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DIAMETER DISTRIBUTION OF *Nauclea diderrichii* (D Wild) Merr. PLANTATIONS IN A RESTRICTED TROPICAL RAINFOREST OF NIGERIA

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ABSTRACT

Distribution of tree diameter in a specific stand provides basic information for forest management. Little attention is given to the use of probability distribution functions in characterising the stem diameters of most Nigeria indigenous species. This study used Three-parameters lognormal and Weibull probability distribution functions in characterising the diameter of Nauclea diderrichii plantations within a restricted tropical rainforest of Nigeria for sustainable management. Thirty temporary sample plots of dimensions 20 m × 20 m were randomly laid in three age strata (42, 43 and 46 years). Stem diameters of all living Nauclea diderrichii trees with diameter at breast height ≥10.0 cm in all the sampled plots were enumerated. The performance and suitability result revealed that lognormal distribution gave superior description of the stem diameter for the Nauclea diderrichii species in the study area, with least values of Bias (0.00001), Mean absolute error (0.1214), Mean square error (0.7421), Kolmogorov-Smirnov (0.02781) and Anderson Darling (0.49562) statistics. Hence, three-parameter lognormal using maximum likelihood method was recommended for distributing stem diameter of Nauclea diderrichii in the study area.

Keywords: Indigenous species; Lognormal; Maximum likelihood; Probability distribution function; Weibull

INTRODUCTION

Forest plantations in African countries are dominated by exotic than indigenous tree species (Pandey, 1995). Onyekwelu (2001) reported that exotic tree species account for over 80% of total plantations in Nigeria. However, exotic species dominance in the region was attributed to ease of establishment, rapid growth rate and consequently short rotation length. Nevertheless, previous studies (Akindele and Fuwape, 1998; Onyekwelu, 2001) asserted that Nigerian wood consumers have higher preference for the products of indigenous tree species. *Nauclea diderrichii* (De Wild) Merr.is one of the 22 major economic trees species of importance (Redhead, 1971). It belongs to the family of Rubiaceae. The stem is of good form, slender, straight, usually branching and cylindrical up to the height of between 24 – 30 m and has low effect of buttresses (Wagefuhr, 2000).

Tree size distributions are simple yet effective tool to describe tree populations and forest stands. They are used to value forests, plan harvest activities, predict forest growth, and thus, enhance forest productivity (Bailey and Dell, 1973; Burkhart and Tome, 2012). Tree diameter distributions can also be used to infer past disturbance events, forest successional status, and aboveground biomass stocks (Coomes and Allen, 2007). When assessed at the species level, they can be used to provide information on species-specific regeneration strategies, demographic rates, and population trends (Knight, 1975; Wright *et al*., 2003).

One of the most common and important tree characteristics used in forest management decision-making is tree diameter at breast height (Dbh). This variable has numerous beneficial attributes. It is easy to measure (Zhang *et al*., 2003) and have strong correlations with other tree growth characteristics. The distribution of trees by Dbh class allows foresters and ecologists to understand stand structure, stand dynamics, selecting tree species for protection, to prescribe silvicultural treatments, estimate cutting cycles and future forest yield. They allow one to project and describe the state of a tree in future time. Nowadays, the number of trees distributed in diameter classes is used in different aspect of forestry researches. For instance, the forest distributed by the help of this probability distribution curve uses the quality of operation progress in thinning and leading the forest towards regulation. With the help of probability distribution curve, statistical distributions can easily be demonstrated, the quality of stand progress or supply in different diameter classes in future can be predicted and it can be used in the programming (Cao, 2004).

Several probability density functions (such as normal, gamma, Johnson's SB, beta and Weibull) have been used in describing the diameter distributions of forest stands, (Bailey and Dell 1973; Maltamo *et al*., 2000; Palahí *et al*., 2007). In this study, Weibull and Lognormal distributions were chosen because Weibull distribution has the ability to fit various distributions from the reversed J-shaped through left skewed and symmetrical distributions to right-skewed distributions (Bailey and Dell 1973, Shifley and Lentz, 1985).On the other hand, lognormal distribution is used for modeling positively skewed data, depending on the values of its parameters; the lognormal distribution can have various shapes including a bell-curve similar to the normal distribution (Aristizabal, 2012). Thus, the selection of the distribution function is entirely up to the researcher (Siipileh to and Mehtätalo, 2013).

The study therefore, aimed at comparing the statistical distribution of trees in diameter class to determine and select the most appropriate distribution function for modelling the distribution of tree stem sizes (diameter at breast height) for *Nauclea diderrichii* in the study area for sustainable management.

MATERIALS AND METHODS

Study Area

This study was carried out in three age series (42, 43 and 46 years) of *Nauclea diderrichii* plantations in Omo Forest Reserve, Nigeria with a total land area of 139,100 ha, situated between Latitude 6°35' to 7°05' N and Longitudes 4°9' to 4°40' E (Figure 1). The reserve is bounded by Benin-Shagamu expressway to the south and Omo River and Oni River to the east. The Reserve lies within The Reserve lies within the tropical rainforest and has a mean annual rainfall of 1,200 mm and average elevation of about 91.47 m (Chukwu and Osho, 2018).

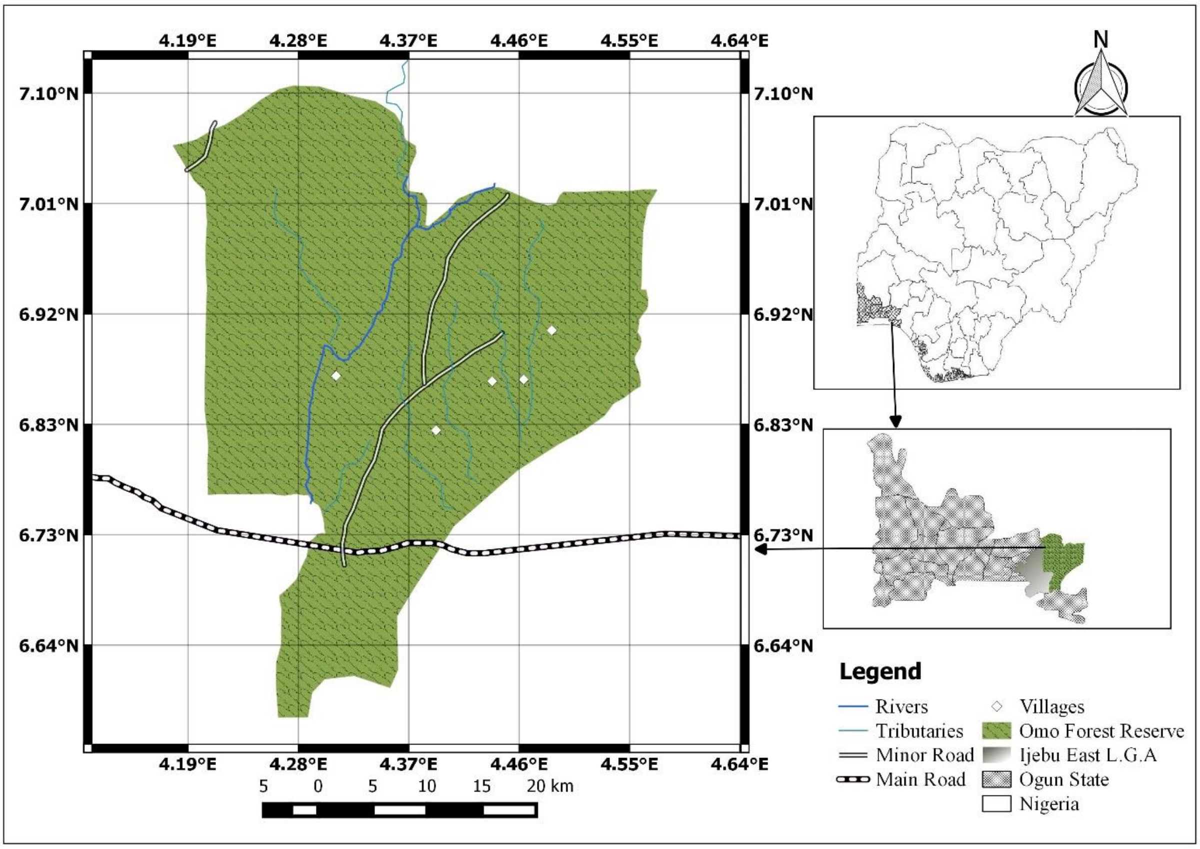

Figure 1: Map of the study area
Source: *Chukwu and Osho (2017)*

Data Collection

Data used for this study were obtained from 30 temporal sampled plots (20 m x 20 m size each) randomly within the three age strata (42, 43 and 46 years) of *N. diderrichii* plantations. Diameter at breast height (Dbh) of all theliving trees with Dbh ≥ 10.0 cm in the selected plots were measuredusing diameter tape calibrated in centimeter.

Model Specification

Weibull (3P) and Lognormal (3P) were used in describing the distributions diameter (Dbh) of *N. diderrichii* stands in the study area.

Weibull distribution function

The probability density function for Three-Parameter Weibull was used in this study. Three-parameter (3P) Weibull distribution (Weibull, 1951) is expressed as:

$$f(x) = \frac{c}{b}\left(\frac{x-a}{b}\right)^{c-1} exp\left[-\left(\frac{x-a}{b}\right)^{c}\right] \quad (1)$$

Where: X = tree diameter (Dbh), a = location parameter, b = scale parameter, c = shape parameter.

The Weibull cumulative distribution was obtained by the integration of its density function in equation (1).

$$F(x) = \int_0^x \frac{c}{b}\left(\frac{x-a}{b}\right)^{c-1} exp\left[-\left(\frac{x-a}{b}\right)^{c}\right] dx = 1 - \exp\left[-\left(\frac{x-a}{b}\right)^{c}\right] \quad (2)$$

Where: F(x) is the cumulative distribution function.

Lognormaldistribution function

The probability density function of the three-parameter lognormal distribution is

$$f(x) = \frac{1}{(x-a)b\sqrt{2\pi}}\, exp\left[-\frac{(ln(x-a)-c)^2}{2b^2}\right] \quad (3)$$

The Lognormal cumulative distribution was obtained by;

$$F(x) = \frac{1}{\sqrt{2\pi}}\int_0^x exp^{-t^2/2}\, dt = exp\left[\frac{ln(x-a)-c}{b}\right] \quad (4)$$

Method of Parameter Estimations

In this study, maximum likelihood (ML) method was used to estimate the three parameters of the Weibull and Lognormal distributions. When applied to a data set and given a statistical model, maximum-likelihood estimation provides estimates for the model's parameters. In this research, all the parameters were assumed to be unknown and were estimated by numerical iteration.

Diameter Characterisation

After the parameters of the distributions have been estimated using the maximum likelihood methods, the parameters were fitted to the distribution functions which was used to obtain the class probabilities (*P*i) and subsequently used to compute the diameter-class frequencies for each class.

Predicted Number of tree per class (Ni)

$$(\mathrm{Ni}) = \mathrm{N} \times \mathrm{Pi} \quad (5)$$

Where: Ni = estimated number of trees per class, N = number of trees per ha and Pi = class probability.

Evaluation and Comparison Criteria

The goodness of fit for the two distributions were assessed by the mean value of bias, mean absolute error (MAE), mean square error (MSE), Kolmogorov Smirnov (D_n) and Anderson Darling (A^2). They are mathematically expressed as:

$$Bias = \frac{\sum_{i=1}^{N} Y_i - \hat{Y}_i}{N} \quad (6)$$

$$MAE = \frac{\sum_{i=1}^{N} |Y_i - \hat{Y}_i|}{N} \quad (7)$$

$$MSE = \frac{\sum_{i=1}^{N} (Y_i - \hat{Y}_i)^2}{N} \quad (8)$$

$$D_n = Supx|F(x_i) - F_0(x_i)| \quad (9)$$

$$A^2 = -N - \sum_{i=1}^{N} \frac{(2i-1)}{N}[\ln F(Y_i) + \ln(1 - F(Y_{N+1-i}))] \quad (10)$$

Where: MAE= mean absolute error, MSE= mean square error, D_n= Kolmogorov Smirnov statistics and A^2= Anderson Darling statistics, Supx= supremum value, F (x_i) = cumulative frequency distribution observed for the sample x_i (i = 1, 2… n), F_0 (x_i) = probability of the theoretical cumulative frequency distribution, F (Y_i) = cumulative frequency distribution observed for the sample Y_i (i = 1, 2… n), N= total number of observations. Diameter classes of 10 cm intervals were selected.

RESULTS

The results of the Lognormal and Weibull distributions for characterizing tree diameter distribution of *Nauclea diderrichii* in Omo Forest Reserve, Nigeria using Maximum likelihood estimators are as presented Table 1. Figure 2 showed the graphical analysis of the fitted distributions and their observed number of trees. From the graph (Figure 2), bell shape patterns were observed for both of the distributions, which are of typical of plantation forest. From the result also, it can be seen that the lognormal distribution did not display ample difference from the Weibull distribution in fitting the diameter distribution of the all-inclusive plots evaluated. The produced probable number of trees (per class) by Lognormal and Weibull distributions exhibited trivial variation with the observed number of trees; hence, the Lognormal estimate superiority over Weibull function and this can be appreciated in some of the diameter classes.

The goodness of fits for the distribution functions were evaluated and compared using mean values of bias, mean absolute error (MAE), mean square error (MSE), Kolmogorov Smirnov (D_n) and Anderson Darling (A^2) statistics. The results (Table 1) revealed that the Lognormal distribution had the smallest mean values of Bias, MAE, MSE, D_n and $A^2$of 0.00001, 0.1214, 0.7421, 0.02781 and 0.49562, respectively. These criteria (bias, MAE and MSE) for evaluation and comparison were also graphically represented in Figures 3 – 5 and their patterns of variation were exposed.

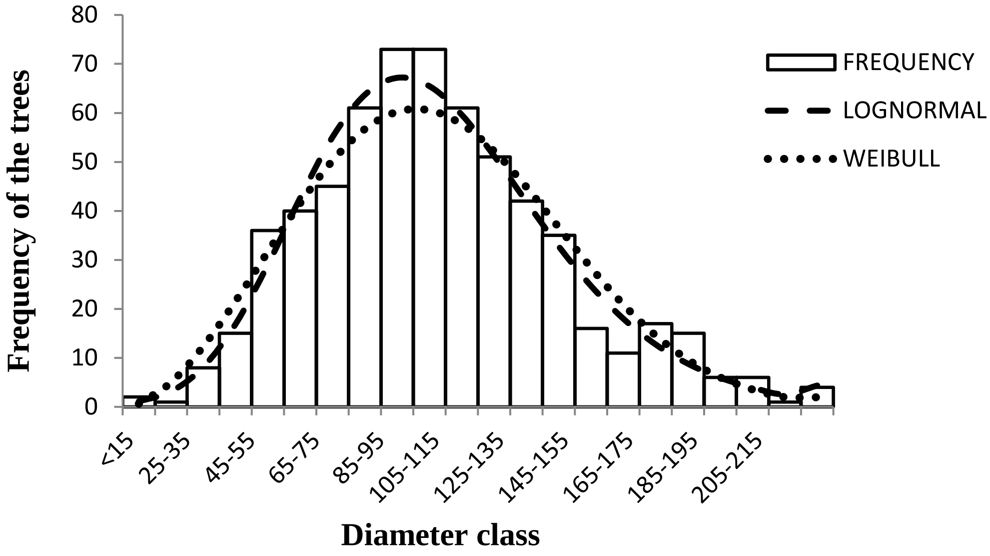

Figure 2: Observed diameter distributions, fitted 3-parameters Weibull and Lognormal distributions by maximum likelihood method.

Table 1: Goodness of fit test for compared distributions

Distribution	**Bias**	**MAE**	**MSE**	**D_n**	**A^2**
Lognormal (3P)	0.00001	0.12140	0.74210	0.02781	0.49562
Weibull(3P)	0.00002	0.16413	1.27105	0.04775	1.71290

Where: MAE= mean absolute error, MSE= mean square error, D_n= Kolmogorov Smirnov statistics and A^2= Anderson Darling statistics

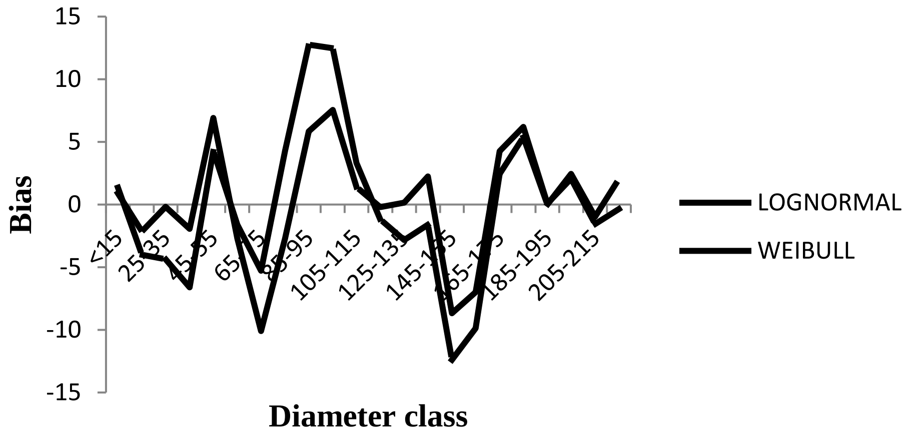

Figure 3: Mean values of bias in number of trees per diameter class obtained by Maximum likelihood method of the two fitted distributions.

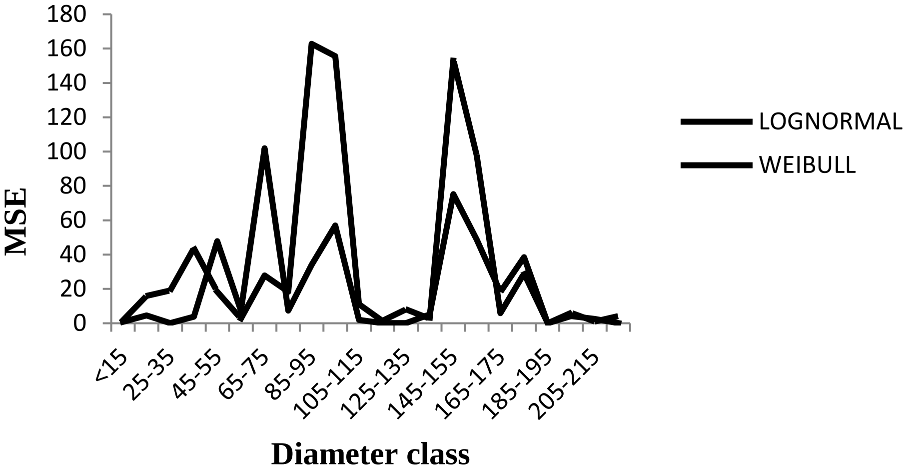

Figure 4: Mean square error (MSE) in number of trees per diameter class obtained by Maximum likelihood method of the two fitted distributions.

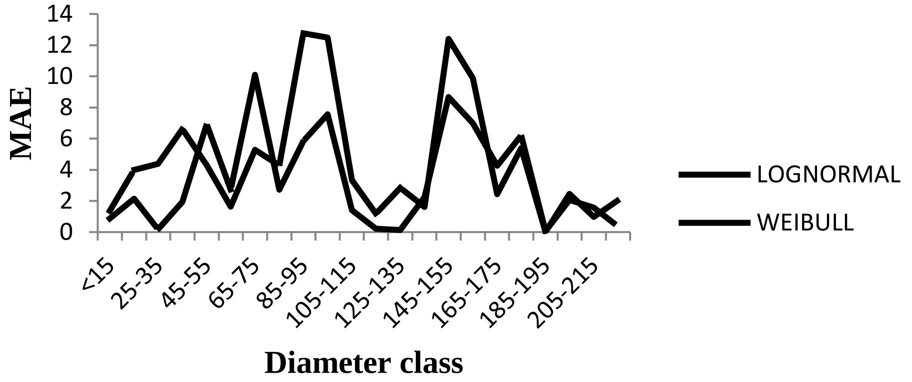

Figure 5: Mean absolute error (MAE) in number of trees per diameter class obtained by Maximum likelihood method of the two fitted distributions.

DISCUSSION

The study provided information on tree population trend and regeneration strategies with reference to stem diameter classes of *Nauclea diderrichii* species in Omo Forest Reserve, Nigeria. The maximum likelihood estimators (MLE) were used in estimating the parameters Weibull and lognormal distributions. The MLE is the most popular and best estimation technique for many distributions, for the reason that it selects the values of the distribution's parameters that give the observed data the greatest probability (Aristizabal, 2012).Consequently, MLE have ability to account for stem diameter measurement errors (Taubert *et al.*,2013).Zhang *et al*. (2003) obtained better results with MLE than moments and percentile for fitting the 3-parameter Weibull distribution to mixed Spruce-fir stand in north-eastern North America. Ginos (2009) in a study on parameter estimation for the lognormal distribution also reported MLE the best overall of all other estimators used.

Maximum likelihood estimators were calculated for the two distributions, the observed and predicted fits were compared using mean values of bias, mean absolute error (MAE), mean square error (MSE), Kolmogorov Smirnov (D_n) and Anderson Darling (A^2) statistics. These criteria for testing the goodness of fit displayed preference for lognormal distribution. This was similar to the findings of Nanang (2002) on statistical distribution for modelling stand structure of Neem *(Azadirachta indica)* plantations, reported that lognormal distribution was the appropriate choice for the diameter and height distributions. Weibull distribution been unsuitable for fitting the data used for this study was rather surprising. Kayes *et al*. (2012) reported that both lognormal and Weibull distributions were equally effective for describing the diameter distributions of Akashmoni (*Acacia auriculiformis* A. Cunn. ex Benth.) plantations grown in the north-eastern region of Bangladesh.

However, both of the distribution functions used for the study depicted bell shaped structures which were typical of a plantation forest, with most of the trees within the middle diameter classes. This indicates that most of the trees in the plantation were growing at similar rate and with less competition. Hence, only few numbers of trees towards the right and left tails of the bells were of dominant and supressed diameters, respectively. Furthermore, lognormal distribution showed superiority over Weibull distribution and can be chosen if only one distribution function is needed for diameter characterisation of *N. diderrichii* in the study area.

CONCLUSION

Tree diameter distribution is an effective method for describing stand properties. Tree volume, value, conversion cost, and product specifications are dependent on stem diameter. The study provided information on tree population trend and regeneration strategies with reference to stem diameter classes.

The study reveals that both Lognormal (3-parameter) and Weibull (3-parameter) distribution functions can be used for describing Dbh of *Nauclea diderrichii* in Omo Forest Reserve, Nigeria. Hence, 3-parameter Lognormal distribution using maximum likelihood estimators gave superior description of the stem diameter for the *N. diderrichii* species in the study area. The study therefore, recommends three-parameter lognormal distribution function for fitting diameter at breast height of *N. diderrichii* species in Omo Forest Reserve, Nigeria for sustainable management.

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