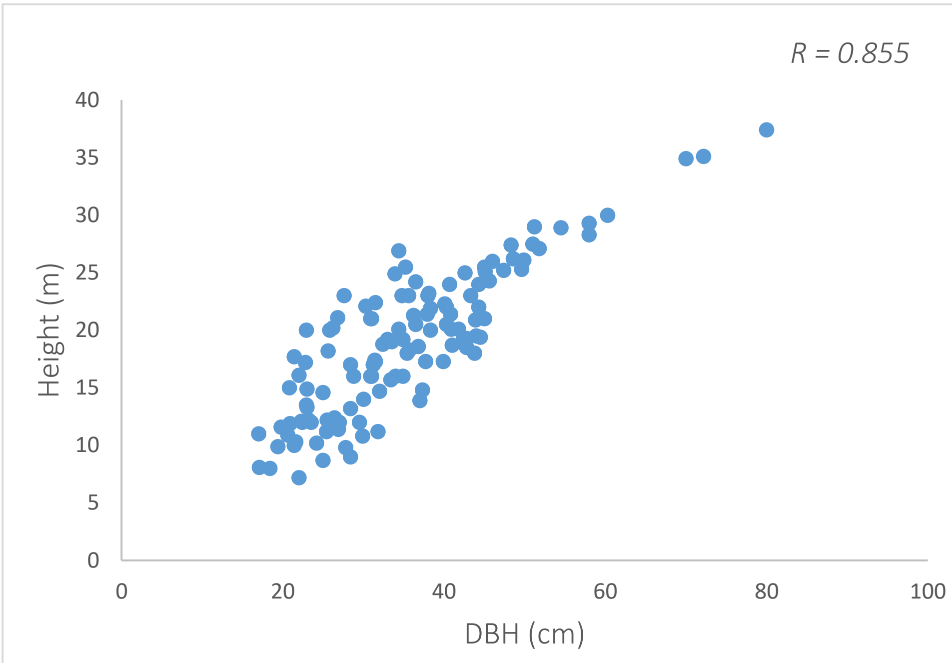


Figure 1: Scatter plot showing the relationship between Tree Height and DBH

Table 2: Parameter estimates of models developed in this study.

Model	Input	Parameters of Models		
		a	**b**	**c**
Model 1 (Power)	*D*	0.676	0.916	-
Model 2 (Gompertz)	*D*	48.104	2.639	0.522
Model 3 (Logistics)	*D*	41.051	7.387	0.048
Model 4 (Naslund)	*D*	-42.244	17.019	-
Model 5 (Chapman-Richards)	*D*	86.309	0.007	1.079
Model 6 (Weibull)	*D*	84.222	0.005	1.064

Where: *H = Height, D = Diameter at Breast Height (DBH), a, b and c= Parameters.*

The evaluation statistics for six models developed in the study area are reported in Table 3. The evaluation metrics are calculated for both the training and validation data sets, which was divided into 80% and 20%, respectively. The metrics used to evaluate the performance of the models are R^2, RMSE, Bias, and AIC.

The results of the fitted Height-Diameter models showed that Chapman-Richards and Weibull model were the best function for estimating tree height of *Gmelina aborea* (Table 3) for the training dataset. They had the smallest RMSE 3.223, a Bias of -0.005, R^2 value of 0.734, AIC of 510.314 and 510.321 respectively. The Power model also performed well. It had RMSE and AIC of 3.229, 508.666 with a R^2 value of 0.734 and a Bias of -0.014. The Naslund Height-Diameter function had the largest RMSE of 3.302. For the validation dataset as shown in Table 3 the Logistics model was the best with the smallest RMSE 2.906 and Bias of 0.013. The RMSE ranged from 2.906 to 3.252 in the validation dataset.

Table 3: Evaluation statistics for models developed in the study area.

Model	**Training (80%)**				**Validation (20%)**	
	R^2	***RMSE***	***Bias***	***AIC***	***RMSE***	***Bias***
Model 1 (Power)	0.734	3.229	-0.014	508.666	2.945	0.034
Model 2 (Gomperz)	0.732	3.238	-0.006	511.221	2.930	0.047
Model 3 (Logistics)	0.729	3.255	-0.012	512.217	2.906	0.013
Model 4 (Naslund)	0.721	3.302	2.4E-07	512.991	3.252	0.161
Model 5 (Chapman-Richards)	0.734	3.223	-0.005	510.314	2.962	0.071
Model 6 (Weibull)	0.734	3.223	-0.005	510.321	2.961	0.070

Where: *R^2 = Coefficient of Determination, RMSE = Root Mean Squared Error, AIC = Akaike's Information Criterion.*

Figure 2 shows the combined curves for all six models developed in the study. The x-axis represents the diameter at breast height (DBH) in centimeters, and the y-axis represents the height of the tree in meters. Each model is represented by a different coloured curve. It highlights the differences in shape among the models. The curves show the predicted relationship between tree height and DBH according to each model. From the graph, five models follow a similar trend, with an increasing relationship between tree height and DBH. However, the shape of the curve varies among the models, except Model 4 (Naslund) showing a gradual increase with a slower increase in tree height at lower DBH values, followed by a faster increase in tree height as DBH increases and a decreasing trend in tree height at higher DBH values. It shows a curved line.

As shown in Figure 2 the six height-diameter functions created tree height trajectories that are conformable with biological fact which is monotonic growth, deviation point and asymptote.

The models had good estimates for the trend of the observed data (Figure 3). From the graph, we can see that all the models except Naslund follows a linear relationship between tree height and DBH. For Naslund height increases rapidly as DBH increases up to a certain point, after which the rate of increase for height gradually slows down.

The curve fit is based on the parameters of the model estimated using the data, and it represents the predicted relationship between tree height and DBH according to the model. The curve fit can be useful in predicting the height of trees based on their DBH values, especially for trees within the range of DBH values used in the study. However, it is important to note that the curve fit may not be accurate for trees outside this range or in different geographic regions or forest types.

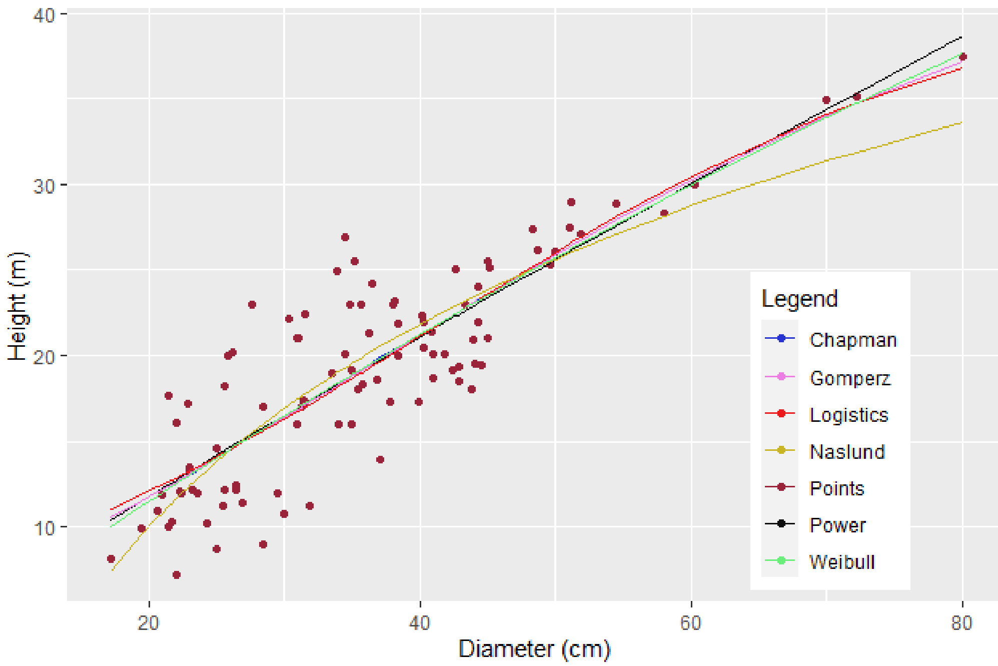


Figure 2: Combined Curves for all Models

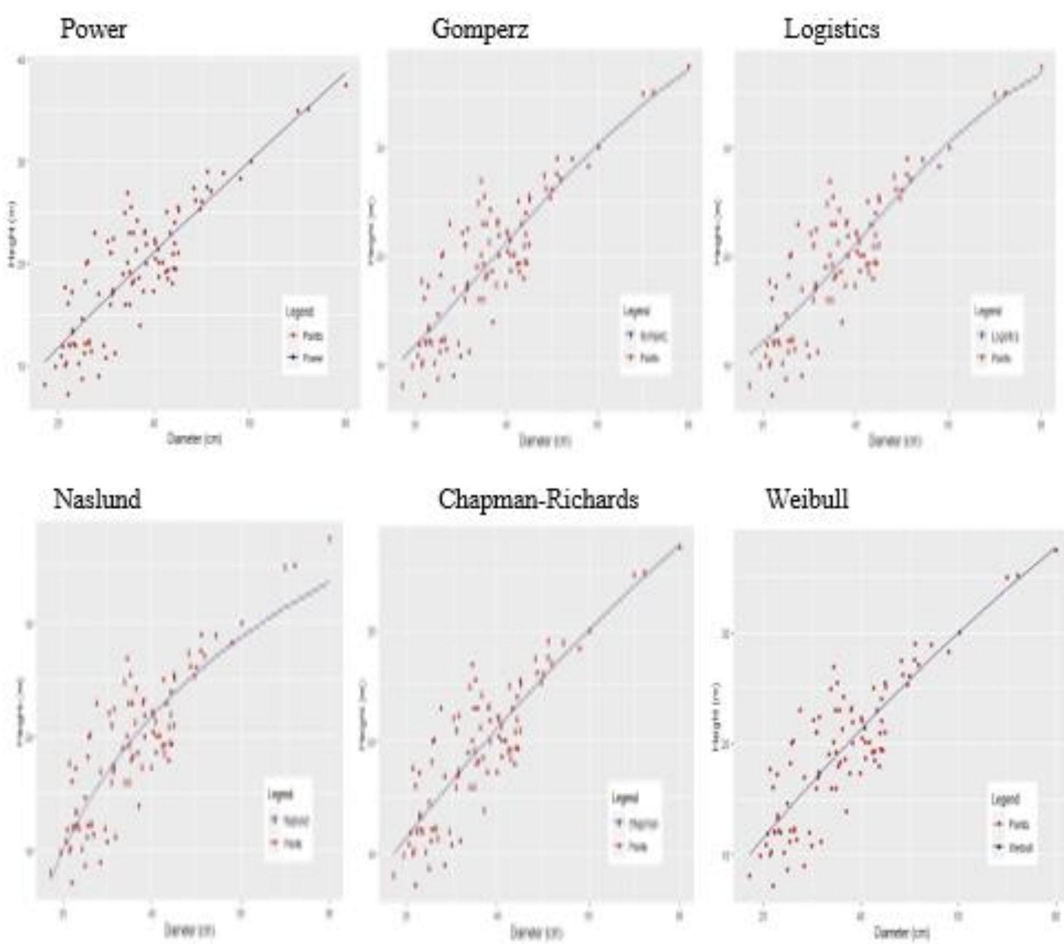
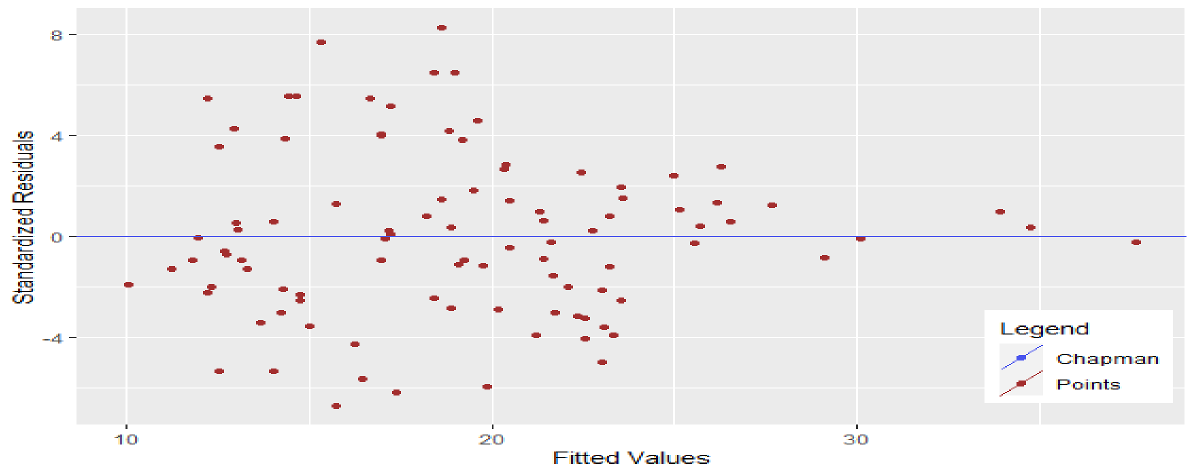


Figure 3: Curve fit

The residual plot of the selected height-diameter i.e Model 5 and 6 developed for the study area (Figure 4) the x-axis represents the predicted tree height values, and the y-axis represents the residuals (i.e., the difference between the observed and predicted tree height values) in meters. From the plot, it showed a relative horizontal band within the range of 9 to -6 m respectively, which corroborates the fitting parameters (Table 2).

The residuals are randomly distributed around zero, indicating that there is no systematic pattern or trend in the residuals, the lack of heteroscedasticity and the points in the chosen models did not lean in one direction This suggests that Model 5 and 6 are good fit for the data and that the assumptions of linearity and constant variance have been met. Additionally, there are no outliers or influential points that could significantly affect the model fit. It can be used to predict tree height based on DBH values within the range used in the study.



Residuals vs Predicted values of Tree Height for Model 5 (Chapman-Richards)

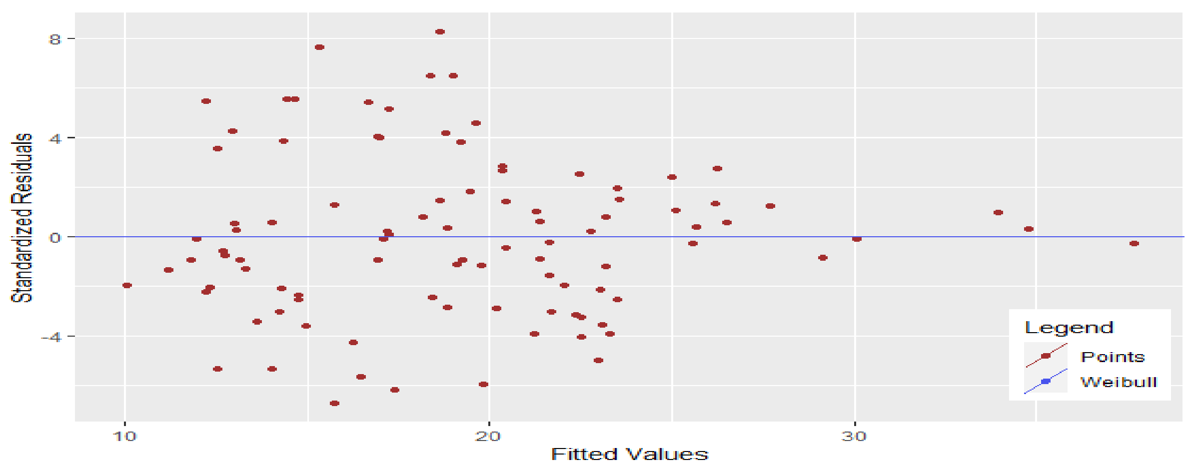


Figure 4: Residuals vs Predicted values of Tree Height for Model 6 (Weibull)

DISCUSSION

Akaike's information criterion has a crucial role in choosing the best model (Sanquetta *et al.*, 2018). The Weibull height-diameter function shows a considerable reduction in errors in the estimates of height when it is fitted for local-scale (Ledo *et al.*, 2016; Kearsley *et al.*, 2017; Sullivan *et al.*, 2018). The Weibull equation is ecologically consistent and is a representative of biological realism in a forest that is dense

(Mehtätalo *et al*., 2015). The Weibull and Chapman-Richards height-diameter models were great with the Weibull model having the lowest mean bias for *Tectona grandis* in Oluwa forest reserve, Nigeria (Egonmwan and Ogana, 2020). Popoola and Uii (2023) also revealed that the Chapman-Richards was the best model for *Tectona grandis* among the six selected non-linear height-diameter models for Mbavaa forest reserve in Nigeria.

Mengesha *et al*. (2018) carried out research for *Cupressus lusitanica* in Gergeda forest Ethiopia, the evaluation criteria used were Coefficient of determination (R^2), root mean square error (RMSE), bias (E), absolute mean deviation (AMD), and coefficient of variation (CV%) the Weibull model was the best fit in terms of the equation's goodness of fit and predictive ability which is in line with this study. The best fitting model for the calibration dataset is the logistic height-diameter model for the study area. The logistic height-diameter function used by Seki (2022) for *Pinus brutia* and *Pinus nigra* in Turkey was the best among the functions used. This equation is superior to its counterparts for Tatarian maple in Hungary (Misik *et al*., 2016), *Gmelina arborea* in Nigeria (Ogana, 2018), Maritime pine in Spain (Ogana *et al*., 2020). The research carried out by Deng *et al*. (2019) for Masson pine, the logistic height-diameter function did not perform well compared to other functions. This scenario could be because the performance of an equation is connected with the equation attribute to capture the variation in the ecological system of the study area of the data which implies that selecting a reliable model that can fit into the data is the number one step of modeling studies (Seki, 2022). Eby *et al*. (2017) conducted a study on nonlinear models for height- diameter relationship for *Gmelina arborea* in Ibadan, Nigeria, the logistic height-diameter function performed well while the Weibull equation was the best with a sigmoidal form. When the dataset used is very small, the equations may not perform well in predicting height. It could be because of: (i) non-linear relationships, like asymptotic maximum heights, is not in the small validation data sets (Duncanson *et al*., 2015). (ii) equations can be greatly influenced by outliers (trees that are excessively tall or extremely short for their diameter).

It is crucial to note that while diameter at breast height is a commonly applied measure for estimating tree growth, it may not be the only characteristics that affects tree height. Other characteristics like climate, soil quality and tree species have an important stake in determining tree growth patterns. It is therefore crucial to consider different variables when developing models to estimate tree height. The tree height-diameter relationship depends on a series of physiological, biogeographic and environmental factors (Gorgens *et al*., 2021). Besides climatic conditions and altitude (Jackson *et al*., 2021) could also influence the height-diameter relationship.

Wood density is important for the growth and survival of trees, it indicates the efficiency of transporting water as well as nutrients (Borghetti *et al*., 2020). This attribute is related to the height of trees and radial growth. The climatic condition, management history of the forest, type and range of dataset employed for modeling the site characteristics such as the productivity of the site of the sampled forest has an influence on the tree allometry as well as growth dynamics (Baral *et al*., 2022; Koulelis *et al*., 2022; Seki and Sakici, 2022b; Wu *et al.*, 2022). An attribute that positively affects tree growth is precipitation (Mondek *et al*., 2021; Koulelis *et al*., 2022).

CONCLUSION

Six non-linear height-diameter relationship model forms were selected. Selecting a suitable model that can fit a data perfectly is the number one step of modeling studies. In the data set of *Gmelina arborea* in the study area it was observed that the relationship between tree height and diameter at breast height was strong. Chapman-Richards and Weibull Height-Diameter models were the best function for estimating tree height of *Gmelina arborea* for training dataset in Achusa, Makurdi Local Government Area, these models can be used by the forest manager to estimate tree height while for the validation dataset the Logistics Height-Diameter model was the best.

RECOMMENDATIONS

i. The research recommends that variables such as soil fertility, density, spacing, crown area, age and silvicultural practices can be incorporated in height-diameter models so as to improve the accuracy and reliability of the models. However, such information cannot be obtained in a single measurement, therefore there is need for a permanent sample plot which can be used for remeasurements over time.

ii. The height-diameter models should be evaluated and, if necessary, revisited or calibrated when they are applied in a different location.

iii. New technologies like LiDAR terrestrial or LiDAR aerial should be considered in measuring the attributes of the individual tree.

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