



## NONLINEAR HEIGHT-DIAMETER MODELS FOR *Tectona grandis* (Linn. F.) IN MBAVAA FOREST RESERVE, BENUE STATE, NIGERIA

Popoola, V. D. \* and Uii, J. N.

Department of Forest Production and Products, Joseph Sarwuan Tarka University, Makurdi, Nigeria

\*Corresponding Author: [popoola.victoria@uam.edu.ng](mailto:popoola.victoria@uam.edu.ng); +234 809 856 6488

### ABSTRACT

The research was carried out at Mbavaa Forest Reserve in Benue State. Height and diameter at breast height are the most important measures of tree growth and their relationship is useful in determining site-index, calculating tree volume, evaluating site-quality and predicting future growth of the stand. The measurement of total tree height is relatively complex, time consuming and expensive. The aim of the study was to develop models for predicting height of *Tectona grandis*. The variables measured were; Diameter at base, Diameter at breast height and total tree height. The different height-diameter models used are Exponential, Weibull, Chapman-Richards, Lundqvist-Korf, Naslund and Modified logistic. The asymptotic coefficient  $a$  of the Exponential model was the highest compared to the five other functions. Modified Logistic had the highest Standard Error (SE) compared to the other five height-diameter models. Chapman-Richards had the highest  $R^2$  value. The fit statistics and coefficients estimation indicated that the Chapman-Richards model is the most suitable out of the six models considered for predicting height-diameter relationships in Mbavaa Forest Reserve. Height growth normally declines as the trees grow larger in size which was attributed to less competition at the upper level of the stand distribution. It is recommended that stands variables like spacing, crown area, soil fertility density, age and silvicultural practices should be incorporated in order to improve the accuracy and reliability of height-diameter models. The potential tree height-diameter equations should be validated for their predictive capabilities (e.g. accuracy, precision, and flexibility) across a range of tree diameters.

**Keywords:** height-diameter models, *Tectona grandis*, Nonlinear models, Chapman-Richards

### Correct Citation of this Publication

Popoola, V. D. \* and Uii, J. N. (2023). Nonlinear height-diameter models for *Tectona grandis* (Linn. f.) in Mbavaa forest reserve, Benue State, Nigeria. *Journal of Research in Forestry, Wildlife & Environment* Vol. 15(1): 20 – 27.

### INTRODUCTION

Forest plantations are particularly important for economic and social development of rural areas. They also play a crucial role in maintaining environmental functions. Teak (*Tectona grandis*) is a tropical tree species with a significant economic potential (Hall, 2010). In forestry, it is important to be able to make accurate future predictions of the mean values of growth variables based on repeated measurements through time made on units grouped hierarchically. In many forest management practices, decisions are based on yield projections that crucially depend on projections of plot level averages of tree height, basal area, and other morphometric

variables (Sharma, 2013). One of the greatest challenges in model development is measuring tree variables and data gathering. Diameter at breast height and tree height are two important used parameters for sustainable forest management (Karatepe, 2022). It requires estimates of growing stock, flexible and accurate models that can determine the volume of standing trees, individual log and the entire stands. Such information guides forest managers in timber valuation as well as in allocation of forest areas for harvest (Akindele and LeMay, 2009).

Teak (*Tectona grandis* Linn. F. *Verbenaceae*) is one of the most important plantation species both in the high forest and the savannah zones

of Nigeria. The species was introduced into Nigeria between 1900 and 1910 (FAO and UNEP, 2008). Large scale plantations of teak in Nigeria started in the late 1960s, under a plantation programme that was initiated with the help of the Food and Agriculture Organization (FAO) of the United Nations (Prah, 2012). According to (Jiang, 2010), teak alone cover about 15,000 ha. Several studies have shown the importance of the revenue generated by the production of teak. These benefits also involve job creation, added value and revenue generated through downstream processing and distribution of the product (Ahmadi, 2012).

A widely used metric for the characterization of the productive capacity of forest stands is the site index, defined by the mean height of the dominant trees at a reference age (Sharma, 2011). Modeling the dominant height along stand age is an effective way of characterizing and quantifying tree growth. It is the advantage of allowing the comparison of the growth between plantations with different densities and under different management regimes (for example, thinned versus non-thinned stands), since spacing and thinning from below have little impact on this variable. *Tectona grandis* (Teak) is a fast-growing exotic tree species used for a wide variety of purpose such as pole and furniture making and it has been one of the most preferred species for investment opportunities, due to its high wood quality and excellent growth performance. Effective management of Teak plantation requires information on volume of the growing stock. Therefore the need to increase the supply of Teak timber for socio-economic development through adequate measurement of its stands to determine the volume and improve the quality and quantity of the stands is very important.

Measuring diameter at breast height (Dbh) is quite easy, accurate and cost effective (Ferraz-Filho *et al.*, 2018; Corral-Rivas *et al.*, 2019). Therefore, with many permanent and temporary sample plot systems, Dbh is conventionally measured for all trees sampled, but height is measured for only a sub-sample of trees selected across the range of diameters observed. Height-diameter relationship models are used to estimate the heights of trees measured only for diameter (Ozkal *et al.*, 2021). Height-diameter models can be simple

or generalized. Simple models use Dbh to estimate height, as the relationship of the two variables is known (Bronisz and Mehtatalo, 2020). Kearsley *et al.* (2017) describes tree height estimation models, using non-linear regression models. Height and diameter at breast height are the most important measures of tree growth and their relationship is useful in determining site-index, calculating tree volume, evaluating site-quality and predicting future growth of the stand (Wagle and Sharma, 2012).

Knowledge of diameter at breast height (DBH) and tree height is fundamental to both developing and applying many growth and yield models. One of the greatest challenges in model development is measuring tree variables and data gathering. In forest measurement, DBH of a tree can be measured quickly, easily, and accurately, but the measurement of total tree height is relatively complex, time consuming and expensive. Furthermore, some site conditions and tree composition especially in tropical forests may prevent accurate height measurements on all trees measured for Dbh as it may not be possible to unambiguously observe a given tree, or reach an appropriate vantage point. A number of height-diameter models have been developed using only Dbh as a predictor variable for estimating total or merchantable tree height. However, the relationship between the diameter and its height varies among stands and depends on the growing environment and stand conditions. The study was aimed to develop models for predicting height of *Tectona grandis*. The study was limited to Mbavaa forest reserve in Konshisha Local Government, Benue state, Nigeria.

## MATERIALS AND METHODS

### Study Area

Mbavaa Forest Reserve was established from 1985 till 1991 on grazed land. At present, the area is a mosaic of Teak plantations of different ages, cultivated under various management practices, surrounded by small grazed patches, small villages and secondary forests close to streams. The plantation is located in Konshisha Local Government Area on a levelled land in the North East of Benue State.

### Sampling and Data Collection

The selected plots and stands were located by the use of prismatic compass, GPS, 50-meter tape, cutlass, and ranging poles. Ten plots were randomly selected each measured 20 x 20 m. Within each randomly selected sample plot, preference was given to the enumeration of healthy trees with more typical growth forms; dead trees and trees with abnormal form were avoided. This is because in developing growth and yield models for effective management of forest plantations, only such trees that meet grade, soundness and size requirements for commercial logs or poles are considered relevant. While this sampling guideline appeared to introduce a bias in favour of better-formed trees, it is justified because only healthy trees with good form are of commercial value and therefore, require volume computation. The following measurements were carried out on each selected tree:

- i. Stump diameter (0.3m above ground level) and diameter at breast height (at 1.3 m above ground) using the diameter tape (in cm);

- ii. Total height of trees using the Spiegel relaskop (in meters).

**Data Analysis**

Data collected were organized and screened (removing the outliers) for analysis. Descriptive statistical analysis was further carried out in order to summarize the data.

**Height-diameter Models**

For the construction of height-diameter models, the first step was to plot the data from field measurements of height against the Dbh in a scatter form. This gives an outlook of the relationship which appears to be nonlinear hence the choice of nonlinear modelling. Different height-diameter models used as shown in table 1 are Exponential, Weibull, Chapman-Richards, Lundqvist-Korf, Naslund and Modified logistic. The Chapman-Richards and Weibull functions were selected for the height-diameter modeling due to their simplicity, accuracy, and precision (Shamaki *et al.*, 2016).

**Model Evaluation**

The developed models were evaluated to know how well the model fit the data. This was carried out using the following references: Huang *et al.*, 1992; Thomas *et al.*, 1977; Zeide, 1989; Thomas, 1977). The formula below was used to compute it.

**Table 1.** Height-diameter models

Model no.	Model	Equation	References
1	Chapman Richards	$h = 1.3 + a \cdot \exp(-b \cdot dbh^c)$	Huang <i>et al.</i> , 1992
2	Weibull	$h = 1.3 + a \cdot \exp(-b \cdot dbh^c)$	Thomas <i>et al.</i> , 1977
3	Exponential	$h = 1.3 + \frac{a}{1 + b \cdot dbh^c}$	Zeide, 1989
4	Modified Logistic	$h = 1.3 + a \cdot \exp\left(\frac{b}{1 + b \cdot dbh^c}\right)$	Huang <i>et al.</i> , 2009
5	Lundqvist -Korf	$h = 1.3 + a \cdot \exp\left(\frac{b}{1 + b \cdot dbh^c}\right)$	Thomas, 1977)
6	Naslund	$h = 1.3 + \left(\frac{dbh}{a + b \cdot dbh}\right)^c$	Naslund, 1936

*h = tree height; dbh = Diameter at breast height; a, b, c = parameters; exp = base of natural logarithm raised to power of a number (exponent)*

Where R<sup>2</sup> = Coefficient of determination, SSE = Error sum of squares or Residual sum of squares, SST = Total sum of squares and 1= regression line.

- b. Standard Error: The value should be relatively small for the model to be considered valid. Standard error is given as

$$SE = \sqrt{\frac{\sum_{i=1}^n (H_i - \hat{H}_i)^2}{n-k}} \text{ ----- [8]}$$

*H<sub>i</sub>* is the observed height for the *i*th tree;  
*Ĥ<sub>i</sub>* is the predicted height for the *i*th tree; *k*

is the number of model parameters; n is the number of observations.

**RESULTS**

The dataset from the field inventory were carefully organised and analysed so as to detect and display underlying patterns. The dataset comprises of tree growth variables measured from individual trees of *Tectona grandis* plantation. A total of 159 trees were measured and summary statistics of the dataset

used in this study is presented in table 2. The distribution of diameter at breast height (DBH) ranged from 8 – 27.4 cm; diameter at the base (DB) ranged from 9.10 – 33.9 and (TH) from 1.60 – 12.9 cm. The table shows that diameter at the base (DB) has a mean value of 17.63 cm, (DBH) 14.25 cm and (TH), 7.26m. The standard deviation values of the variables (DBH, DB, and TH) are: 3.64, 4.47 and 2.33 respectively.

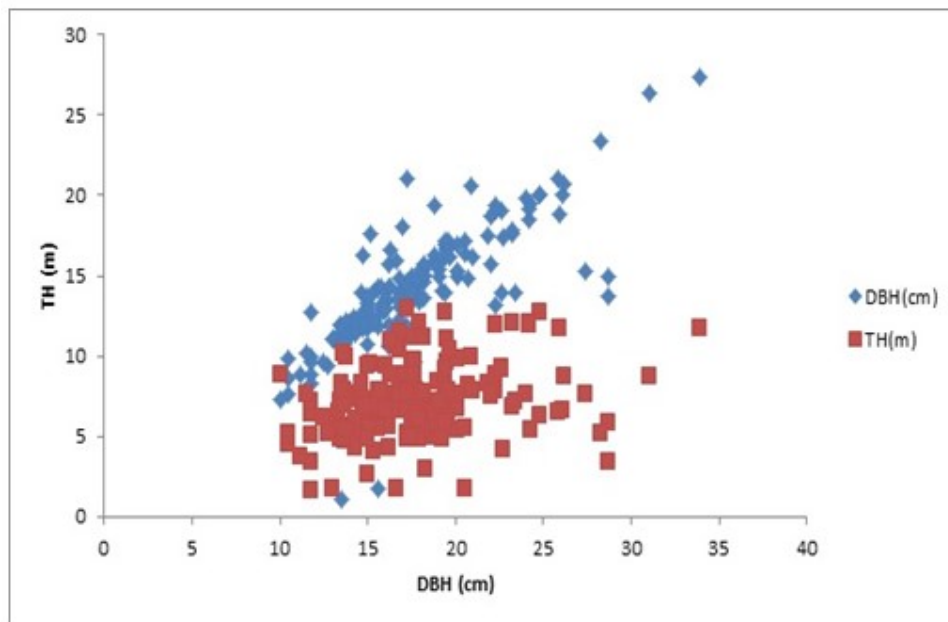
**Table 2:** Descriptive Statistics for Tree Variables of *Tectona grandis* in the Study Area.

Tree variables	N	Minimum	Maximum	Range	Mean	St. Deviation
DB (cm)	159	9.10	33.9	32.80	17.63	4.47
DBH (cm)	159	8.00	27.4	26.30	14.25	3.64
TH (m)	159	1.60	12.9	11.30	7.26	2.33

**Height-diameter Models**

Figure 1 shows the scatter relationship between Diameter at Breast height and Total height using the data gotten from the plantation before model fitting. It displays

non-uniformity of the height distribution as the teak increase in height; it is because there is less competition at the upper level of the stands.



**Figure 1:** Scatter plot of Diameter at Breast height and Total height

Chapman-Richards, modified Logistics, Korf, Weibull and Exponential have three parameters in the fit models. Naslund has two parameters in the fit model, Asymptote coefficients of the six height-diameter models (coefficient a and b in table 3), Modified Logistic and Korf models had similar asymptotic coefficients. The asymptotic

coefficient a of the Exponential model was the highest compared to the five other functions. The value is more than thrice as large as the asymptotic coefficients of the Weibull function. Modified Logistic had a highest Standard Error (SE) compared to the other five height-diameter models.

**Table 3: Regression coefficients and the standard errors for the developed models for *Tectonia grandis***

Model	Parameters	Estimate	Std. Error
chapman-richards	a	9.073	0.000
	b	1.639	0.000
	c	1.790	0.000
modified logistic	a	5.962	0.186
	b	1.817	1773777738966350340.00
	c	54.078	10243586601539381000.00
Korf	a	5.320	17544729.16
	b	1.868	5831503.52
	c	-0.494	2956057.63
Naslund	a	-1.653	0.396
	b	-0.291	0.027
Weibull	a	639.490	108599.18
	b	-0.002	0.370
	c	-0.003	0.496
Exponential	a	2218.024	0.000
	b	0.100	0.000
	c	-112.404	0.000

From the model performance for height-diameter models for *Tectonia grandis* (Table 4), the  $R^2$  values are low for Exponential, Lundqvist-Korf, Weibull and Modified logistic models. It confirms that height-diameter models can be influenced by various factors therefore height-diameter models can produce better output if factors such as; age, stand level density, spacing and some climatic variation are incorporated to the equation. Chapman-Richards had the highest  $R^2$  value.

**Table 4: Model Performance for height-diameter models for *Tectonia grandis***

Model	$R^2$
Chapman-Richards	1
Modified Logistic	0.00
Korf	0.18
Naslund	0.76
Weibull	0.069
Exponential	0.3

## DISCUSSION

This study provided information on tree growth variables of Teak (*T. grandis*) plantation in the study area. The result of the descriptive statistics (Table 2) is similar to the researches of Shamaki and Akindele (2003) and Shamaki *et al.* (2016). From the scattered graph it shows that non uniformity of the height distribution especially as the trees increase in size, this is as a result of

competition at the upper levels of the stands. Consistent increase in height with increase in diameter was noted up to 20cm in diameter similar relation was observed by Guimareas *et al.* (2009) and Krisnawati *et al.* (2010). The parameter estimate revealed a relatively higher Standard Error (SE) for modified logistic, Korf and Weibull. The Weibull SE was high as observed by Guimareas *et al.* (2009) and Shamaki *et al.* (2016). Korf model has the highest estimate which is unreasonably high as noted in other researchers for tree height and diameter models (Kriswanati *et al.*, 2010; Kourosh *et al.*, 2013).

Sharma and Paton (2007) obtained corresponding results. Studies carried out by (Jiang, 2010; Kriswanati, 2010) suggested that Chapman-Richards over other Models for height-diameter models. Shamaki *et al.* (2016) also revealed Chapman Richard was a better model for *Tectona grandis* height-diameter relationship because of the low standard error values. The results of Krisnawati *et al.* (2010) and Kourosh *et al.* (2013) reported that the models have 75% of the total variation in heights. Sharma (2009); Lumbres *et al.* (2011), noted the total explained variation was higher in comparison. Sharma (2009) observed that biological logistics is paramount and need to be considered in selecting the best models.

The Chapman-Richards model had good performance in terms of SE and  $R^2$ . Chapman-Richards model is recommended for this study area because of its performance. The result is similar with outcomes noted by Zhang (1997) and Peng *et al.* (2001). The Chapman-Richards model should be used as the best model for modelling height growth for Teak in the Plantation.

Guimaraes (2009) developed height-diameter models with the addition of covariates and noted goodness-of fit statistics in comparison with the equation that excluded covariates. Jiang (2010) developed mixed-effects models to estimate height from diameter. He derived the models from each stand data fix-effects before the combination of stands attributes as mixed-effects the results indicated that fix-effect model performed poorly compared to the mixed-effect models.

## CONCLUSION

Tree height is an important parameter for forest inventory and even to characterize tree, stand, and site conditions. However, measuring tree height is less reliably and very costly than measuring tree dbh, it is therefore limited to a sample of trees. Nonlinear growth models have been increasingly used for modelling tree height-diameter relationships. Findings of this study confirmed height growth normally decline as the trees grow larger in size, this is attributed to less competition at the upper level of the stand distribution. The fit statistics and coefficients estimation indicated that the Chapman-Richards model is the most suitable out of the six models considered for

predicting height-diameter relationships in Mbavaa Forest Reserve. The suggested model allows the natural variability in height within diameter class to be mimicked and therefore provides realistic height prediction at stand level.

Therefore, using dbh and total height can give precise estimates of total height of teak tree.

The application of Chapman-Richards height-diameter model will improve the accuracy of height prediction. The result of this study has shown that the nonlinear model can be used to model the tree height-diameter relationship.

## RECOMMENDATIONS

- i. The work recommends that stands variables like spacing, crown area, soil fertility density, age and silvicultural practices should be incorporated in order to improve the accuracy and reliability of height-diameter models.
- ii. However, such information cannot be gotten in a single measurement, there is a need for a permanent sample plots that can be used for remeasurements over time.
- iii. The models should be evaluated and, if necessary, revisited or calibrated when they are applied in different regions.
- iv. More importantly, the potential tree height-diameter equations should be validated for their predictive capabilities (e.g. accuracy, precision, and flexibility) across a range of tree diameters. This useful information can help forest researchers and managers to select and apply the appropriate models. Model validation can be conducted using independent validation data sets.

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