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A LINEAR PROGRAMMING APPROACH FOR SUSTAINABLE HARVESTING SCHEDULE FOR *TECTONA GRANDIS* LINN F. IN OMO FOREST RESERVE, NIGERIA

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ABSTRACT

Linear programming (LP)is a useful analytical tool for determining future sequence of harvests from the forest. It has been an inherent part of forest management planning and decision-making.This study was carried out to determine the optimal harvesting schedules for Tectona grandis plantations in Omo Forest Reserve, Southwestern Nigeria using Linear Programming. The plantations are being managed for timber production and are to be exploited within twenty-five (25) years based on a 5-year harvesting period. Data was collected from five compartments established in the year 1994, 1995, 2001, 2003 and 2005, and covered an area of 59.5 hectares. A simple random sampling technique was used to select 6 Temporary Sample Plots of size 25 m x 25 m (0.0625 ha) from each compartment. The maximization problem of wood volume on compartmental level was formulated and solved. The solution of the LP showed that about 61,374.5 m^3 of wood was maximized. The equivalent numbers of stems (N) of the volume value was 45,985.0. The optimal solution prescribed 17.9 ha (22065.0 m^3, 14,181 N) to be cut in period 1(2017-2021); 13.0 ha (9451.0 m^3, 9,911 N) is to be cut in period 2 (2022-2026); 10.0 ha (13744.4 m^3, 7,809 N) is to be cut in period 3 (2027-2031); 9.5 ha (8879.7 m^3, 7,190 N) is to be cut in period 4 (2032-2036) while 9.1 ha (7234.4 m^3, 6,894 N) will be cut in period 5 (2037-2041). Sensitivity analysis and opportunity costs were used to highlight the effects of changing some of the model parameters on the prescribed optimal solution.

Keywords: Linear Programming, Optimal harvesting schedules, *Tectona grandis,* Omo Forest Reserve, Sensitivity analysis.

INTRODUCTION

Forest management is the art and science of making decisions with regards to the organization use and conservation of forests (Buongiorno and Gilless, 1987). Such decisions could be based on a short or long term projection of the forest productivity. Furthermore, forest management problems are complex because of the diversity of the landscape, the unpredictability of the natural processes that occur in forests, the inter-relationships between the different components of the forest, and most importantly, because of the diversity of values associated with natural resources (Buongiorno and Gilless, 1987). In other to tackle this complexity, forest management plans are often developed using mathematical programming techniques, including linear programming, non-linear programming, inter programming, etc.

A great variety of timber and non-timber products and services are being produced by the forest (Ticktin, 2004; Galatsidas, 2012). With the importance of non-timber forest services in the past decades, and the increase in oil prices and current economic crisis, fuel wood is considered as a major heating source. Thus, necessitate the need for forest management planning. Forest management planning is best described by the use of mathematical programming (Dykstra, 1984). Forest production models are aimed towards maximizing the volume (m^3/ha) that will be eventually harvested. Volume maximization will lead to larger marginal profits if accompanied by restrictions on harvested quantities

(Ware and Clutter, 1971). Common criteria used in forest management modelling in addition to volume control are; area control and non-declining even flow of timber (Buonogiorno and Gilles, 2003).

To achieve sustainable management of a forest, one needs to know the maximum potential of the forest and must have studied the processes of degeneration and recovery (Osho, 1988). Complex problems involving hundreds, even thousands of management areas and a wide range of concerns at multiple scales are solved using mathematical programming. Management decisionsin forestry have the potential to impact greatly both in the financial well-being of the forest owner and the future ecological condition resulting from timber management activities. Linear programming (LP) can be defined as the method of allocating limited resources to competing activities in an optimal manner (Buonogiono and Gilles, 1987). It is also known as a common analytical tool in the management and planning of renewable resources, which has gained wide acceptance in forestry, and other fields. LP serves as a tool for future sequence of harvests from the forest and also helps in evaluating existing forest conditions as an analytical technique (Leuschener, 1990). LP can also be defined as a mathematical method for determining the best outcome (such as maximum profit or lowest cost) in a given mathematical model for some list of requirements represented as linear relationship (George, 2002).

Tectona grandis (Teak) is one of the world's premier hardwood timbres because of its physical and aesthetic qualities. Teak ranks among the top five tropical hardwood species established in plantations worldwide (Krishnapillay, 2000). Though it takes only a marginal position in the volume of world timber production and trade, it is among the tropical hard woods most in demand for the luxury market and for heavy duty applications. Larsen and Scott, (2009) attributed the high market value of teak to its stem form, excellent durability in tropical climates and its resistance to termite attack and fungi. Teak is known to be used for various purposes, some of which are; ship building and yacht furnishing, high-class furniture, decorative building components, flooring, utility poles for transmission lines etc. In view of the importance of teak and the important role of LP in achieving forest sustainability, this study seeks to prescribe timber harvest in the management of even-aged *Tectona grandis*in Omo Forest Reserve, South-western Nigeria.

MATERIALS AND METHODS

Study Area

This study was carried out in Omo Forest Reserve located between Latitudes 6°35’ and 7°05’N and Longitudes 4°19’ and 4°10’E of Ijebu-East, and North Local Government Areas of Ogun State, Nigeria (Chima *et al.*, 2009 cited in Ogana *et al*., 2017). It lies within the high forest zone in south-western Nigeria. The climate of the area is moist monsoon with the annual rainfall varying from about 1,500 mm to 1,800 mm, most of it from March to October. Rainfall distribution is bimodal with a marked decline in August. This reserve has a long history of uncontrolled exploitation and is described in the plantation project document as low-value logged over high forest.

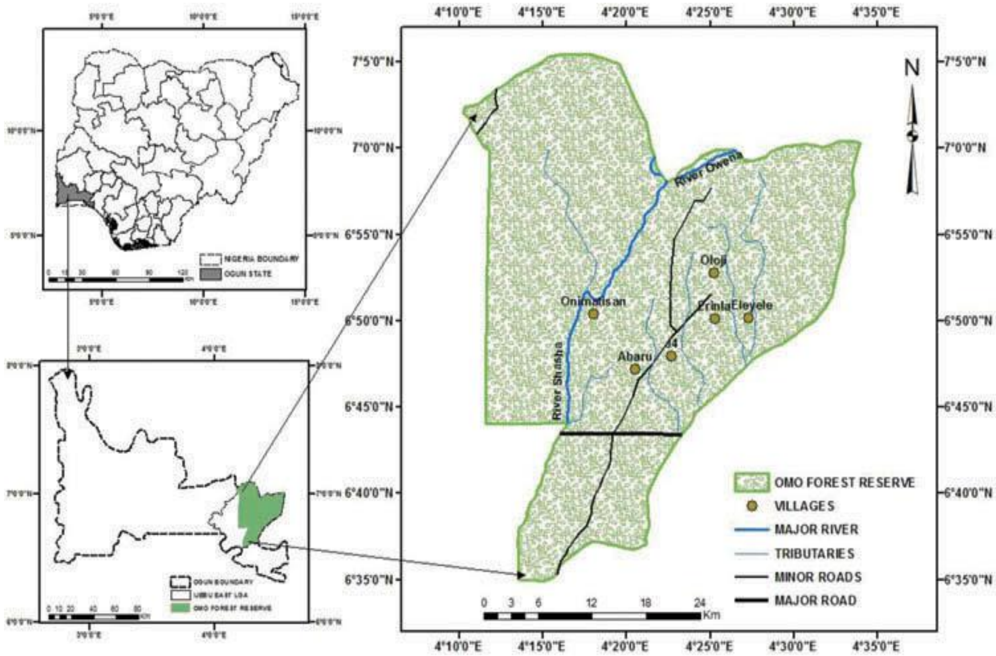

Figure 1. Map of Omo Forest Reserve; **Source:** Adesoye and Olayinka (2016)

The data for this study were collected from the *Tectona grandis* stands in the Forest Reserve. A total of 30 sample plots of size 25 m x 25 m were laid in the five age series (i.e. 23, 22, 16, 14 and 12). These age series were treated as five compartments. In each compartment, six sample plots were randomly laid.Complete enumeration of all trees in each plot was carried out. The height and diameter at breast height (dbh) of each standing tree was measured using spiegel relaskop and conventional diameter tape, respectively.

Model Development

The compartments are to be exploited over a rotation age of *Tectona grandis* for timberthat is,25-year period. This means that a single cutting over the exploitation period is to be carried out in each compartment i.e. the planning is to be carried out on five (5) year basis. By implication the schedule is to be done for 5 cutting periods of five (5) years each. However, a compartment that is not completely harvested within a cutting period can be extended to another period.

The model is patterned after linear programming model proposed by Buonogiorno and Gilles (1987). It involves: identifying the decision variables, formulating the objective function, identifying and formulating the constraints and writing out the non-negativity constraints.

Decision Variables: the symbol **X** is used to denote the decision variable which is the area as shown:

X_{ij}, i = 1, 2, 3, 4, 5

j = 1, 2, 3, 4, 5 (2017-2021, 2022-2026, 2027-2031, 2032-2036, 2037-2041)

Where: i = Compartment; j = Period.

Objective function: the objective function in this study was based on optimizing volume. The objective function therefore expresses the total volume of timber cut during the period of the plan (i.e. 25 years in this study) as a linear function of the decision variables. The object of the management planning model is to find the values of the decision variables such that the total amount of timber produced throughout the plan is optimum. The general form of the objective function is given as:

$$Z = \sum_{i=1}^{m}\sum_{j=1}^{p} C_{ij}X_{ij} \tag{1}$$

Where: Z = objective function (stem Volume per hectare);C_{ij} = the amount of wood (volume) as a result of cutting a hectare of land from compartment i in period j,X_{ij} = the fraction (area) of compartment i in period j,$C_{ij}X_{ij}$= the amount of wood taken (volume) from the *ith* compartment in the *jth* year;

$\sum C_{ij}X_{ij}$ = total wood taken from all the compartments in all the years.
Constraints: the decision variable $X_{ij}$was constrained in several ways. Firstly, the area cut from each compartment throughout the planning horizon cannot exceed the area available. This is expressed as:

$$\sum_{j=1}^{p} X_{ij} \leq a_i \qquad (2)$$

Where X_{ij}= area cut from compartment i in period j (ha); a_i = total area in compartment i (ha). There are as many constraints of this kind as there are compartments. Secondly, the specified area that must be cut during each period from the entire forest cannot exceed the total area of the entire forest. This is expressed as;

$$\sum_{j=1}^{m} X_{ij} \leq y_j \qquad (3)$$

Where X_{ij} = area cut during each period from the entire forest;y_j = total area of the entire forest which can be cut in compartment j.
Non negativity constraint: $X_{ij} \geq 0$ for all i and j.
The whole exploitation process can now be formulated as a linear programming problem thus:

$$\text{Maximize } Z = \sum_{i=1}^{m}\sum_{j=1}^{p} C_{ij}X_{ij} \qquad (4)$$

Subject to:
$\sum X_{ij} \leq a_i$ (The area cut from each compartment throughout the planning horizon cannot exceed the area available.)
$\sum X_{ij} \leq y_j$ (The specified area that must be cut during each period from the entire forest cannot exceed the total area of the entire forest.)
$X_{ij} \geq 0$ for all i and j (All values of i and j must be positive definite)

Estimation of Parameters of the Model (C_{ij}, A_i, Y_j)

The five plantations were established in the year 1994, 1995, 2001, 2003 and 2005. These plantations were as five compartments aforementioned. The projection of wood volume per compartment and period was carried out by the use of multiple linear regression model:

$$V = \beta_0 + \beta_1 A + \beta_2 G + \beta_3 H_d$$

Where:A = age of trees in the compartment (years); G = basal area per ha (m^2/ha); H_d= dominant height (m); $\beta_0, \beta_1, \beta_2, \beta_3$ = regression coefficient;
The projected wood volumes per compartment and by periods are presented in table 1

Maximise:
1399S1P1 + 1404S1P2 + 1409S1P3 + 1414S1P4 + 1419S1P5 + 1290S2P1 + 1296S2P2 + 1301S2P3 + 1305S2P4 + 1310S2P5 + 887S3P1 + 893S3P2 + 899S3P3 + 904S3P4 + 909S3P5 + 970S4P1 + 976S4P2 + 982S4P3 + 987S4P4 + 992S4P5 + 720S5P1 + 727S5P2+ 732S5P3 + 738S5P4 + 743S5P5

Subject to:
i) Land Availability
$6.9X_{1.1} + 6.9X_{1.2} + 6.9X_{1.3} + 6.9X_{1.4} + 6.9X_{1.5} \leq 6.9$

$19.3X_{2.1} + 19.3X_{2.2} + 19.3X_{2.3} + 19.3X_{2.4} + 19.3X_{2.5} \leq 19.3$
$6.5X_{3.1} + 6.5X_{3.2} + 6.5X_{3.3} + 6.5X_{3.4} + 6.5X_{3.5} \leq 6.5$

$4.8X_{4.1} + 4.8X_{4.2} + 4.8X_{4.3} + 4.8X_{4.4} + 4.8X_{4.5} \leq 4.8$

$22X_{5.1} + 22X_{5.2} + 22X_{5.3} + 22X_{5.4} + 22X_{5.5} \leq 22$

ii) Area Control
$6.9X_{1.1} + 19.3X_{2.1} + 6.5X_{3.1} + 4.8X_{4.1} + 22X_{5.1} \leq 17.9$
$6.9X_{1.2} + 19.3X_{2.2} + 6.5X_{3.2} + 4.8X_{4.2} + 22X_{5.2} \leq 13.0$
$6.9X_{1.3} + 19.3X_{2.3} + 6.5X_{3.3} + 4.8X_{4.3} + 22X_{5.3} \leq 10.0$
$6.9X_{1.4} + 19.3X_{2.4} + 6.5X_{3.4} + 4.8X_{4.4} + 22X_{5.4} \leq 9.5$

$6.9X_{1.5} + 19.3X_{2.5} + 6.5X_{3.5} + 4.8X_{4.5} + 22X_{5.5} \leq 9.1$

$X_{ij} \geq 0$ for all i and j
The Linear programming problem was solved using the LINDO Version 6.0 Software. A useful link

between Number of stems per hectare (N) and Stem volume per hectare (SV) is provided by the model:

$$lnN = \beta_0 + \beta_1 \ln SV + \beta_2 A \quad (6)$$

Where: N = Number of stems (N/ha); SV = Stem volume (m^3/ha); A = Age; $\beta_0, \beta_1, \beta_2$ = regression coefficient

RESULTS

The results of the scheduled solutions are presented in Table 2, 3 and 4; and Fig. 2. The solution of the LP showed that about 61,374.5 m^3 of wood was maximized. The equivalent numbers of stems (N) of the volume value was 45,985. The optimal solution prescribed 17.9 ha (22065.0 m^3, 14,181 N) to be cut in period 1(2017-2021); 13.0 ha (9451.0 m^3, 9,911 N) is to be cut in period 2 (2022-2026); 10.0 ha (13744.4 m^3, 7,809 N) is to be cut in period 3 (2027-2031); 9.5 ha (8879.7 m^3, 7,190 N) is to be cut in period 4 (2032-2036) while 9.1 ha (7234.4 m^3, 6,894 N) will be cut in period 5 (2037-2041).

The sensitivity analysis performed on the coefficients of the objective function and the Right hand side (RHS) values of the constraints are presented in Table 5 and 6, respectively. The sensitivity analysis for the objective coefficients gave the range of values over which the objective coefficients could vary without affecting the values of the decision variables in the optimal solution. Similarly, for the RHS gives the range of values within which the RHS of the constraints could vary without affecting the values of the decision variables in the solution. Thus, within the ranges specified, the solution (optimal harvesting schedules) would still hold. The negative infinity implies infinitely small minimum range values. Conversely, the positive infinity indicated very large maximum range values of the objective coefficients.

Opportunity cost for deviating from the prescribed optimal schedule can be computed using the wood volume coefficients contained in Table 7. The coefficient is multiplied by the corresponding area (ha.) to determine the loss in wood volume.

Table 1: Projected Wood Volume per Hectare by Compartment and 5-Year Management Period (m^3/ha)

		Volume (m^3/ha)					
Compartment Number	**Stand**	**1st Period 2017-2021**	**2nd Period 2022-2026**	**3rd Period 2027-2031**	**4th Period 2032-2036**	**5th Period 2037-2041**	**Area per Compartment (Ha)**
1	**1994**	1399	1404	1409	1414	1419	6.9
2	**1995**	1290	1296	1301	1305	1310	19.3
3	**2001**	887	893	899	904	909	6.5
4	**2003**	970	976	982	987	992	4.8
5	**2005**	720	727	732	738	743	22

Table 2: Harvest Schedule for Maximum Wood Volume: Area to be cut data (ha)

		Area cut (ha) per period					
Stand		**1st Period 2017-2021**	**2nd Period 2022-2026**	**3rd Period 2027-2031**	**4th Period 2032-2036**	**5th Period 2037-2041**	**Total Area cut Per compartment (ha)**
1994	Stock	6.9	6.9	6.9	0.1	0	
	Cut	0	0	6.8	0.1	0	6.9
1995	Stock	19.3	3.2	3.2	0	0	
	Cut	16.1	0	3.2	0	0	19.3
2001	Stock	6.5	6.5	6.5	6.5	0	
	Cut	0	0	0	6.5	0	6.5
2003	Stock	4.8	4.8	4.8	4.8	1.9	
	Cut	0	0	0	2.9	1.9	4.8
2005	Stock	22	20.2	7.2	7.2	7.2	
	Cut	1.8	13.0	0	0	7.2	22
Total Area Cut		17.9	13.0	10.0	9.5	9.1	59.5

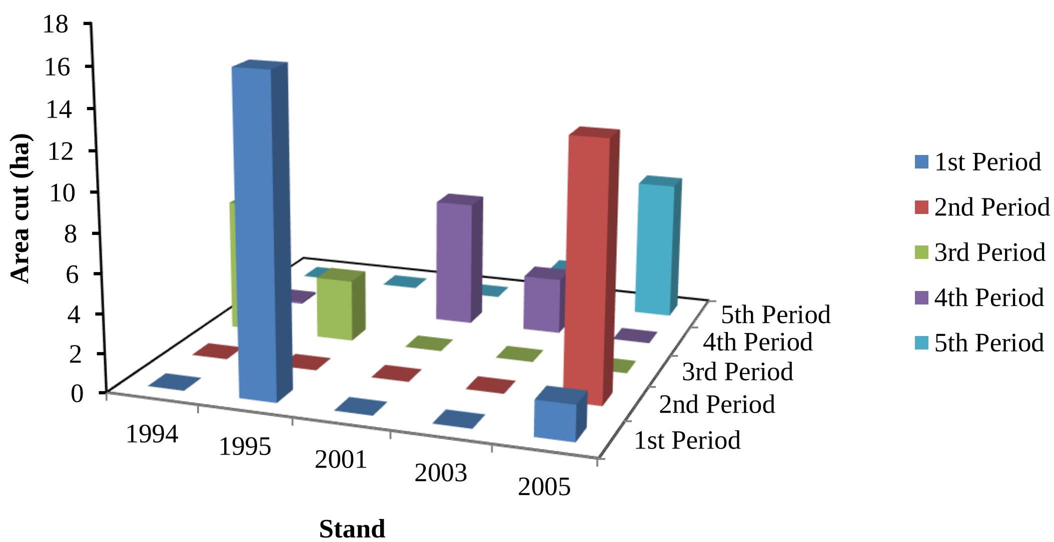

Fig. 2: Optimal Management Plan (Volume Optimization) for *Tectona grandis* Stand in terms of Cutting area by Stand and Period

Table 3: Harvest Schedule for Maximum Wood Volume; Volume Data (m^3)

	Volume cut (m^3) Per Period					
Stand	**1st Period 2017-2021**	**2nd Period 2022-2026**	**3rd Period 2027-2031**	**4th Period 2032-2036**	**5th Period 2037-2041**	**Total Volume (m^3)**
1994	0	0	9581.2	141.4	0	9722.6
1995	20769	0	4163.2	0	0	24932.2
2001	0	0	0	5876	0	5876.0
2003	0	0	0	2862.3	1884.8	4747.1
2005	1296	9451	0	0	5349.6	16096.6
Total	22065.0	9451.0	13744.4	8879.7	7234.4	61374.5

Table 4: Logging Plan that Optimizes Volume: Equivalent Number of Stems to be cut

	Number of Stems Cut Per Period					
Stand	**1st Period 2017-2021**	**2nd Period 2022-2026**	**3rd Period 2027-2031**	**4th Period 2032-2036**	**5th Period 2037-2041**	**Total Stems**
1994	0	0	5,263	77	0	5,340
1995	12,809	0	2,546	0	0	15,355
2001	0	0	0	4,968	0	4,968
2003	0	0	0	2,145	1,405	3,550
2005	1,372	9,911	0	0	5,489	16,772
Total	14,181	9,911	7,809	7,190	6,894	45,985.0

Table 5: Sensitivity Analysis for the Objective Coefficients

Variable	Allowable Decrease	Current Coefficient	Allowable Increase
$X_{1.1}$	-infinity	1399.000	6.067358
$X_{1.2}$	-infinity	1404.000	3.262812
$X_{1.3}$	0.3076923	1409.000	3.287188
$X_{1.4}$	3.287188	1414.000	3.5976923
$X_{1.5}$	-infinity	1419.000	939.5697
$X_{2.1}$	0.1409091	1290.000	0.4727273
$X_{2.2}$	-infinity	1296.000	0.1409091
$X_{2.3}$	0.4727273	1301.000	9.126416
$X_{2.4}$	-infinity	1305.000	9.985507
$X_{2.5}$	-infinity	1310.000	2647.043
$X_{3.1}$	-infinity	887.0000	8.585192
$X_{3.2}$	-infinity	893.0000	4.653374
$X_{3.3}$	-infinity	899.0000	0.2898551
$X_{3.4}$	0.2898551	904.0000	INFINITY
$X_{3.5}$	-infinity	909.0000	884.8120
$X_{4.1}$	-infinity	970.0000	10.78599
$X_{4.2}$	-infinity	976.0000	6.313261
$X_{4.3}$	-infinity	982.0000	1.521739
$X_{4.4}$	1.521739	987.0000	3136.318
$X_{4.5}$	653.3996	992.0000	INFINITY
$X_{5.1}$	0.5388601	720.0000	0.1606218
$X_{5.2}$	0.1606218	727.0000	INFINITY
$X_{5.3}$	-infinity	732.0000	0.5388601
$X_{5.4}$	-infinity	738.0000	10.48089
$X_{5.5}$	-infinity	743.0000	3017.152

Table 6: Sensitivity Analysis for the Right Hand Side (RHS)

Row	Allowable Decrease	Current RHS	Allowable Increase
B1	6.90	6.90	8.69
B2	19.30	19.30	21.09
B3	6.50	6.50	6.59
B4	4.80	4.80	4.89
B5	22.00	22.00	INFINITY
B6	11.79	17.90	17.90
B7	13.00	13.00	13.00
B8	9.79	10.00	10.00
B9	7.09	9.50	9.50
B10	7.46	9.10	9.10

Table 7: Coefficients for Calculating opportunity cost (m^3/ha)

Compartment No	1st Period 2017-2021	2nd Period 2022-2026	3rd Period 2027-2031	4th Period 2032-2036	5th Period 2037-2041	Area Per Compartment/ha
1 (1994)	6.07	3.26	0	0	939.60	6.9
2 (1995)	0	0.14	0	9.98	2647.04	19.3
3 (2001)	8.58	4.65	0.28	0	884.81	6.5
4 (2003)	10.78	6.31	1.52	0	0	4.8
5 (2005)	0	0	0.54	10.48	3017.15	22

DISCUSSION

The management planning model developed and solved in this study is equation 5 and 6 (based on stem volume optimization and the equivalent number of stems of the volume values). The model was used to obtain a harvest schedule that optimizes stem volume of *Tectona grandis* throughout the period of the management plan, that is, 25 years. Loucks (1964), Lappi (1992), Johnson *et al*. (1997) and Ajayi (2013) obtained linear programming models similar to the management planning model obtained in this study. The solution to the LP problem developed in this study shows the sequence of harvest in terms of areato be cut (ha.) that would optimize the stem volume of *Tectona grandis* harvested through the period of the management plan (i.e. 25 years). The LP also shows the corresponding volume data with respect to the area cut; and the number of stems which is the equivalence of volume value. This offers some valuable information on logging, regeneration and environmental protection. For instance, schedule gives room for clear felling. Logging cost per unit of timber is lower in a clear cutting operation than in selection harvest. The approximate standardization of products (timber) can also allow for mechanization harvest. For the regeneration of cleared area, the schedule provides allowance for harvesting and regeneration activities to be planed and implemented concurrently.

Sensitivity analysis reveals more useful information, as change is inevitable in a dynamic system like forest plantations; hence, sensitivity analysis is tailored to demonstrate how the system or model is affected when a change occurs in one or more of its parameters - coefficients, resources availability and usage of the resources. In a situation whereby the change is so significant, the model would have to be reviewed. For the solution in this study to remain optimal, the range of values for wood volume and hectares (which are of interest to the plantation management) would have to be maintained.

Loss of profit is incurred when operations cannot be undertaken at the ideal time. In postponing clear felling much beyond optimal schedule, there is a 'cost of delay' (an opportunity cost). A loss (cost) can as well be incurred, if one is earlier than scheduled. The opportunity cost here in wood volume, implies the volume of wood used in a particular cutting period and not the other periods (i.e. it is valued by the amount of wood in the period it is not used), an information that is very useful in plantation management. For this study, if the management decide to delay the harvesting of 4.1 hectares say, from compartment 2 till the 5thcutting period, the loss in wood volume would be (2647.04 x 4.1) m^3 = 10,852.864 m^3. Similarly, if the harvesting of the whole area of compartment 3 (6.5 ha) is to be delayed till the 5th cutting period, the loss in wood volume would be (884.81 x 6.5) m^3 = 5,751.265 m^3. Harvesting earlier within the 1st period would have incurred a loss of only (8.58 x 6.5) m^3 = 55.77 m^{3}of wood. This is in accordance with Price (1989) who stated that a long delay entails greater costs than a short delay.

CONCLUSIONS

The management planning models are useful for establishing strategic logging goals with spatially feasible plan that can be implemented. The additional information on the corresponding number of stems to be harvested ensures practical implementation. The harvest schedules obtained in this study have shown that the management-planning model provides rational and practicable results. Clear felling will lead to reduction in the cost of logging per unit timber as indicated by the schedule, especially as mechanized logging can be employed. The schedule provides for a steady and close source of planting stock (seeds and seedlings), apart from the allowance created for regeneration activities to go side by side with harvesting. Additionally, sensitivity analysis offer ranges of coefficient values within which fluctuations can be accommodated. Analysis based on opportunity cost suggests that, to avoid loss in wood volume, it is good to stick to the optimal schedule. If deviation becomes inevitable, it should not be far from optimal schedule in terms of period.

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