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## CHARACTERISING TRUNCATED TREE DIAMETER DATA FROM TROPICAL FOREST STANDS IN NIGERIA

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### ABSTRACT

*In forest inventory, diameter threshold is often set during data collection and as such lead to left truncation. Such data are preferably fitted with a truncated distribution. In this study, newly introduced truncated Log-Logistic and well-known truncated Weibull were used for characterising tree diameter from natural forest. The truncated distributions were compared with Burr, Logit-Logistic, generalized Weibull, Johnson's  $S_B$ , 3-parameter Log-Logistic, 3-parameter Weibull, 2-parameter Log-Logistic and 2-parameter Weibull distributions. Data were obtained from 11, 10, 10 and 7 sample plots of size 0.25 ha in Akure, Cross River South, Ikrigon and Oluwa forest reserves, respectively. Distributions were fitted with maximum likelihood. Model assessment was based on Kolmogorov-Smirnov, Reynold's error index, mean square error, mean absolute error and Akaike information criterion. The result showed that the performance of the truncated distributions was comparable to other distributions considered in this study. The truncated distributions were more suitable than the commonly used Weibull and Johnson's  $S_B$  distributions. Thus, whenever a diameter threshold is fixed, truncated distribution should be used for modelling the data set.*

**Keywords:** truncated distributions; Akure forest reserve; Cross River South forest reserve; Ikrigon forest reserve; Oluwa forest reserve

### INTRODUCTION

Nigeria is blessed with unique forest ecosystem especially in the Southern and Western part of the country. In them are found the relics of some of the world tropical rainforests characterised with diverse species composition. The Nigerian tropical rainforest ecosystem is a main source of timber for the wood industries (Akindele, 2002). However, a large proportion of the country's forest estate has been lost to uncontrolled forest exploitation, poor logging practices, fuel wood extraction, bush burning, and agricultural expansion etc. while the remaining are the not sustainably managed (Adekunle, 2006). A good understanding of stand developmental patterns is essential in sustainable management and planning of cultural treatment for any forest (Oliver and Larson, 1996).

Trees Diameters and mainly its structure characterise any forest stand very well. Diameter

structure is a very significant stand characteristic for evaluating growth and volume production of a stand as well as the structure of assortments, maturity, etc. Diameter structure may be expressed by frequency distributions of tree diameters that quantify their distribution in diameter classes (Petras *et al.*, 2010). A typical structure of a natural forest has an inverse J-shaped, i.e. large number of trees in the lower diameter classes with a decreasing frequency from left to right of the curve. Inventory data from natural forest are usually left truncated because minimum measurement limit is fixed. Diameter limit is usually fixed during inventory either "to reduce measurement time or because only merchantable trees are of interest" (Zutter *et al.*, 1986). Whatever the reason might be, such data are usually left truncated. In forest plantation, a diameter (i.e. diameter at breast height, dbh) limit of 7.5 cm is usually used (Palahi *et al.*, 2007). However, in

natural forest a Dbh limit of  $\geq 10$  cm is often used (Adekunle *et al.*, 2013).

Modelling truncated data was first introduced to forestry by Hyink (1979). And since then few authors have used the truncated Weibull distribution to model diameter/basal area distribution(s) of truncated data (e.g. Zutter *et al.*, 1986; Palahi *et al.*, 2007 etc.). In Nigeria, different studies on diameter distributions of natural forests have used either 2-parameter, 3-parameter or 4-parameter distributions such as the Weibull and Johnson's  $S_B$  distributions with no truncation point (e.g. Ige *et al.* 2013; Aigbe and Omokhua 2014; Ogana *et al.*, 2015; Ogana and Gorgoso-Varela, 2015; Adekunle and Akharume, 2017).

Fitting truncated data with a distribution without a specification of truncation point will invariably result to bias in predicting the stand diameter distribution. To date, no published literature exists on characterising tree diameter distribution with truncated distribution functions in Nigeria. Also, no study exists in forestry literature where the truncated Log-Logistic distribution was used to describe stand diameter distributions. The Log-Logistic distribution is the limiting form of the Logit-Logistic distribution (Wang, 2005). It is relatively simple in expression like the Weibull distribution and has a closed-form cumulative distribution function which provides easy estimation of the number of trees per hectare in diameter classes. Therefore, the main objective of this study is to apply

a new truncated Log-Logistic and well-known truncated Weibull distributions to characterise tree diameter data from natural forest stands. The truncated distributions were compared with other established distribution functions in forestry.

## MATERIALS AND METHODS

### Study area

The data for this study came from four distinct Forest Reserves (FR) in Nigeria including Akure FR, Cross River South FR (CRS FR), Ikrigon FR and Oluwa FR. Akure FR and Oluwa FR are located in Ondo State, Nigeria. Akure FR lies between latitude  $5^{\circ}45' - 8^{\circ}15'N$  and longitude  $4^{\circ}30' - 6^{\circ}0'E$  and occupies an area of 10,500 ha (Adekunle and Akharume, 2017). Oluwa FR lies between Latitude  $6.83^{\circ} - 6.91^{\circ}N$  and Longitude  $4.52^{\circ} - 4.59^{\circ}E$ . It covers an area of 629  $km^2$  (Ogunjemite *et al.*, 2006). Ikrigon and CRS FR are located in Cross River State, Nigeria. Ikrigon FR lies between latitude  $6^{\circ}17.597' - 6^{\circ}17.862'N$  and longitude  $8^{\circ}35.597' - 8^{\circ}35.276'E$  and occupies an area of 542.7 ha. CRS FR lies between latitude  $5^{\circ}50.978' - 5^{\circ}51.029'N$  and longitude  $8^{\circ}29.833' - 8^{\circ}29.424'E$  and occupies an area of 80,534.07 ha.

Inventory data were obtained from 11, 10, 10 and 7 sample plots of size 0.25 ha in Akure FR, CRS FR, Ikrigon FR and Oluwa FR, respectively. The following stand variables were computed from the inventory data: quadratic mean diameter, dominant height, density, basal area per ha and volume per ha (Table 1).

**Table 1.** Descriptive statistics of some stand variables of the study area

FR	Stand Variable	Statistics			
		Mean	Max	Min	S.D
Akure	Quadratic mean dbh (cm)	62.54	82.25	47.18	10.99
	Dominant height (m)	42.71	45.21	35.81	2.68
	Density (N/ha)	237.09	336.00	108.00	58.76
	Basal area (m <sup>2</sup> /ha)	72.29	126.89	45.20	26.01
	Volume (m <sup>3</sup> /ha)	3012.15	5379.23	1601.92	1107.54
	Tree species = 39				
CRS	Quadratic mean dbh (cm)	30.44	43.63	18.76	6.18
	Dominant height (m)	24.68	31.26	15.07	4.82
	Density (N/ha)	175.60	312.00	60.00	70.11
	Basal area (m <sup>2</sup> /ha)	12.51	23.20	3.76	5.41
	Volume (m <sup>3</sup> /ha)	357.77	698.26	67.62	175.72
	Tree species = 75				
Ikrigon	Quadratic mean dbh (cm)	33.81	39.59	29.38	3.46
	Dominant height (m)	31.93	35.93	27.56	2.53
	Density (N/ha)	310.40	440.00	184.00	72.82
	Basal area (m <sup>2</sup> /ha)	28.01	40.91	16.94	8.29
	Volume (m <sup>3</sup> /ha)	814.13	1235.85	447.89	264.42
	Tree species = 63				
Oluwa	Quadratic mean Dbh (cm)	29.58	34.46	26.12	2.73
	Dominant height (m)	25.72	33.03	22.06	3.71
	Density (N/ha)	277.14	352.00	200.00	56.56
	Basal area (m <sup>2</sup> /ha)	19.39	31.71	14.53	6.68
	Volume (m <sup>3</sup> /ha)	512.92	906.35	331.83	213.93
	Tree species = 58				

S.D = standard deviation

**Model Specification**

Generally, a left-truncated form of any distribution model with truncation point *t* say, has probability density function pdf (*f<sub>t</sub>(x)*) and cumulative distribution function cdf (*F<sub>t</sub>(x)*) expressed as:

$$f_t(x) = \frac{f(x)}{1-F(t)} \tag{1}$$

$$F_t(x) = \frac{1}{1-F(t)} \{F(t) - F(x)\} \tag{2}$$

Where *f(x)* and *F(x)* are the pdf and cdf of any selected distribution function, *t* = truncation point (10 cm was used in this study). Equation (1) and (2) were used to construct the truncated Weibull (TWeibull) and Log-Logistic (TLogL). The simplified expression of the TWeibull used by Palahi *et al.* (2007) is given by:

$$f_t(x) = \frac{\alpha}{\beta} \left(\frac{x}{\beta}\right)^{\alpha-1} e \left[ \left(\frac{t}{\beta}\right)^\alpha - \left(\frac{x}{\beta}\right)^\alpha \right] \tag{3}$$

$$F_t(x) = 1 - e \left[ \left(\frac{t}{\beta}\right)^\alpha - \left(\frac{x}{\beta}\right)^\alpha \right] \tag{4}$$

Where: *x* = tree diameter;  $\alpha$  = shape parameter ( $\alpha > 0$ );  $\beta$  = scale parameter ( $\beta > 0$ ).

The analytical expression of TLogL is given by:

$$f_t(x) = \frac{\frac{\alpha}{\beta} \left(\frac{x}{\beta}\right)^{\alpha-1} \left[1 + \left(\frac{x}{\beta}\right)^\alpha\right]^{-2}}{1 - \left[1 + \left(\frac{\beta}{10}\right)^\alpha\right]^{-1}} \tag{5}$$

$$F_t(x) = \frac{1}{1 - \left[1 + \left(\frac{\beta}{10}\right)^\alpha\right]^{-1}} \left\{ \left[1 + \left(\frac{\beta}{10}\right)^\alpha\right]^{-1} - \left[1 + \left(\frac{\beta}{x}\right)^\alpha\right]^{-1} \right\} \tag{6}$$

All parameters are previously defined. This is the first application of truncated Log Logistic distribution to forestry to the best of my knowledge.

The TWeibull and TLogL were compared with other distributions including Burr, Generalized Weibull (GW), Johnson's S<sub>B</sub>, Logit-Logistic (LL), 3-parameter Log-Logistic (3P LogL), 3-parameter Weibull (3P Weibull). Also considered are the ordinary 2-parameter Log-Logistic (2P LogL) and 2-parameter Weibull (2P Weibull).

The Johnson's S<sub>B</sub> distribution (Johnson, 1949) is expressed as:

$$f(x) = \frac{\delta}{\sqrt{2\pi}} \cdot \frac{\lambda}{(\xi + \lambda - x)(x - \xi)} \cdot e^{-\frac{1}{2}[\gamma + \delta \cdot \ln(\frac{x - \xi}{\xi + \lambda - x})]^2} \tag{7}$$

Where:  $\xi < x < \xi + \lambda$ ,  $-\infty < \xi < +\infty$ ,  $-\infty < \gamma < +\infty$ ,  $\lambda > 0$ , and  $\delta > 0$ . The S<sub>B</sub> function has location parameter  $\xi$ , the scale parameter  $\lambda$ , and the shape parameters  $\gamma$  and  $\delta$  (asymmetry and kurtosis parameters, respectively).

Logit-Logistic (LL) (Wang and Rennolls, 2005) distribution: the pdf and cdf are expressed as:

$$f(x) = \frac{\lambda}{\sigma} \frac{1}{(x - \xi)(\xi + \lambda - x)} \frac{1}{e^{-(\mu/\sigma)(\frac{x - \xi}{\xi + \lambda - x})^{1/\sigma}} + e^{\mu/\sigma(\frac{x - \xi}{\xi + \lambda - x})^{-(1/\sigma)}}} \frac{\alpha k}{\beta} \left(\frac{x - \gamma}{\beta}\right)^{\alpha - 1} \left(e\left[-\left(\frac{x - \gamma}{\beta}\right)^\alpha\right]\right) \left(1 - e\left[-\left(\frac{x - \gamma}{\beta}\right)^\alpha\right]\right)^{k - 1} \tag{8}$$

$$F(x) = \frac{1}{1 + e^{\mu/\sigma(\frac{x - \xi}{\xi + \lambda - x})^{-(1/\sigma)}}} \tag{9}$$

Where:  $f(x)$  = probability density function,  $F(x)$  = cumulative distribution function,  $x$  = diameter. The parameters  $\mu$  = mu and  $\sigma$  = sigma are the shape parameters. Other parameters are previously defined in equation (1).

The Burr (Burr, 1942) has pdf and cdf expressed as:

$$f(x) = \frac{\alpha k \left(\frac{x - \gamma}{\beta}\right)^{\alpha - 1}}{\beta \left(1 + \left(\frac{x - \gamma}{\beta}\right)^\alpha\right)^{k + 1}} \tag{10}$$

$$F(x) = 1 - \left(1 + \left(\frac{x - \gamma}{\beta}\right)^\alpha\right)^{-k} \tag{11}$$

Where:  $f(x)$  = probability density function (pdf);  $F(x)$  = cumulative distribution function (cdf)  $k$  and  $\alpha$  = two shape parameters ( $k > 0$ ;  $\alpha > 0$ );  $\beta$  = scale parameter ( $\beta > 0$ );  $\gamma$  = location parameter.

The 3-parameter Log-Logistic distribution (3P LogL) has pdf and cdf expressed as:

$$f(x) = \frac{\alpha}{\beta} \left(\frac{x - \gamma}{\beta}\right)^{\alpha - 1} \left[1 + \left(\frac{x - \gamma}{\beta}\right)^\alpha\right]^{-2} \tag{12}$$

$$F(x) = \left[1 + \left(\frac{\beta}{x - \gamma}\right)^\alpha\right]^{-1} \tag{13}$$

Where:  $f(x)$  = pdf;  $F(x)$  = cdf  $\alpha$  = shape parameters ( $\alpha > 0$ );  $\beta$  = scale parameter ( $\beta > 0$ );  $\gamma$  = location parameter. When location the location parameter is equal to zero (i.e.  $\gamma = 0$ ), 2-parameter Log-Logistic (2P LogL) is formed.

The pdf and cdf of the 3-parameter Weibull (3P Weibull) distribution (Weibull, 1951) are expressed as:

$$f(x) = \frac{\alpha}{\beta} \left(\frac{x - \gamma}{\beta}\right)^{\alpha - 1} e\left[-\left(\frac{x - \gamma}{\beta}\right)^\alpha\right] \tag{14}$$

$$F(x) = 1 - e\left[-\left(\frac{x - \gamma}{\beta}\right)^\alpha\right] \tag{15}$$

When  $\gamma = 0$ , 2-parameter Weibull (2P Weibull) is formed.

The generalized Weibull distribution (Wang and Rennolls, 2005) (GW) has pdf and cdf expressed as:

$$f(x) = \frac{\alpha k}{\beta} \left(\frac{x - \gamma}{\beta}\right)^{\alpha - 1} \left(e\left[-\left(\frac{x - \gamma}{\beta}\right)^\alpha\right]\right) \left(1 - e\left[-\left(\frac{x - \gamma}{\beta}\right)^\alpha\right]\right)^{k - 1} \tag{16}$$

$$F(x) = \left(1 - e\left[-\left(\frac{x - \gamma}{\beta}\right)^\alpha\right]\right)^k \tag{17}$$

Where:  $f(x)$  = probability density function (pdf);  $F(x)$  = cumulative distribution function (cdf)  $\alpha$  = shape parameter ( $\alpha > 0$ );  $k$  = exponentiated shape parameter ( $k > 0$ );  $\beta$  = scale parameter ( $\beta > 0$ );  $\gamma$  = the lower boundary parameter i.e. location parameter.

**Fitting Method**

The method of maximum likelihood was used to fit the distributions to the truncated diameter data across the four forest stands. It involves taken the partial derivatives of the log-likelihood function with respect to each of the distribution's parameters and setting the expression equal to zero and then solve by a numerical iterative algorithm to yield the estimates. This was achieved with the 'optim function' in R (R Core Team 2017). Studies have shown that modelling species-specific data provides better fit than pooling the species together (Palahi *et al.*, 2006; Liu *et al.*, 2014). However, because of the complex nature of the forest stands, few individuals of anyone species was found in each plot ( $n < 20$ ). And as such, the species were pooled together by plot and fitted.

The distributions were evaluated based on Kolmogorov-Smirnov (K-S), Reynold's error index (EI), mean square error (MSE), mean absolute error (MAE), bias and Akaike information criterion (AIC). The AIC was used because the number of parameters differs in the ten models. The smaller the values of the fit indices are, the better the distribution.

## RESULTS

The assessment of the overall fitting performance of the distributions across the four stands are presented in Table 2 and 3. The result showed that the new truncated Log-Logistic (TLogL) had the best performance for the Akure FR data with respect to the fit indices. TLogL had mean K-S, EI, MSE, MAE, bias and AIC values of 0.07508, 2.21924, 0.00021, 0.01017, 0.00093 and 548.225, respectively. This was followed by GW, Burr, LL, 3P LogL, 2P LogL, truncated Weibull (TWeibull) and 3P Weibull in that order. Johnson's  $S_B$  and 2P Weibull had the worst fit. The result for the data from Oluwa forest reserve, showed that GW distribution had the best fit with mean K-S, EI, MSE, MAE, bias and AIC values of 0.07865, 1.59067, 0.00019,

0.00794, 0.00015 and 519.285, respectively. The result was followed by Burr, LL, 3P LogL, TWeibull, TLogL and Johnson's  $S_B$ . 2P LogL and 2P Weibull distributions had the worst performance (Table 2). The truncated distributions had smaller AIC values compared to the other distributions.

The performance of the distributions with respect to the data from Cross River South forest reserve (CRS FR), showed that Burr distribution had the best fit with mean K-S, EI, MSE, MAE, bias and AIC values of 0.08453, 1.75194, 0.00035, 0.01135, 0.00053 and 330.928, respectively. The result was followed by TLogL, GW, LL, TWeibull, Johnson's  $S_B$ , 3P LogL and 3P Weibull. 2P LogL and 2P Weibull distributions had the worst performance (Table 3). In the case of data from Ikrigon forest reserve, GW had best fit. The mean values for the fit indices were 0.06718, 1.26791, 0.00019, 0.00985, 0.00023 and 614.5, respectively. This was followed by LL, TWeibull, Burr, Johnson's  $S_B$ , 3P Weibull, TLogL, 3P LogL, 2P LogL and 2P Weibull in that order.

**Table 2:** Mean fit indices of the ten distributions for Akure and Oluwa forest reserves (FR)

FR	Distributions	Fit indices					
		K-S	EI	MSE	MAE	Bias	AIC
Akure	Burr	0.08154	2.28495	0.00021	0.01025	0.00089	550.814
	GW	0.08316	2.38958	0.00021	0.01029	0.00093	550.822
	LL	0.09087	3.23786	0.00022	0.01027	0.00113	551.275
	SB	0.10354	4.40481	0.00022	0.01043	0.00125	554.624
	2P LogL	0.08487	2.75218	0.00022	0.01023	0.00118	554.252
	3P LogL	0.07487	2.47612	0.00022	0.10109	0.00112	552.412
	2P Weibull	0.13768	5.28643	0.00023	0.01068	0.00202	567.153
	3P Weibull	0.09807	3.11736	0.00022	0.01044	0.00092	550.695
	TLogL	0.07508	2.21924	0.00021	0.01017	0.00093	548.225
	Tweibull	0.09668	2.99946	0.00022	0.01042	0.00104	549.507
Oluwa	Burr	0.08614	1.56222	0.00019	0.00804	0.00018	520.541
	GW	0.07865	1.59067	0.00019	0.00794	0.00015	519.285
	LL	0.07852	1.68936	0.00019	0.00831	0.00009	521.109
	SB	0.08776	1.94527	0.00019	0.00833	0.00004	520.765
	2P LogL	0.10248	2.41581	0.00023	0.00839	0.00103	534.997
	3P LogL	0.08016	2.10526	0.00019	0.00803	0.00036	520.623
	2P Weibull	0.16787	3.52561	0.00029	0.00942	0.00183	559.748
	3P Weibull	0.10237	1.96201	0.00021	0.0084	0.00016	521.384
	TLogL	0.08514	1.74696	0.00021	0.00822	0.00023	518.109
	Tweibull	0.08556	1.65579	0.00021	0.00839	0.00009	517.697

**Table 3:** Mean fit indices of the ten distributions for CRS and Ikrigon forest reserve (FR)

FR	Distributions	Fit indices					
		K-S	EI	MSE	MAE	Bias	AIC
CRS	Burr	0.08453	1.75194	0.00035	0.01135	0.00053	330.928
	GW	0.09174	1.87826	0.00036	0.01166	0.00047	330.269
	LL	0.08263	1.94422	0.00037	0.01219	0.00039	329.507
	SB	0.0912	2.08533	0.00037	0.01206	0.00022	329.926
	2P LogL	0.10604	2.64116	0.00043	0.01207	0.00156	339.845
	3P LogL	0.09173	2.22074	0.00037	0.0114	0.00077	331.585
	2P Weibull	0.16358	3.55162	0.00051	0.01343	0.00236	352.901
	3P Weibull	0.09926	2.11008	0.00038	0.01181	0.00057	330.625
	TLogL	0.08848	1.89809	0.00038	0.01159	0.00069	328.679
	Tweibull	0.08686	1.90972	0.0008	0.01188	0.00049	327.438
Ikrigon	Burr	0.07329	1.38152	0.0002	0.00994	0.00029	616.181
	GW	0.06718	1.26791	0.00019	0.00985	0.00023	614.500
	LL	0.07055	1.55025	0.0002	0.01006	0.00012	614.481
	SB	0.07472	1.67348	0.0002	0.01005	0.00013	616.135
	2P LogL	0.08912	1.88912	0.00023	0.01033	0.00102	626.202
	3P LogL	0.08725	2.88168	0.00022	0.01009	0.00094	626.279
	2P Weibull	0.09813	2.01147	0.00024	0.01049	0.00126	631.738
	3P Weibull	0.07509	1.42023	0.0002	0.00998	0.00027	614.413
	TLogL	0.08232	1.94409	0.00021	0.01007	0.00062	617.761
	Tweibull	0.07271	1.31856	0.00021	0.01004	0.00029	612.735

The graph of the diameter distributions of the forest reserves (FR) are presented in Fig 1 to 4. A representative sample plot with highest stand density from each reserve was selected. The graphs showed the observed number of trees per hectare (N trees per ha) and the fitted distributions for 10 cm diameter class interval. Analyses of the observed N trees per ha and fitted distributions were typical of natural forest (inverse J-shaped) with large proportion of trees in the smaller diameter classes and decreasing frequency as the diameter increases. The estimated N

trees per ha produced by truncated Log-Logistic (TLogL) and truncated Weibull (TWeibull) distributions were comparable to the other distributions considered in this study. The fitted distributions did not show much difference with the observed diameter distributions except in plot 8 of Akure forest reserve (Fig. 1). The distribution underestimated N trees per ha in the 20 cm diameter class in the data from Akure forest reserve. The 2P Weibull had the poorest fit to the data across the four species.

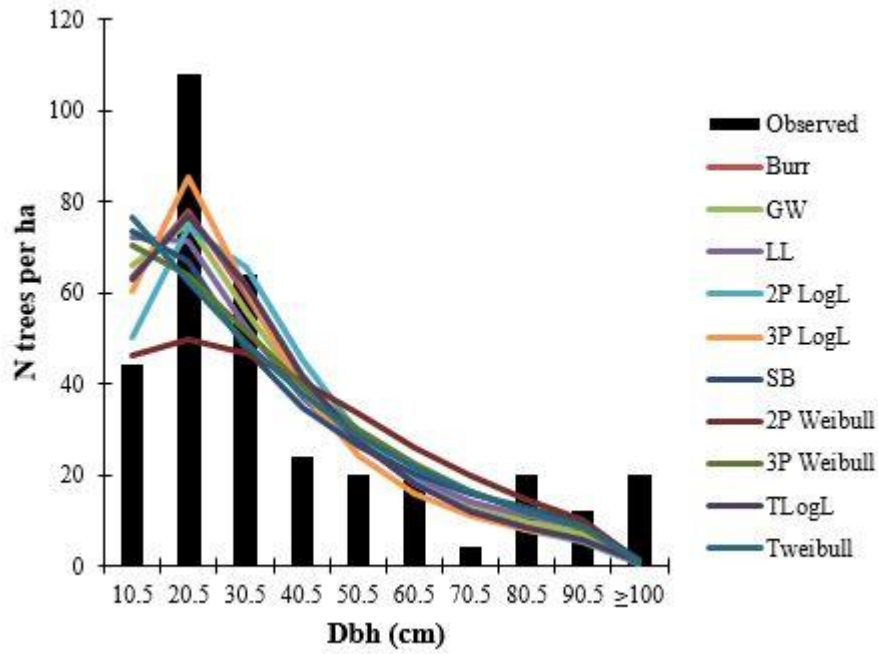


Fig 1. Observe number of trees per ha and fitted distributions for plot 8 in Akure forest reserve

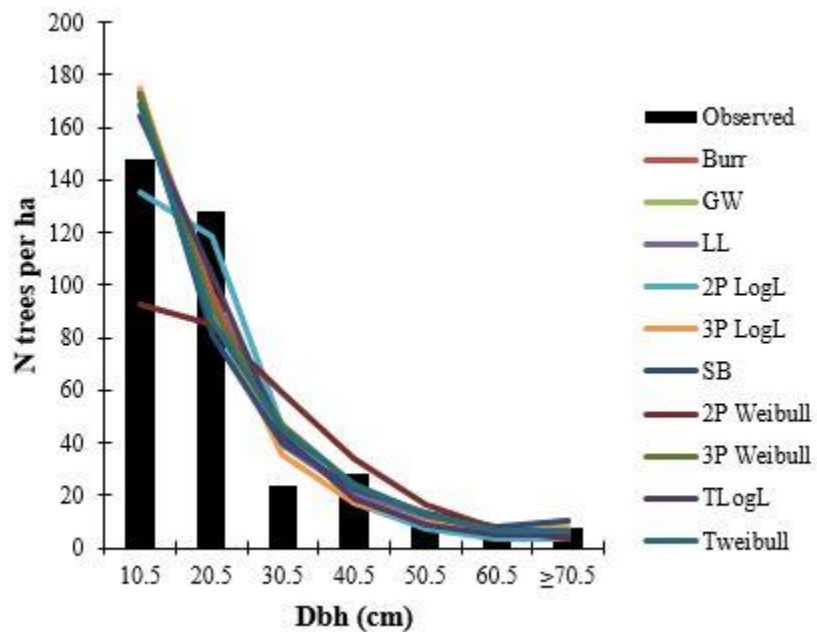
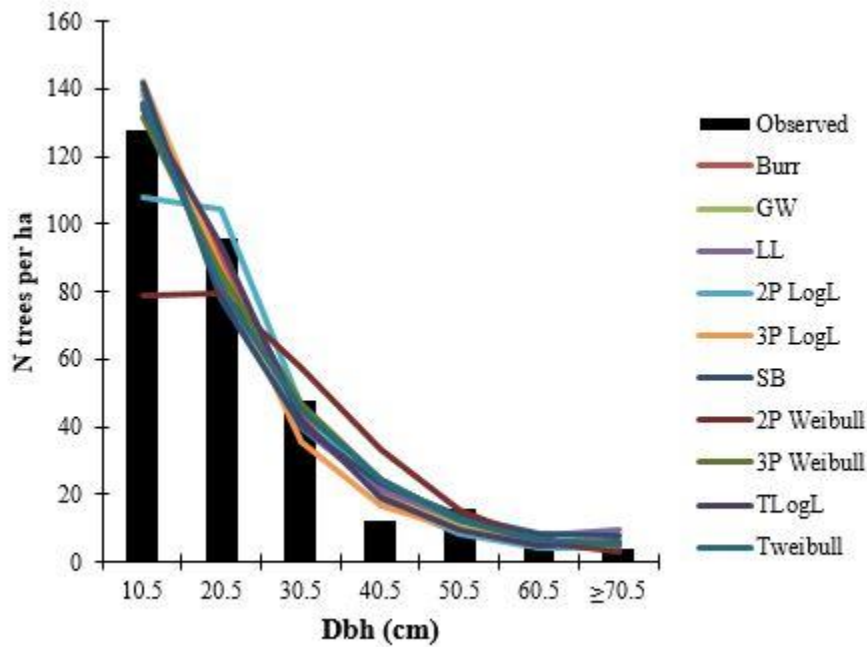
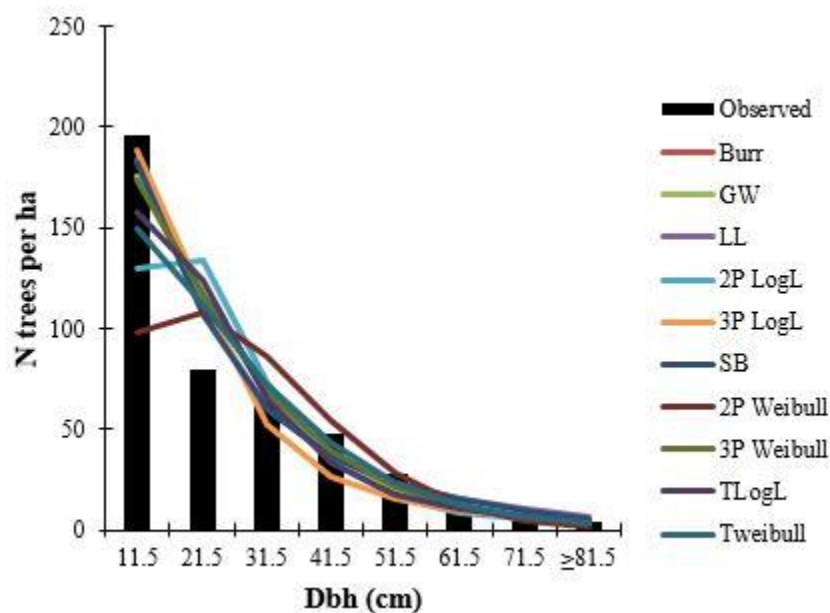


Fig 2. Observe number of trees per ha and fitted distributions for plot 8 in Oluwa forest reserve





**Fig 3.** Observe number of trees per ha and fitted distributions for plot 4 in CRS forest reserve



**Fig 4.** Observe number of trees per ha and fitted distributions for plot 8 in Ikrigon forest reserve

**DISCUSSION**

The characterisation of truncated diameter data from four natural forest stands have been investigated and the results obtained are quite interesting. The truncated distributions (i.e. TLogL and TWeibull) provided better fits to the data sets compared to some of the commonly used

distributions in quantitative forestry. For example, the truncated distributions were better than Johnson’s  $S_B$ , 3P Weibull and 2P Weibull distributions. Also, the performance of the truncated distributions was comparable to other recently introduced 4-parameter distributions e.g. generalized Weibull (GW), Logit-Logistic (LL) and Burr distributions. Considering the AIC values (best index

for comparing distributions with different number of parameters) of the distributions, one could observe that the truncated distribution had best result. As a rule of thumb, two distributions are indistinguishable if the difference in their AIC values ( $\Delta AIC$ ) is  $\leq 2$  (Ogana 2018). The differences in the AIC values of the truncated distributions were more than 2 for most of the commonly used distributions in forestry.

The result of this study agrees with Palahí *et al.* (2007) who compared the fit of beta, Johnson's  $S_B$ , 3P Weibull and truncated Weibull functions to characterised diameter distributions of forest stands in Catalonia. The distributions were fitted to the observed diameter distributions of the number of stems and the stand basal area based on truncated data, with the diameter threshold equal to 7.5 cm. In both instances, the authors reported more accurate and consistent results with the truncated Weibull distribution compared to beta, Johnson's  $S_B$  and 3P Weibull distributions. A diameter threshold i.e., truncation point of 10 cm was used in this study, because only trees with diameter  $\geq 10$  cm were measured. The beta distribution did not produce fit in some of the plots in the data set; as such it was not included in this article.

The truncated Log-Logistic and Weibull distributions are favoured over the well-known 3-parameter forms of the Weibull (3P Weibull) and Log Logistic (3P LogL) distributions, because a threshold diameter limit leads to censored data. Mehtätalo (2013 51p) asserted that "such data are realistically modelled with a truncated distribution, having a jump in the density at  $x = t$ ". For example, similar data set from Oluwa FR was used by Ogana *et al.* (2015) wherein a better result was reported for the 3P Weibull distribution. However, in this study, the truncated distributions (TWeibull and TLogL) were superior and more suitable than 3P Weibull distributions. Furthermore, attempt was made to fit the truncated version of the 3-parameter and 4-

parameter of the distributions considered in this study. However, difficulty was encountered in assigning initial values to the parameters during model-fitting process; and the parameter estimates in some plots were unrealistic. Although, a good result was observed for the 4-parameter distributions e.g. Burr, GW and LL across the different forest stands without specification of the truncation point in the model; their complexity could be a major challenge in modelling forest growth and yield. Model simplicity is a prerequisite in adopting any model in quantitative forestry.

The importance of diameter distribution modelling cannot be overemphasised. It provides information on the stand structure of the forest and the regeneration potential of the stand. For example, the stand structures of Akure, Oluwa, Cross River South and Ikrigon forest reserves represented in Fig. 1 to 4 show that there are enough trees in the lower diameter class that can grow into the larger classes. Information on product size and volume of the stand can be derived from the diameter distributions (Gorgoso-Varela and Rojo-Alboreca, 2014). The proportion of trees that satisfy utilisation standard can be specified from the diameter distributions and their yield computed by using appropriate height-diameter and volume equations. This information is needed for sustainable forest management.

## CONCLUSION

Truncated distributions have provided a simple and more robust method of characterising truncated or censored tree diameter data resulting in better approximation of the forest stand structure. The truncated distributions (TLogL and TWeibull) can be extended to model stand basal area distributions where truncation point is required especially in natural forest. The truncated Log Logistic distribution can also be applied to model censored/truncated data from temperate forest.

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