VOLUME ESTIMATION FROM PROBABILITY DISTRIBUTION MODELS

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ABSTRACT

Two Statistical distribution functions; 3parameter Weibull and 3parameter Dagum were compared to evaluate the most suitable for fitting the diameter distribution and subsequently estimating the volume of Tectona grandis Plantation at Omo Forest Reserve, Ogun State, Nigeria. Fifteen temporary sample plots of 25m by 25m were randomly established and complete enumeration of the diameter at breast height over back and total height was carried out. The diameter distributions were then processed into 1cm diameter classes. A 3parameter Weibull function with α, β and γ (representing shape, scale and location respectively) and 3parameter Dagum with k, α (shape) and β (scale) were fitted into the size-class dataset using maximum likelihood method. Weibull recorded smaller mean goodness-of-fit values of 0.9962, 0.0882 and 0.5565 for Kolmogorov-Smirnov, Cramer Von Misser and Anderson Darling statistics respectively hence its preference over Dagum. Stand volume estimation was done using Weibull distribution which estimated 7580.977m3/ha (predicted) as against 8010.146m^3/ha (observed). This therefore indicates a shortfall of 429.169m^3/ha. Overall, 3Parameter Weibull is suitable volume estimation and fitting diameter distribution of the Tectona grandis plantation at Omo forest Reserve.

Keyword: Weibull, Dagum, *Tectona grandis*, Volume estimation

INTRODUCTION

Teak (*Tectona grandis*) is one of the most valuable timber yielding species in the world, with predominant distribution in tropical or sub-tropical countries. Most forest plantations in Nigeria are composed of Teak due to its rapid growth, moderate heaviness, rapid turn-over with quick and high returns on investments per hectare (Agbeja, 2004). However, by virtue of its economic significance, it is important to closely monitor plantations of Teak to ensure best management practices.

While single tree models may be considered too complex and stand models considered oversimplified for growth studies, diameter distribution modeling (DMD) has become an intrinsic part of sustainable forest management planning as it plays a major role in bridging the gap (Nord-Larsen and Cao., 2006). It is a less expensive and practical management means for forest managers as it indicates whether the density of smaller trees in a stand is sufficient to replace the current density of older trees (Rubin and Manoin., 2006). In time past, yield tables were found adequate in meeting this management need; to show empirical size distributions, expected stand structure and size distribution for any specific silvicultural regime (Carbonnier, 1971). However, such yield tables are grossly limited in their capacity to handle changes in silvicultural regimes (Nor-Larsen and Cao., 2006).

In recent forestry practices, different probability density functions have been used to model the diameter distribution of trees in stands e.g. Beta (Loetsch *et al.*, 1973; Gorgoso *et al.*, 2008, 2012), Gamma (Nelson, 1964; Mohammed, *et al.*, 2009; Zheng and Zhous, 2010; Eslami *et al.*, 2011), Johnson SB (Johnson and Kitchen, 1971; Knoebel and Burkhart, 1991), Lognormal (Sheykholeslami *et al.*, 2011), Normal (Nanang, 1998) and Weibull distribution (Bailey and Dell, 1973;Zutter *et al.*, 1986; Maltamo *et al.*, 1995; Palahi *et al.*, 2007; Ajayi, 2013; Ogana *et al.*, 2015; Mayrinck *et al.*, 2018) and have exploited the correlation between model parameters and stand variables to predict future scenarios with respect to forest types (Ogana *et al.*, 2015; Lorimer and Krug., 1983), management objectives (Cheng., 2004) and forest dynamics (Ogana, 2018). Although, the selection of any distribution function and estimation method is entirely the choice of the researcher (Siipilehto and Mehtatalo, 2013) and there is no *a priori* biological basis for choosing to use any statistical function to characterize forest diameter structure (Shiver 1988). In this study, 3-Parameter Weibull and 3-Parameter Dagum distribution are used to assess the most suitable for fitting the diameter structure of *Tectona grandis* plantation at Omo Forest Reserve and subsequently estimate stand volume.

MATERIALS AND METHODS

Data Collection

The data used in this study was collected from 15 temporary sample plots established in the *Tectona grandis* stand at Omo Forest Reserve, Nigeria. We established square plots with a 25m x 25m dimension ($625m^2$ plot size). In each sample plot, all living trees with DBH ≥ 5cm were enumerated and assessed for diameter at breast height (1.3 meters from the ground) and their corresponding height using a diameter tape and a Spiegel Relascope, respectively. A total of 834 trees were available for the analysis.

Table 1: below presents the summary statistics of the data from the sample plots (n=15)

	Statistics			
Stand Variables	**Mean**	**Minimum**	**Maximum**	**Standard Deviation**
Mean Diameter(cm)	19.605	17.255	21.706	1.303
Dq(cm)	20.185	17.823	22.355	1.299
Mean Height(m)	18.059	16.507	20.041	1.010
Density(Nha)	889.600	704.000	1728.000	262.475
Basal Area, G (m^2/ha)	28.415	17.564	50.388	7.874
Volume (m^3/ha)	256.735	146.027	459.310	76.019
Hdom (m)	22.493	20.400	24.000	1.282

Dg is Quadratic mean diameter; Hdom is Dominant Height

Probability Density Functions

Weibull Distribution

The probability density function (PDF) of the three-parameter model for the Weibull random variable x, utilizing notation by Dubey (1967), is

$$f(x) = \frac{\alpha}{\beta}\left(\frac{x-\gamma}{\beta}\right)^{c-1} e^{-\left(\frac{(x-\gamma)}{\beta}\right)^{2}} \quad \text{.......... (1)}$$

x≥γ; β>0; α>0

where α = shape parameter, β = scale parameter, γ = scale parameter

It's preference over other distribution is as a result of its ability to assume the required variety of shapes wholly dependent on the value of the shape parameter, α (Harter., 1964). If α < 1, the curve is a reversed J-shape. When α = 1, the exponential distribution results:

$$f(x) = \frac{1}{\beta}\exp\left(\frac{-x}{\beta}\right) \quad \text{............ (2)}$$

X ≥ 0, β > 0

Dagum Distribution

The probability density function of 3 parameter Dagum distribution is;

$$f(x) = \frac{\alpha k\left(\frac{x}{\beta}\right)^{\alpha k-1}}{\beta\left(1+\left(\frac{x}{\beta}\right)^{-\alpha}\right)^{-k}} \quad \text{...........(3)}$$

The cumulative distribution function is;

$$F(x) = \left(1+\left(\frac{x}{\beta}\right)^{-\alpha}\right)^{-k} \quad \text{......... (4)}$$

k>0; α > 0; β > 0

Where k and α are continuous shape parameters; β is the scale parameter

Fitting Volume Model

To predict volume, the "general nonlinear" model below which describe individual tree volume as a regressant of the diameter at breast height (dbh) and total (ht) was fitted. As used in previous studies e.g. Mugasha *et al.* (2016), Magalh˜aes (2017), Schumacher and Hall. (1993), it is considered to have satisfactory mathematical properties (Malata *et al.*, 2017).

$$V = b_0 \times dbh^{b_1} \times ht^{b_2} \quad \text{......(5)}$$

Where V is total tree stem volume over bark (m^3), dbh is the diameter at breast height (cm), ht is total tree height (m), and b_0, b_1, and b_2 are regression parameters.

Estimating Volume with the best Diameter Distribution Model

Volume estimation using best Diameter Distribution Model was calculated by:

i. multiplying predicted Relative Frequencies for each diameter class by total number of trees in each plot then divided by plot size(0.0625m^2) to estimate density (Nha).

$$Nha = \frac{\text{Relative Frequancy} * \text{Total Number of Trees}}{\text{plot size}} \quad . (6)$$

i. Basal area (equation 6) multiplied by Nha gives per hectare estimation of the basal area of each diameter class

$$G_i = \frac{\pi d^2}{4} \text{ X Nha} \quad \text{.................. (7)}$$

Note: $\sum G_i$ gives plot by plot basal area estimation

ii. Volume is calculated by multiplying G_i for

each diameter class by mean height (Ht) as estimated by the Height-Diameter model.

RESULTS

Diameter Distribution

This study sought to characterize the tree diameter structure of *Tectona grandis* plantation at Omo Forest Reserve and estimate volume. The performance of 3-parameter Weibull and 3-parameter Dagum distribution was assessed to evaluate the distribution function that best predicts the diameter structure of the forest stand.

The graphs plotted for number of trees per hectare against relative frequency of the two statistical distribution considered in this study are atypical of a managed plantation; a dumb bell shape with most of the observations clustering around the mean. The skewness and kurtosis of the distributions (within-plot observations) were negative further confirming that the distributions have light tails i.e. fewer numbers of trees towards the right (large-size trees) and left (small-size trees). This confirms the even aged nature of the forest. From the graphs, it is evident that 3-parameter Weibull distribution performed better than 3-parameter Dagum distribution in fitting the diameter distribution of the *Tectona grandis* plantation at Omo Forest Reserve. Also, judging by the values of the goodness of fit indices in Table 3 below, 3-parameter Weibull distribution recorded the lowest rank sum in all 15 plots.

Table 2: Goodness of statistics for 3P Weibull and 3P Dagum

Plot	Kolmogorov-Smirnov Test Values				Cramer Von Misser Test Values				Anderson Darling Test Values				Rank Sum	
	3P Weibull	Rank	3P Dagum	Rank	3P Weibull	Rank	3P Dagum	Rank	3P Weibull	Rank	3P Dagum	Rank	Weibull	Dagum
1	0.0995	1	1	2	0.1022	1	16.2796	2	0.6427	1	560.0013	2	3	6
2	0.0683	1	0.9929	2	0.0368	1	0.3087	2	0.279	1	1.639659	2	3	6
3	0.1032	1	1	2	0.0693	1	14.6666	2	0.3876	1	16731.17	2	3	6
4	0.1111	1	1	2	0.1081	1	14.2122	2	0.6873	1	186.0389	2	3	6
5	0.1275	1	0.9958	2	0.1412	1	1.7301	2	0.7251	1	11.13999	2	3	6
6	0.0995	1	0.9998	2	0.1460	1	1.0928	2	0.8845	1	5.821821	2	3	6
7	0.0936	1	0.9763	2	0.0815	1	0.5427	2	0.5708	1	2.987278	2	3	6
8	0.1324	1	1	2	0.1479	1	14.0048	2	0.9055	1	165.723	2	3	6
9	0.0737	1	1	2	0.0491	1	19	2	0.3711	1	2579.605	2	3	6
10	0.0555	1	1	2	0.0252	1	13.25	2	0.1807	1	175.9752	2	3	6
11	0.1014	1	1	2	0.0963	1	14.019	2	0.6291	1	256.719	2	3	6
12	0.0736	1	0.9998	2	0.0483	1	6.2826	2	0.3788	1	41.43821	2	3	6
13	0.0534	1	0.9996	2	0.0208	1	6.5518	2	0.192	1	48.84848	2	3	6
14	0.0780	1	1	2	0.1134	1	25.1323	2	0.7196	1	264.905	2	3	6
15	0.0989	1	0.9823	2	0.1364	1	0.0804	2	0.7941	2	0.6052	1	4	5

Table 3: Percentage difference in the observed and predicted values for Density and Volume

Plot	Nha (Observed)	Nha (Predicted)	Difference	% difference	Observed Volume	Predicted Volume	Difference	% difference
1	784	768	16	2.0	572.24	512.71	59.53	10.40
2	880	865	15	1.7	487.82	516.56	-28.74	-5.89
3	704	691	13	1.8	303.74	290.32	13.42	4.42
4	768	748	20	2.6	439.99	445.30	-5.31	-1.21
5	832	819	13	1.6	465.28	465.39	-0.11	-0.02
6	1168	1151	17	1.5	607.90	575.07	32.83	5.40
7	832	784	48	5.8	513.35	453.68	59.67	11.62
8	800	781	19	2.4	623.12	576.36	46.75	7.50
9	912	902	10	1.1	669.32	634.89	34.42	5.14
10	784	741	43	5.5	462.25	444.06	18.19	3.93
11	704	692	12	1.7	448.50	433.11	15.39	3.43
12	992	977	15	1.5	667.37	635.65	31.71	4.75
13	736	702	34	4.6	362.67	293.80	68.87	18.99
14	1728	1694	34	2.0	955.36	901.82	53.55	5.60
15	720	703	17	2.4	431.24	402.24	29.00	6.73
			Sum = 8010.15m^3/ha		**Sum= 7580.977m^3/ha**		**Deficit = 429.17 m^3/ha**	

Weibull recorded mean goodness-of-fit values of 0.9962, 0.0882 and 0.5565 for Kolmogorov-Smirnov, Cramer Von Misser and Anderson Darling Statistics while 3-Parameter Dagum recorded higher values as presented in table 2.

Volume Estimation using 3Parameter Weibull

The expected number of trees (N/ha) predicted by Weibull distribution shows slight variation from the observed. This variation has a direct impact on the basal area and volume estimation by virtue of the strong correlation between these stands' variables. Although slight, Weibull underestimated density (number of trees per hectare) in all 15 plots ranging from 1.1 percent in plot 9 to 5.8 percent underestimation in plot 7 as shown in Table 3. Similar trend was noticed in the volume estimation as all plots except Plots 2, 4 and 5 were overestimated with percentage difference of 5.89, 1.12 and 0.02 respectively. In total, stand volume estimated using Weibull distribution was 7580.977m3/ha (predicted) as against 8010.146m^3/ha (observed). This therefore indicates a shortfall of 429.169m^3/ha.

Table 3 and Figures 1 and 2 present the trend of density and volume estimation.

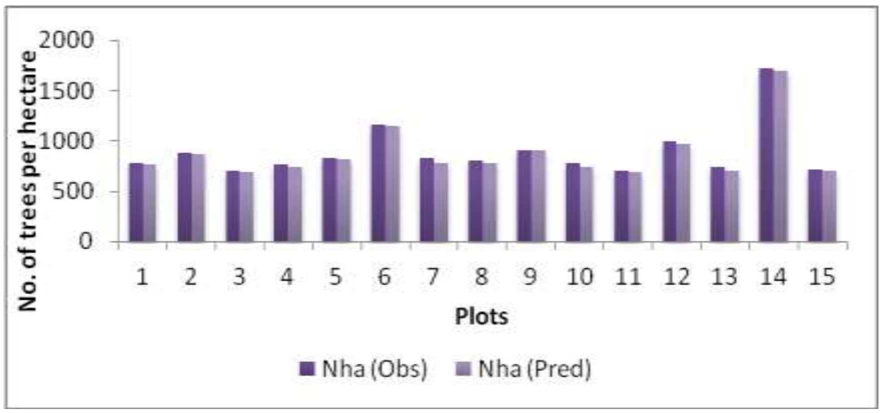

Figure 1: Graph showing the Observed Density and Predicted Density per plot using 3P Weibull

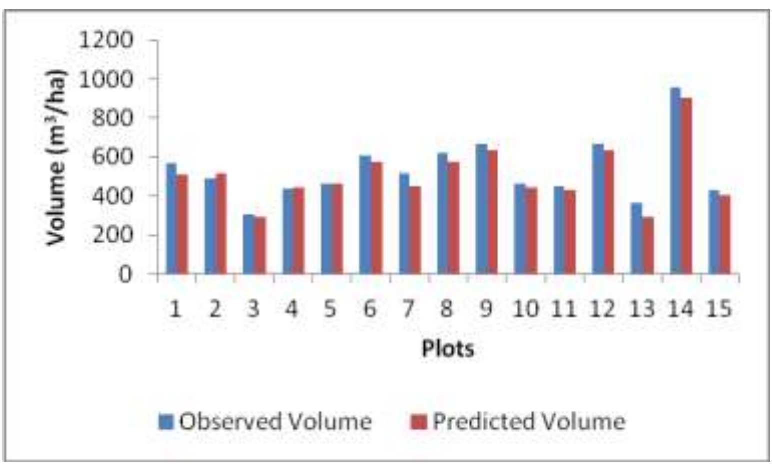

Figure 2: Graph showing the Observed Volume and Predicted Volume per plot using 3P Weibull

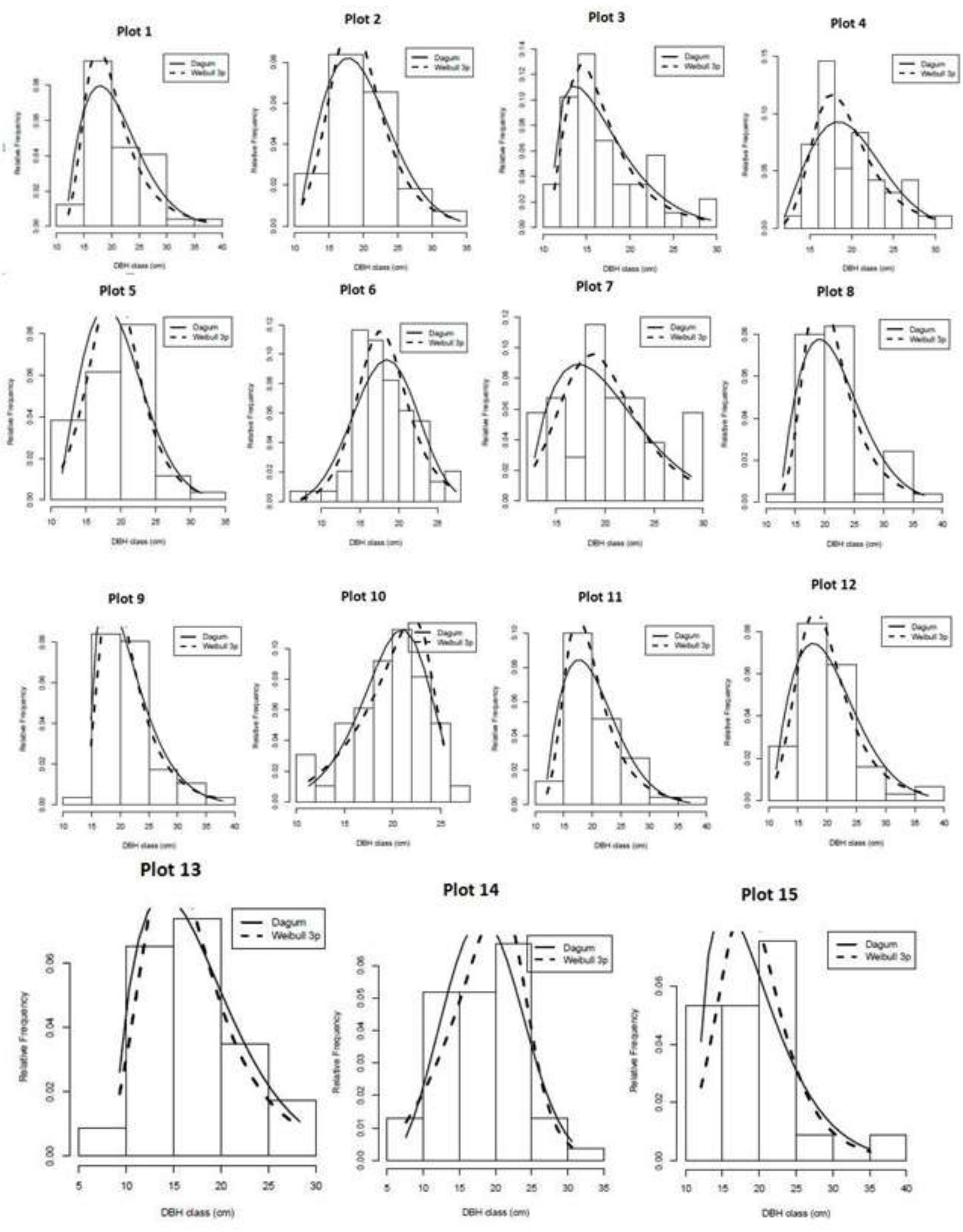

Figure 3: Graphical analysis showing the predictive ability of 3parameter Weibull and Dagum distribution from Plot 1 to 15

DISCUSSION

Therefore, 3-Parameter Weibull was favored for estimation of number of trees per hectare and volume estimation (on plot basis). This agrees with Alo et al. (2017) who compared Beta, 3-Parameter lognormal, 3-Parameter gamma and 3-Parameter Weibull for describing diameter structures of second rotation plantations of Teak (*Tectona gandis*) at Eda Forest Reserve and found 3parameter Weibull most suitable. Ogana and

Gorgoso-Varela (2015) also found 3-Parameter Weibull suitable for describing the natural stands of Oluwa Forest Reserve which is also a tropical rainforest ecosystem in Southwestern Nigeria. Ajayi., 2013 also found 3-ParameterWeibull suitable for diameter characterization of Ukpon Forest Reserve in Cross River Nigeria and opined that its suitability is due to its flexibility in depicting positive and negative skewness.
This estimation methodology gives a leverage to managers to estimate stock volume per diameter class rather than oversimplified stand models or complex single tree model that may not be found applicable in practical situations. Depending on the thinning and harvesting cycle adopted by the management of Omo Forest Reserve, the graphical representation will aid decision-making of allowable cut, type of machinery and cost-benefit ratio of silvicultural operations.

CONCLUSION

This study successfully compared the ability of two statistical functions to predict diameter distribution at Omo Forest Reserve and found 3Parameter Weibull more suitable. The results from the goodness of fit statistics i.e. Kolgomorov-Smirnov (K-S), Cramer Von Misses and Anderson Darling statistics indicated that both 3Paramter Weibull and 3parameter Dagum successfully fitted the data. However, the Weibull distribution was more accurate in all the diameter classes for the individual plots in that it gave closer estimates to the real distribution.

In the same vein, Weibull distribution gave volume and density estimation that are close to observed estimation with an underestimation deficit of 429.17 m^3/ha (as shown in Table 3). Although, T-test revealed a statistically significant difference between the observed and Weibull-predicted values, we therefore still conclude that Weibull distribution is suitable for fitting the diameter distribution and estimating volume and other stand variables of Teak Plantation at Omo Forest Reserve and recommend further studies on the suitability of other probability distribution functions. We also recommend further studies involving parameter prediction and parameter recovery methods taking information provided in this study as a foundational resource.

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