



HEIGHT-DIAMETER MODELS FOR *Tectona grandis* (Linn F.) IN TSE-DEI, BENUE STATE, NIGERIA

Popoola, V. D. * and Ademoyegun, M. S.

Department of Forest Production and Products, Joseph Sarwuan Tarka University, Makurdi.

*Correspondent Author: popoola.victoria@uam.edu.ng; +2348098566488

ABSTRACT

*Height-diameter models are important in forest assessment. Height-diameter models are used to estimate tree volume, survival analysis, to predict missing tree height and biomass and to describe stand growth dynamics and succession. Tree height measurement are expensive and time consuming. The objective of the research was to develop equation for estimating tree height as a function of diameter at breast height for *Tectona grandis* (Teak). The study was conducted at Tse-Dei, simple random sampling technique was employed and four linear models were selected. Coefficient of determination (R^2), Root Mean Square Error (RMSE), Significance of Regression (F-val) and Confidence Interval (CI) were the criteria applied to evaluate the predictive performance of the models. The model of Curtis $\text{Log}H = b_0 + (b_1D^{-1})$ was the best of the four models because it attained a rank of one and has the highest R^2 (0.610) and the lowest RMSE (0.063). The selected model can accurately predict tree height when fitted to stands with diameter at breast height. The selected model can be used in similar condition of teak species in the north-central region of Nigeria for effective decision making and for future prediction of teak height.*

Keywords: Teak, height, diameter, linear models

Correct Citation of this Publication

Popoola, V. D. * and Ademoyegun, M. S. (2023). Height-diameter models for *Tectona grandis* (Linn f.) in Tse-Dei, Benue State, Nigeria. *Journal of Research in Forestry, Wildlife & Environment*, 15(4): 198 - 207

INTRODUCTION

Measurement of tree height and diameter is important in forest assessment and modeling. Tree height are used for estimating timber volume, site index and various variable related to forest growth and yield, succession and carbon budget models (Peng, 2001). Diameter at breast height (dbh) can be more accurately measured, and at lower cost compared to total tree height, just a subsample of tree height is normally measured instead of measuring all tree height. Height-diameter models have been applied to estimate tree volume, to know the social position of the tree within the stand (Colbert *et al.*, 2002), survival analysis (Saud *et al.*, 2016), to predict missing tree height and biomass (Mukti *et al.*, 2018) and to describe stand growth dynamics and succession (Curtis, 1967). Various growth and

yield models use height and diameter as the two basic input variables, with all or some of the tree height predicted from measured diameters (Huang *et al.*, 2000). Diameter at breast height and tree height data is necessary in development of equations for forest growth and yield (Dubenok *et al.*, 2023). Patricio *et al.* (2022) stated that developing of easy and reliable equations to accurately know tree height in a forest from the diameter information is an important task of forest management.

Teak is one of the most important timbers in the world because of its salient properties (Adekanmbi and Saka, 2022). It has a great economic valued timber in the global market (Pachas *et al.*, 2019). It has been selected in reforestation programs for decades (Moya and

Tenorio, 2021). *Tectona grandis* is one of the few trees that has gotten some attention and research on techniques of production forecasting. It grows fast and has the ability to regenerate (Adekanmbi *et al.*, 2023). Nigeria has the largest *Tectona grandis* plantation in Africa, with an estimate of 70,000 ha, that is 52.7% of the total forest under teak plantation in Africa (Dantani *et al.*, 2019). It is widely utilised in plantation forestry by individuals, government and private sector in Benue State, Nigeria (Popoola and Mbasanga, 2023). It is utilised for carpentry works, construction of buildings, furniture, electric poles and to manufacture high-value products (Amusa and Adedapo, 2020).

Forest inventory normally measure the diameter of all trees in a sample plot and the height of some trees. Tree diameter is measured easily and at a small cost. Total tree height, is more difficult and expensive to measure because of the time needed to complete measurement, observer error and visual obstruction (Wang *et al.*, 2015). A significant amount of information exists regarding height-diameter relationships in different species and different forest regions but it is lacking in the study site.

A major obstacle in modeling the height-diameter relationship is the large number of variables influencing it and thus obstructing the creation of generic models based on empirical method such

as linear and non-linear regression. The variables used in height-diameter models often display a lack of independence among measurement. This can be because of various observation that is taken at the same sampling unit. Height-diameter models can be used to predict tree heights from the rest of tree belonging to the same plots there by reducing data acquisition expenditure in forest inventories. Therefore, developing appropriate height-diameter models is considered as one of the most significant components in forest design and monitoring. Individual tree height-diameter models are significant in order to improve the accuracy of the models on which forest management decisions is based. The objective of this study was to develop height-diameter models for *Tectona grandis* in Tse-Dei for forest management and planning purposes.

MATERIAL AND METHODS

Study Area

This research was carried out at Tse-Dei located in Makurdi local government area of Benue state as showing the location of the plantation (Figure 1). It lies at latitude $7^{\circ} 25'N$ to $7^{\circ} 33'N$ and longitude $8^{\circ}25'E$ to $8^{\circ}36'E$. The plantation is located in Daudu along Makurdi-Lafia road. The objective of the plantation was to supply electric poles and fuel wood for domestic purpose. The plantation is made up of these species *Tectona grandis* (Teak) and *Gmelina arborea* (Gmelina).

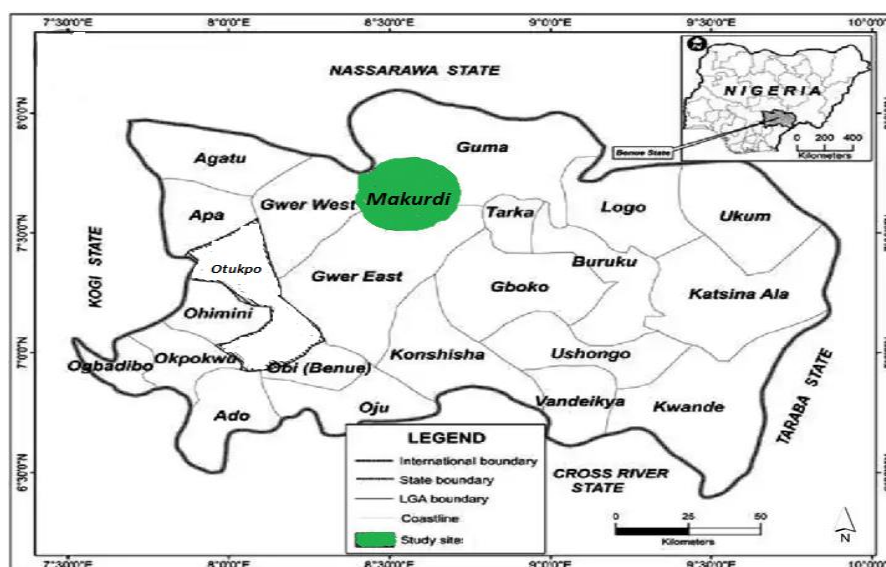


Figure 1: Map of the study area

Data Collection

Simple random sampling was used in order to ensure that each plot in the plantation has equal chance of being selected. The total size of the plantation is one hectare and the age of the teak plantation is thirteen. A sampling frame consisting of sampling units of size 20m by 20m (0.04ha) was established. The total area of the plantation was divided into sample plots of equal size (0.04ha) and five sample plots was randomly selected using random table.

Complete enumeration of trees larger than 10cm was carried out in the sample plots. Within each plot, the following tree growth variables was measured:

- i. Total height
- ii. Diameter at breast height (Dbh)

Measurement of Tree Growth Variables

Diameter at breast height

Diameter at breast height (dbh) was measured for all tree individuals by means of a diameter tape. Trees with deformities at 1.3m, the measurement was taken at the sound point on the stem above the abnormality. Trees with buttress, a point of measurement was chosen approximately 0.5m above the convergence of the buttress (Husch *et al.*, 2003).

Tree height

Haga altimeter was used to measure total height.

Tested Models

$$H = b_0 + (b_1 D^2) \quad \text{Curtis, 1967 (1)}$$

$$H = b_0 + (b_1 \text{Log}D^2) \quad \text{Curtis, 1967 (2)}$$

$$\text{Log}H = b_0 + (b_1 D^{-1}) \quad \text{Curtis, 1967..... (3)}$$

$$H = b_0 + (b_1 D^2) + (b_2 D^3) \quad \text{Parresol, 1992... (4)}$$

Where;

H = Total tree height (M)

D = Diameter of breast height (M)

b_0, b_1, b_2 = Parameter to be estimated

Model Comparison

The height-diameter models were assessed in order to recommend those with good fit for further uses. The models were ranked in terms of High R^2 and low RMSE value. The following statistical criteria were used:

Coefficient of Determination (R^2)

This is the measure of the proportion of variation in the dependent variable that is explained by the behaviour of the independent variable (Thomas, 1977). For the model to be accepted, the R^2 value must be high.

$$R^2 = 1 - \frac{SSE}{SST} \quad \dots \dots \dots (5)$$

Where R^2 = Coefficient of determination, SSE = Error sum of squares or Residual sum of squares, SST = Total sum of squares and 1= regression line.

Regression Mean Square Error (RMSE)

This is also referred to as the standard deviation or residual of the error variance of the estimate. It measures the spread of data and is a good indicator of precision. The value must be small.

$$RMSE = \sqrt{\frac{\sum(Y_i - Y)^2}{n - p}} \quad \dots \dots \dots (6)$$

Note: Y_i = observed value of the dependent variable

Y = predicted value of the dependent variable

n = number of observations

p = number of parameters

Significance of Regression (F-ratio)

This was used in testing the overall significance of the regression equation. The critical value of F (that is, F-tabulated) at $p < 0.05$ level of significance was compared with the F-ratio (F-calculated). Where the variance ratio (F-calculated) is greater than the critical values (F-tabulated) such equation is considered significant and can be accepted for prediction.

Confidence Interval

Confidence interval (CI) is a key output of many statistical analyses, and it has an important role to play in the interpretation of estimates of parameters. A narrower confidence interval indicates a more precise estimate, while a wider confidence interval indicates a less precise estimate.

$$CI = \bar{x} \pm z \frac{s}{\sqrt{n}} \quad \dots \dots \dots (7)$$

CI = Confidence interval
 \bar{x} = sample mean
 z = confidence level value
 s = sample standard deviation
 n = sample size

RESULT

Descriptive statistics of Sample trees of *Tectona grandis* (Linn F.)

The summary statistics of the data set for the tree variables is presented in Table 1. The table shows

the mean, standard error, standard deviation, minimum and maximum of each growth variable. A total of one hundred and seventeen trees was measured. The tree height (H) and Diameter at breast height (D) of the species ranges between 4 - 12.7 m and 0.42 - 1.00 m.

Height-diameter model

The constant of model 2 (10.665) is quite wide while model 1 had the least constant of 1.103 as shown in Table 2.

Table 1: Diameter at breast height and height statistics

Variables	Mean	S.E	S.D	Min.	Max.
D	0.54	0.04	0.18	0.42	1.00
H	8.54	0.36	1.68	4.00	12.70

D = Diameter at breast height (m), *H* = Height (m)

Table 2: Height-diameter equations

Model Number	Model form	b ₀	b ₁	b ₂
1	$H = b_0 + b_1 D^2$	6.879	4.712	
2	$H = b_0 + b_1 \text{Log} D^2$	10.665	3.824	
3	$\text{Log} H = b_0 + b_1 D^{-1}$	1.103	-0.089	
4	$H = b_0 + b_1 D^2 + b_2 D^3$	5.412	18.624	14.404

D = Diameter at breast height (m), *H* = Height (m)

Model Evaluation

Comparison of the model estimate was based on the following statistics, viz; R², RMSE, F-val, CI, Rank total. The best model was ranked one and the poorest model was ranked four. Each of the model was ranked on the base of its statistic and the model with the lowest RMSE and CI was ranked one and the model that had the highest RMSE and CI was ranked four. The model with the highest R² and F-Val was ranked one and the model with the lowest R² and F-Val was ranked four. The rank for R², RMSE, F-val and CI was combined and was added to identify the best model. The ranking value was added to have a single value for overall model ranking.

The coefficient of determination values of all the developed equations ranges between 0.361 and

0.610, while the root mean square error ranges between 0.063 to 1.370 as shown in Table 3. The result of the statistics and ranking of all model show that the model of Curtis 1967 $\log H = b_0 + (b_1 D^{-1})$ was the best of the four models because it attained a rank of one and has the highest R² = 0.610 and had the lowest RMSE = 0.063 which is a good way to measure the accuracy of the predictive abilities of the model and shows that the model predicted well therefore showing the best goodness of fit of the height-diameter relationship model as shown in Table 3. The study concludes that this model can best predict height diameter relationship for *Tectona grandis* (teak) at Tse-Dei compared to the other three models.

Table 3: Model ranking

Model No.	R ²	RMSE	F-Val	CI	Rank Total
1.	0.361 (4)	1.370 (4)	65.636 (3)	0.911 (2)	13 (4)
2.	0.504 (2)	1.207 (3)	117.994 (2)	0.924 (3)	10 (2)
3.	0.610 (1)	0.063 (1)	181.480 (1)	0.059 (1)	4 (1)
4.	0.486 (3)	1.206 (2)	52.953 (4)	1.52 (4)	13 (3)

Tables 4 - 7 showed the correlation between the estimated parameters for each model. Each parameter was computed for calculation of least squares summary statistics viz. sums of squares, the degrees of freedom and mean squares for each of the model. Model 1 is the best and can predict height diameter relationship of *Tectona grandis*

in Tse-Dei. The probability associated with F (Fisher's test) for all the models is in this case less than 0.0001 (Tables 4-7). This implies that for a given species, there is a risk of having 0.01% wrong when predicting the height of an individual. This explanatory variable is highly significant for all the models for teak in Tse-Dei.

Table 4: The least squares summary statistics of model 1

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	123.172	1	123.172	65.636	.000 _a
	Residual	217.684	116	1.877		
	Total	340.856	117			

Table 5: The least squares summary statistics of model 2

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	95% Confidence Interval for B	
		B	Std. Error	Beta			Lower Bound	Upper Bound
2	(Constant)	10.665	.233		45.725	.000	10.203	11.127
	LogD2	3.824	.352	.710	10.862	.000	3.127	4.521

Table 6: The least squares summary statistics of model 3

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	95% Confidence Interval for B	
		B	Std. Error	Beta			Lower Bound	Upper Bound
3	(Constant)	1.103	.015		73.822	.000	1.074	1.133
	D1	-.089	.007	-.781	-13.471	.000	-.102	-.076

Table 6: The least squares summary statistics of model 4

Model		Unstandardized Coefficients		Standardized Coefficients	T	Sig.	95% Confidence Interval for B	
		B	Std. Error	Beta			Lower Bound	Upper Bound
4	(Constant)	5.412	.384		14.104	.000	4.652	6.172
	D2	18.624	3.260	2.455	5.713	.000	12.164	25.083
	D3	-14.404	3.356	-1.845	-4.292	.000	-21.054	-7.755

The cumulative probability of the observed and predicted tree height data along a straight-line pattern of the Normal probability plot also

displayed the goodness of fit of the models (Figure 2-5).

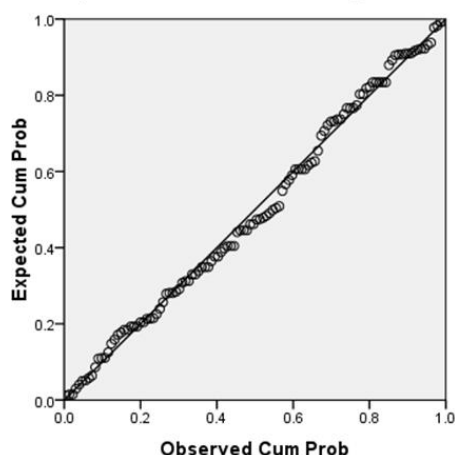


Figure 2. Model 1 Normal probability plot for *Tectona grandis* in Tse-Dei

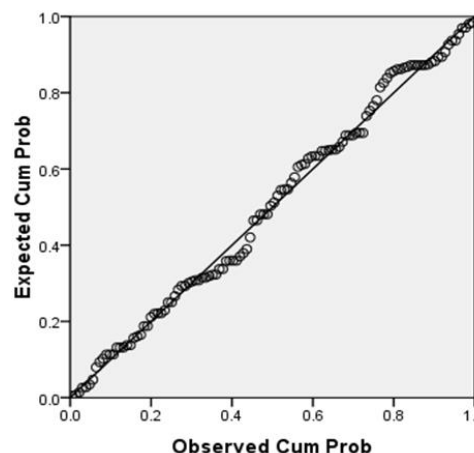


Figure 4. Model 3 Normal probability plot for *Tectona grandis* in Tse-Dei

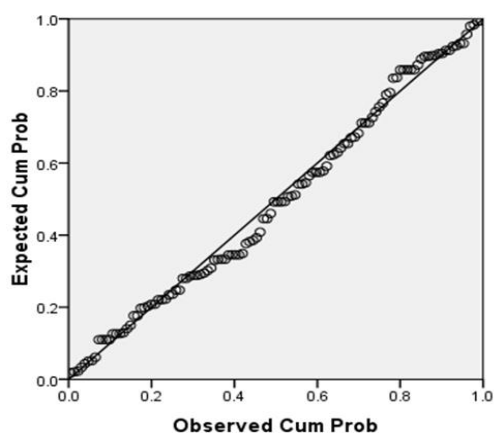


Figure 3. Model 2 Normal probability plot for *Tectona grandis* in Tse-Dei

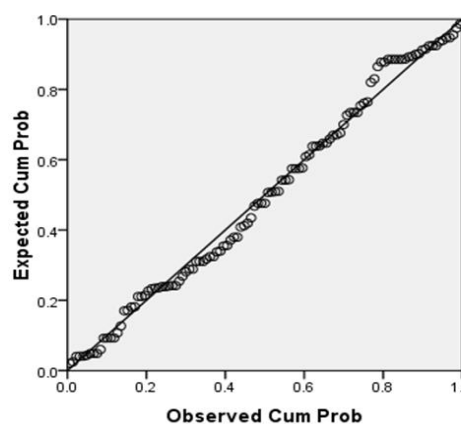


Figure 5. Model 4 Normal probability plot for *Tectona grandis* in Tse-Dei

DISCUSSION

Tree height is important in incorporating ecological characteristics of the species constituting the canopy and in knowing the

dynamics of forest ecosystems. Koirala *et al.* (2017) reported coefficient of determination (R^2) values of linear Height-Diameter relationship of Teak (*Tectona grandis* L. F.) in Central Lowlands

of Nepal with values of 0.907 to 0.928 and root mean square error RMSE, with the lowest value of 1.45 and highest value of 1.66.

The coefficient of determination obtained in this study is also in line with that reported by Achille *et al.* (2022) in their work of height-diameter relationship of *Piptadeniastrum africanum* with R^2 of 0.603 in the forest-savannah transition zone of the Congo Basin. According to Saka *et al.* (2021), juvenile height equation had R^2 value of 69.1 for Plantation-Grown Date Palm (*Phoenix Dactylifera* L.). Mohamed and Ahmed (2014) reported R^2 value of 0.699369 for height diameter relationship model for *Acacia Nilotica* in Riverine Forests Blue Nile using Arabatzis and Burkhart 1992 model. Oyebade and Ebitimi (2011) reported R^2 values less than 0.70 for Nonlinear (logarithm) height - diameter predictive equations, Monserud's (1975) model form for rubber plantation, in Choba, Port Harcourt, Nigeria. Colbert *et al.* (2002) noted a sigmoidal curve relationship for height-diameter equations of thirteen Midwestern bottomland hardwood species of riparian forests along major rivers in Missouri, Illinois and Iowa with high coefficient of determination (R^2).

The low coefficient of determination obtained in this study is the same as reported by Achille *et al.* (2022) in their study on height-diameter relationship in dense humid semi-deciduous forest for *Entandophragma cylindricum* with $R^2 = 0.47$. Adekunle (2007) reported the existing high negative correlation between dominant height and logarithm mean in a natural forest in Nigeria. Oyebade and Ebitimi (2011) noted the low coefficient of determination values of the model in some plots could be because of site quality deficiency or the requirement of silvicultural treatments that can be investigated further for optimal and sustainable management of the rubber plantation. Further research on the effects of site parameters (biotic and abiotic) on the height-diameter relationship like those carried out by (Sharma and Parton, 2007; Trincado *et al.*, 2007; Yang & Huang, 2014) can better elucidate this difference.

Adekanmbi and Saka (2022) noted root mean square error values of between 0.026 to 1.638

which is similar to what was obtained in this research. The root mean square error value gotten in this research is small compared to Mabvurira and Miina (2002) in their research on height growth of *Populus tremula* stands with RMSE values which ranges between 1.226 to 1.832.

CONCLUSION

Height measurements takes longer time compared to diameter measurements that can be easily access. Height-diameter models can be used for tree volume estimation and can be used for sound management of the forest. To achieve these models are applied to meet challenges of cost, shorten time and error in the estimation of height from diameter. Models are very important to quantify tree species. Teak is the most planted species in Nigeria by both farmers and individuals it is therefore important because of its environmental benefits such as carbon sequestration and economic value. The research has paved way for height-diameter relationship in the plantation of Tse-Dei by determining the equation that applies to the height-diameter relationship for *Tectona grandis*.

Four height-diameter models were selected for the study site. Among the four models evaluated, the variable model of Curtis 1967 $\text{LogH} = b_0 + (b_1 D^{-1})$ had the best fit compared to other three models and had a better precision and was ranked number one in height estimate. The model statistic suggested that $\text{LogH} = b_0 + (b_1 D^{-1})$ model was equally well fitted to the tree height-diameter for *Tectona grandis* species at Tse-Dei. The selected model can accurately predict tree height when fitted to stands with diameter at breast height.

RECOMMENDATIONS

- i. The height-diameter models, are normally functions of tree diameter it can be used in similar condition of teak species in the north-central region of Nigeria for effective decision making and for future forecast.
- ii. Various parameters such as age, site and spacing, can be applied in future research for height – diameter modeling.
- iii. *Tectona grandis* planted at the forest plantation site is purely for the purpose of

production of poles, sawn wood and fuel wood. Feasibility study should be carried out to ensure continuous

production of timber successful, so as to be of benefit to the communities around and increase income for the state.

REFERENCES

- Achille, A. N. Y., Beeckman, H., Din, N., Borgia, A. A. F., Christian, Z. J. and Marguerite, M. M. (2022). Height-diameter relationship of some forest species exploited for wood in the Natural Tropical Forest of the Congo Basin. *Open Journal of Forestry*, 12, 235-247.
- Adekanmbi, D. I., Fandohan, A. B., Tchoumado, M. A. and Djossa, A. B. (2023). A regression modelling approach for stem volume estimation of two exotic plantations within Dogo-Ketou Forest Reserve, Benin Republic. *American Journal of Agriculture and Forestry*. Vol. 11, No. 4, pp. 169-175.
- Adekanmbi, D. I. and Saka, M. G. (2022). Development of Height Equations for Teak (*Tectona grandis* L.f.) Plantation in Ketou Commune, Ketou, Republic of Benin. *Asian Journal of Environment & Ecology*, 18(2): 11-21.
- Adekunle, V. A. J. (2007). Non linear Regression models for timber volume estimation in Natural Forest Ecosystem, Southwest Nigeria. *Research Journal of Forestry*, 1(2):40-54.
- Amusa, T. O. and Adedapo, S. M. (2020). Growth and yield characteristics of *Tectona grandis* (Linn. F.) in different age series at University of Ilorin, North Central Nigeria. *Forestist*, 10.5152/forestist.2020.20022.
- Arabatzi, A. A. and Burkhart, H. E. (1992). An evaluation of sampling methods and model forms for estimating height diameter relationships in loblolly pine plantations. *Forest Science*, 3.
- Colbert, K. C., Larsen, D. R. and Lootens, J. R. (2002). Height diameter equations for thirteen Midwestern Bottomland hardwood species. *Northern Journal of Applied Forestry*, 19: 171-176.
- Curtis, R. O. (1967). Height-Diameter and Height-Diameter-Age Equations for Second-Growth Douglas-fir. *Forest Science*, 13: 365-375.
- Dantani, A., Shamaki, S. B., Gupa, M. A., Zagga, A. I., Abubakar, B., Mukhtar, R. B. and Sa'idu, M. (2019). Growth and volume estimates of teak (*Tectona grandis*, Linn F.) in Kanya Forest Plantation, Kebbi State, Nigeria. *Asian Journal of Research in Agriculture and Forestry*, 4 (2): 1-10.
- Dubenok, N. N., Lebedev, A. V., Gostev, V. V., Gemonov, A. V., and Gradusov, V. M. (2023). Height-Diameter fixed effects models for the pine in European Russia. *Earth and Environmental Science*. 1154 012025.
- Hutch, B., Beers, T. W., Kershaw Jr., J. A. (2003). *Forest Mensuration*, 4th ed. John Willey and Sons, Inc., New Jersey, USA, 443p.
- Huang, S., Price, D. and Titus, S. J. (2000). Development of ecoregion-based height-diameter models for white spruce in boreal forests. *Forest Ecology and Management*, 129(1-3): 125-141.
- Gregory, T. G. (1987). Generalized Error Structures for Forestry Yield Models. *Forest Science*, 33(2):423-444.
- Jayaraman, K. and Lappi, J. (2001). Estimation of height-diameter curves through multilevel models with special reference to even-aged teak stands. *Forest Ecology and Management*, 142: 155-162.
- Koirala, A., Kizha, A. R. and Baral, S. (2017). Modeling height-diameter relationship and volume of Teak (*Tectona grandis* L. F.) in Central Lowlands of Nepal. *Journal of Tropical Forestry and Environment*, 7(1):28-42.
- Mabvurira, D. and Miina, J. (2002). Individual-tree growth and mortality models for *Eucalyptus grandis* (Hill) Maiden plantations in Zimbabwe. *Forest Ecology and Management*, 161(1-3):231-45.
- Monserud, R. (1975). Methodology for simulating Wisconsin northern hardwood stands dynamics. PhD dissertation (Univ. Microfilm No.7602496), University of Wisconsin, Madison, WI.156p.

- Mohamed, O. A. and Ahmed, E. (2014). Height Diameter Relationship Model for *Acacia nilotica* in Riverine Forests - Blue Nile. *Journal of Forest Products & Industries*, 3(1), 50-55.
- Moya, R. and Tenorio, C. (2021). Wood properties and their variations in teak, in: Y. Ramasamy, E. Galeano, T. T. Win (Eds.), *The Teak Genome. Compendium of Plant Genomes*, Springer, Cham, https://doi.org/10.1007/978-3-030-79311-1_8.
- Mukti, R., Bishwa, N. O., Surendra, S. and Sophan, C. (2018). Height-Diameter modeling of *Cinnamomum tamala* grown in natural forest in mid-hill of Nepal. *International Journal of Forestry Research*, 6583948.
- Oyebade, B. A. And ebitimi, O. (2011). Height - diameter predictive equations for rubber (*Hevea brasiliensis*-A. Juss- Muell) plantation, Choba, Port Harcourt, Nigeria. *Journal of Agriculture and Social Research*, 11(1): 173-183.
- Pachas, A. N. A., Sakanphet, S., Soukkhy, O., Lao, M., Savathvong, S., Newby, J.C., Souliyasack, B., Keoboualapha, B. and Dieters, M. J. (2019). Initial spacing of teak (*Tectona grandis*) in northern Lao PDR: impacts on the growth of teak and companion crops. *Forest Ecology and Management*, 435: 77–88.
- Parresol, B. R. (1992). Baldcypress height-diameter equations and their prediction confidence interval. *Canadian Journal of Forest Research*, 22: 1429-1434.
- Patrício, M., Dias, C. and Nunes, L. (2022) Mixed-effects generalized height-diameter model: A tool for forestry management of young sweet chestnut stands. *Forest Ecology and Management*, 120209.
- Peng, C., Zhang, L. and Liu, J. (2001). Developing and validation nonlinear height-diameter models for major tree species of Ontario's Boreal Forest. *North Journal of Applied Forestry*, 18: 87–94.
- Popoola, V. D. and Mbasanga, S. S. (2023). Crown diameter models for *Tectona grandis* (Linn. f.) in Agan forest plantation, Makurdi, Nigeria. *Journal of Research in Forestry, Wildlife & Environment*, 15(1): 28 – 36.
- Saka, M. G., Kenan, T. and Yakubu, M. (2021). Juvenile height equations for plantation-grown Date Palm (*Phoenix dactylifera* L.). *International Journal of Research and Innovation in Applied Science*, 6(5): 91-96.
- Saud, P., Lynch, T. B and Guldin, J. M. (2016). “Twenty five years long survival analysis of an individual shortleaf pine trees,” in *Proceedings of the 18th biennial southern silvicultural research conference*, J. S. Callie, K. C.Wayne, and C.M. Oswalt, Eds., pp. 555–557.
- Sharma, M. and Parton, J. (2016). Height-diameter equations for boreal tree species in Ontario using a mixed-effects modeling approach. *Forest Ecology and Management*, vol. 249, no. 3, pp. 187–198.
- Wang, Y. F., Yue, T. X., Du, Z. P. and Zhao, M. W. (2015). Improving the accuracy of the height–diameter equation using the classified factors method. *Environmental Earth Sciences*, vol. 74, no. 8, pp. 6471–6480.
- Thomas, J. J. (1977). *An introduction to statistical analysis for economists*. Weidenfeld and Nicholson Ltd, London. 286 pp.
- Trincado, L., Vanderschaaf, C. L., and Burkhart, E. H. (2007). Regional mixed-effects height-diameter models for Lobolly Pine (*Pinus taeda* L.) Plantations. *European Journal of Forest Research*, 126: 253-262.
- Yang, Y. and Huang, S. (2014). Suitability of five cross validation methods for performance evaluation of nonlinear mixed-effects forest models – A case study. *Forestry*. 87.