# Application of minimal spanning tree search algorithms to the resolution of graph problems: Case of salting and snow removal of the road network of the city of Tiaret

## تطبيق خوارزميات البحث عن الشجرة الممتدة ذات الوزن الأدنفي حل مسائل البيان: حالة تمليح و ازالة الثلوج لشبكة طرق مدينة تيارت

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**Abstract:**

*The main objective of this study is to treat a primordial subject of the optimization of graph and network problems, namely the problem of finding the spanning tree of minimum weight, which consists of identifying and determining the tree which connects all vertices of a graph using a set of edges whose cost is minimal.*

*Through this research study we address the problem of salting and snow removal from the road network of the city of Tiaret during winter by the municipal authorities and services. Our mission is to determine a partial road network (sub-network) from the initial road network of the town of Tiaret, which will have to be salted and cleared of snow by these authorities and services.*

*The application of graph theory techniques for modeling the problem, and the use of the main search algorithms for the minimum weight spanning tree, allowed the study to propose the optimal subnetwork which contains the main roads of the road network of the city of Tiaret, to salt and clear snow at the lowest possible cost.*

***Key words:*** *graph theory, minimal spanning tree, road salting, snow removal, deicing.*

**ملخص:** الهدف الرئيسي من هذه الدراسة هو معالجة موضوع أساسي يتعلق بأمثلية مسائل نظرية البيان والشبكات، وهي مشكلة البحث عن الشجرة الممتدة ذات الوزن الأدنى، التي تقوم على تحديد الشجرة التي تربط جميع قمم بيان باستخدام مجموعة من الاحرف بأقل تكلفة.

من خلال هذه الدراسة البحثية نتناول مشكلة التمليح و إزالة الثلوج من شبكة الطرق لمدينة تيارت خلال فترة الشتاء من قبل السلطات والمصالح البلدية. مهمتنا هو تحديد شبكة طرق جزئية و فرعية من شبكة الطرق الأولية لمدينة تيارت، والتي سيتعين تمليحها وإزالة الثلوج منها .

إن تطبيق كل من تقنيات نظرية البيان لنمذجة المشكلة، واستخدام الخوارزميات الرئيسية للبحث عن الشجرة الممتدة ذات الوزن الأدنى، أتاح للدراسة اقتراح الشبكة الفرعية و الجزئية المثلى التي تتضمن الطرق الاساسية لشبكة طرق مدينة تيارت، الواجب على السلطات و المصالح البلدية تمليحها وإزالة الثلوج منها بأقل تكلفة ممكنة.

***الكلمات المفتاحية:*** *نظرية البيان، الشجرة المثلى، تمليح الطرق، ازالة الثلج، ازالة الجليد.*

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## 1. INTRODUCTION

Operations research, abbreviated OR, also known as decision science, appeared during the Second World War thanks to the combined efforts of eminent mathematicians, the field of application of OR expanded to various sectors and fields such as economics, finance, marketing, health, education and other areas of public interest OR is the discipline of applied mathematics that deals with questions of optimal use of resources available in these different sectors and areas.

Graph theory is part of this discipline, it occupies a very important place in OR, this theory is one of the main modeling tools used for solving many optimization problems. Indeed, graph modeling makes it possible to represent sets of elements which are connected in a certain way, so the graph is a basic mathematical structure which makes it possible to model certain real-life problems.

One of the essential problems in graph theory is the minimum spanning tree problem, abbreviated MST, also called the minimum weight spanning tree (MWS). This problem consists of finding a set of edges in a connected undirected and weighted graph, which forms a tree and which connects all the vertices whose weight is as small as possible. The minimum spanning tree problem finds various areas of practical and varied applications, we cite for example road, rail, air, telephone, computer networks, etc.

According to what has been presented above, this research work inspires its problem, which can be formulated and translated by the following main question.

**What is the role of the use of graph theory, in particular the minimum weight covering tree, in minimizing the costs of salting and snow removal from the road network of the city of Tiaret?**

The answer to this question is the main objective of this article. The latter is subdivided into five sections. Through the first, we will address the definition and presentation of the fundamental notions of graph theory. In the second section we will present the minimum weight spanning tree problem. The third section will present a summary of the three main algorithms for searching the minimum spanning tree, namely the Borùvka algorithm, the Kruskal algorithm and the Prim algorithm. In the fourth section, we will discuss the experimental study that we carried out to apply the notions of graph theory cited above, in particular the minimal spanning tree to the problem of snow removal and salting of the road network of the city of Tiaret, with a view to to reach the covering tree with minimum weight, which allows municipal authorities and municipal services to identify the road sub-network to be salted and cleared of snow. This leads us to discuss the results from the fifth and final section and draw some remarks and conclusions.

## 2. Fundamentals notionsof graph theory:

One of the first important results of graph theory appeared in the articles of Leonhard Euler with the problem of the seven bridges of Königsberg, published in 1736. It is also considered one of the first topological results in geometry, in fact, it does not depend of no measure. These facts characterize the deep relationship that exists between graph theory and topology. In 1835, Gustav Kirchhoff published his laws of circuits to calculate voltage and current in an electrical circuit.[1]Many fundamental theoretical terms and concepts in graph theory have arisen from attempts to solve this problem.

Directed or undirected graphs are irreplaceable tools for modeling and solving many concrete problems, or at least for having a precise idea of their nature and their algorithmic difficulties. Indeed, they make it possible, on the one hand to guide intuition during reasoning, and on the other hand to relate to the known results of graph theory.[2] We define them succinctly below and illustrate their interest with a few examples.

The goal through this section is to make a brief introduction to graph theory, and show the interest as a modeling tool, to present the different fundamental definitions as well as the corresponding terminology (graphs, paths, trees, forests, sources, wells, etc.). These definitions are numerous but very intuitive, which makes them easier to learn.

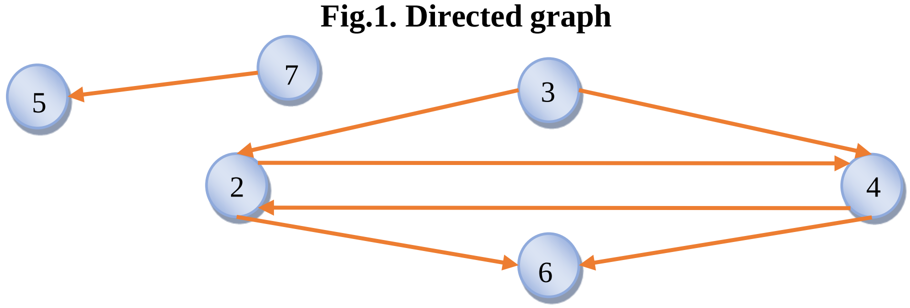
**2.1. The graph:** A graph can be defined as a set of objects or nodes called vertices or ends, some of which are linked together, either by edges or by arcs, oriented case. Graphically, vertices can be represented by points or circles, edges by lines and arcs by arrows.[3] So a noted graph *G(S,E)*is defined by two sets: the first is the set of vertices or nodes, and the second is the set of edges in the case of an undirected graph, or arcs in the case of a graph oriented. The number of vertices in a graph is called the order of a graph. Graphs can be used to model, among other things:[4]

- A large-scale road network: each city is a vertice, each road between two cities is an arc.
- A small-scale road network: each intersection is a vertice, each street section between two intersections is an arc.
- A bus network, a rail network.

There are several types of graph, we cite the following:

**2.1.1. The directed graph:** A directed graph is a pair *G(S,E)*, where is a set whose elements are called vertices and E is a part of whose elements are called arcs.[5]

**Fig.1. Directed graph**

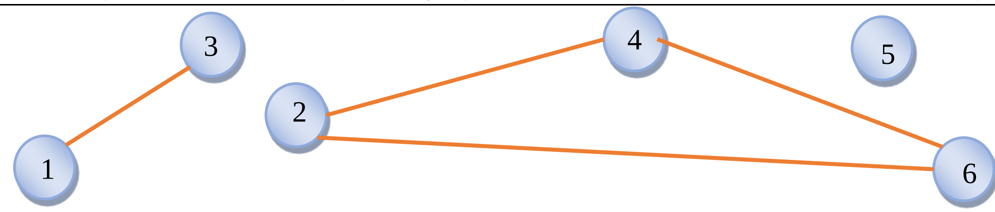



***Source**: Established by researchers*

The figure above represents a directed graph $G(S,E)$of order 7 with $S=\{1,2,3,4,5,6,7\}$and $S=\{(2,4),(2,6),(3,2),(3,4),(4,2),(4,6),(7,5)\}$. A directed graph is a p-graph if it has at most p arcs between two vertices. Most often, we will study 1-graphs. A partial graph of a directed or undirected graph is the graph obtained by deleting certain arcs or edges.[6] A subgraph of a directed or undirected graph is the graph obtained by deleting certain vertices and all arcs or edges incident to the deleted vertices. A directed graph is said to be elementary if it does not contain a loop (an arc or edge connecting a vertice to itself). A directed graph is said to be complete if it includes an arc $(S_i,S_j)$ andan arc $(S_j,S_i)$for any pair of different vertices $S_i,S_j$. Likewise, an undirected graph is said to be complete if it includes an edge $(S_i,S_j)$for any pair of different vertices $S_i,S_j$.[7]

**2.1.2. An undirected graph:** An undirected graph is a pair *G(S,E)*, where*S* is a set whose elements are called vertices and *E*a subset of parts *S*each containing at most 2 elements and whose elements are called edges.[8]

**Fig.2. Undirected Graph**



***Source:*** *Established by researchers*

The figure above represents an undirected graph $G(S,E)$ of order 6 with $S=\{1,2,3,4,5,6\}$ et $S=\{(1,3),(2,4),(2,6),(4,6)\}$. An undirected graph is said to be simple if it does not include a loop, and if it never includes more than one edge between two vertices. An undirected graph that is not simple is a multi-graph. In the case of a multi-graph, $E$ is no longer a set but a multi-set of edges. We will generally restrict ourselves in the following to simple graphs.[9]

**2.2. The chain:** a chain is a finite and alternating sequence of vertices and edges, starting and ending with vertices, such that each edge is incident with the vertices which surround it in the sequence.[10]The first and last vertice are called ends or top of the chain. The length of a chain is the number of edges that make up that chain, that is, the number of vertices minus one. If none of the vertices composing the sequence appears more than once, the chain is called an elementary chain. If none of the edges composing the sequence appears more than once, the chain is a simple chain.

**2.3. The cycle:** a cycle is a path whose ends (vertices) are all linked,as we can define a cycle is a chain whose ends coincide.[11] An elementary cycle is a minimal cycle for inclusion, that is to say strictly containing no other cycle.

**2.4. The path:** A path is a finite and alternating sequence and sequence of vertices $\{X_0, X_1, X_2, X_3,....., X_{n-1}, X_n\}$ et d'arcs $\{(X_0, X_1),(X_2, X_3),.....,(X_{n-2}, X_{n-1}),(X_{n-1}, X_n)\}$, belong to the graph, starting and ending with vertices, such that each edge is leaving a vertice and incident to the next vertice in the sequence.[12] The value of a path is the sum of the evaluations of the edges of this path. The length of a path is its number of arcs.The vertice $X_0$ is called the initial end of the path and the vertice $X_n$ the terminal end. If none of the vertices composing the sequence appears more than once, the path is called elementary,if none of the edges composing the sequence appears more than once, the path is said to be simple.[13]

**2.5. The circuit:** a circuit is a path whose initial end coincides with the terminal end, therefore a circuit is a path whose ends coincide.[14]

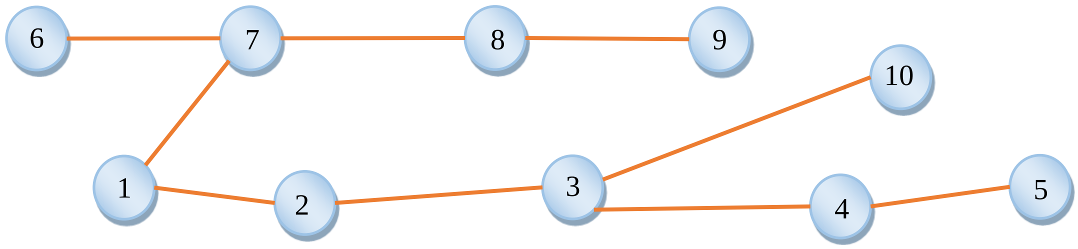
**2.6. Connectedness:** a graph is connected if for any two vertices u and v, that is to say for each pair of vertices u and v, the graph contains and has a path between these two vertices.[15]i.e. if the graph has a path from any vertice to any other.

**3. Tree, Covering tree, Covering tree of minimum weight:**

**3.1. Tree:**a tree can be defined in several ways, we cite a few. A graph $G(S,E)$ is a tree if and only if:[16]

- It is connected and without cycle
- It is minimally connected in the sense of edges, that is to say it is no longer connected if any of its edges are removed
- It is without cycles and maximum in the sense of edges, i.e. we create a cycle by adding an edge making any two of its vertices which were not adjacent
- It is connected (or without cycles) and has $(n-1)$ edges.

**Fig.3. A Tree**




***Source**: Established by researchers*

**3.2. Minimal spanning tree problem:** Given an undirected and connected graph whose graphs are provided with an evaluation which we will indifferently call cost or weight or distance, we seek to construct a partial graph among all the partial graphs of this graph which either a tree is of minimum cost, that is to say the sum of costs or weights or distance is minimum among all the partial trees of the graph. Since it is a partial graph, this tree has as its set of vertices the set of all the vertices of the initial graph. The minimum tree problem consists of connecting N sites for a minimum overall cost so that any site can communicate with all the others. We will see that we must look for a spanning tree with minimal cost and that this problem is easy because it can be solved by a polynomial algorithm.

**3.3. Spanning tree:** Consider $G(S,E)$ a weighted graph (each edge has a weight).A spanning tree, also called a covering tree of this graph, is a set of edges $G(S,E)$ which forms a tree and which contains all the vertices.[17] Its weight is the sum of the weights of the edges of the tree that composes it. So a spanning tree is a tree that connects all the vertices of a graph $G(S,E)$ in such a way that for all pairs (A, B) of points in the graph $G(S,E)$, exists a path that connects A to B.

**3.4. Minimum weight spanning tree:** We consider a set of points in a plane with integer coordinates. These points are considered to be the vertices of a complete graph being evaluated, the weight of an edge being the Euclidean distance between its ends. This involves determining a spanning tree of minimum weight in this graph, in other words connecting the points with line segments so as to have a connected graph, ensuring that the sum of the lengths of the segments is minimum. A spanning tree of minimum weight, also called a minimum spanning tree, sometimes abbreviated MST, is a spanning tree whose total weight, which is the sum of the weights of its arcs or edges, is minimal and there is no spanning tree of minimum weight. lower weight. So the minimum weight spanning tree consists of finding a spanning tree whose sum of edge weights is minimum.[18] The benefit of a minimal spanning tree is as follows: Let's imagine that we want to connect several cities together to any network (gas, electricity, telephone, etc.). A graph represents the possible connections between each of these cities with the cost represented by the connection between these cities. The search for the minimum covering tree will be used to determine the network which will connect all the cities together with a minimum cost.

## 4. Algorithms for determining a minimum weight spanning tree:

To achieve the minimum weight spanning tree (MWST) or minimum spanning tree (MST) in an undirected and weighted connected graph, many algorithms are available.

Through this section we are interested in explaining the operation of the three main classical algorithms:

1. Borůkva algorithm
2. Jarnik (or Prim) algorithm
3. Kruskal algorithm

**4.1. Spanning trees of minimum weights by the Borùvka algorithm:**

The Borûvka algorithm was designed in 1926 by mathematician Otakar Borûvka to find an optimal electricity network for a country located in Central Europe called Moldova.[19] It is the oldest existing algorithm for finding the minimum weight spanning tree (MWST) or minimum spanning tree (MST) in an undirected and weighted connected graph. This algorithm uses the minimum weight edge property. This property gives us the following information:[20] the smallest edge that a vertice has is in the (MWST). So we can start with the full graph, we visit each vertice and we color the smallest edge connected to it.

After completing this first step, there are now quite a few edges that are part of the (MWST). However, even though now all vertices have at least one edge reaching them, the (MWST) is still not found. This step found a good number of disjoint sets in the graph that we can see as small isolated islands. If this step is reapplied a second time, we will have the same result. Instead, we must consider each disjoint set as a single vertice and reapply the same logic. In other words, we no longer consider the least expensive edges to reach each vertice, we are now interested in the least expensive edges which will connect the groups of vertices together.[21]

This step is repeated until all of our vertices are only in one group. With n number of vertices, log n steps are required, because at each step there will be at least half as many groups in the graph.

**4.2. Spanning trees of minimum weights by the Kruskal algorithm:**

The Kruskal algorithm is an algorithm for searching a minimum weight spanning tree (MWT) in an undirected and weighted connected graph. It was designed by Joseph Kruskal following his reading of Borûvka's paper in 1956, this algorithm is considered the simplest algorithm to use and implement.[22]Unlike Borûvka's algorithm and that of Prim, Kruskal's algorithm previously requires the sorting of edges; this condition greatly simplifies the operation of the algorithm.

Kruskal's algorithm is done in two phases! the first called the sorting phase, the latter consists of sorting and arranging all of the edges of the graph in order of increasing weight, the second phase consists of constructing the tree covering of minimal weight by removing the edges one by one in this order and adding them to the MWT sought as long as this addition does not cause a cycle to appear in the MWT.[23]

**4.3. Spanning trees of minimum weights by the Prim algorithm:**

The algorithm was designed and developed in 1930 by Czech mathematician Vojtech Jarnik, and was later rediscovered and republished by Robert C. Prim and Edsger W. Dijkstra in 1959,thus, it is sometimes called DJP algorithm, Jarník's algorithm or Prim– Jarnik algorithm.[24]

The algorithm known as Prim's algorithm closely resembles, in principle, Dijkstra's algorithm, even if the purpose is different (finding the spanning tree of minimum weight, and not the shortest path between two vertices).

If we have $G(S,E)$ an undirected connected graph weighted with vertices $S$, $(S-1)$ iterations will be required to cover all the vertices of the graph and obtain MWT.

The first phase of Prim's algorithm consists of arbitrarily choosing the initial starting vertice of the spanning tree of minimum weight $S$ among the vertices of the graph $G(S,E)$,the second phase must be repeated $(S-1)$ timesto find all the edges of the spanning tree of minimum weight, this phase consists of starting from this initial vertice and choosing an edge of minimum cost from this vertice to go to the next vertice until covering all vertices of the graph $G(S,E)$.[25]At each iteration of $(S-1)$ iterations, we enlarge this subset by taking the edge incident to this subset of minimum cost.[26]Indeed, if we take an edge whose two ends already belong to the tree, the addition of this edge would create a second path between the two vertices in the tree currently being constructed and the result would contain a cycle.

**5. The experimental study:**

The experimental part of this study was carried out with the services of the general resources department of the municipality of Tiaret. Our main mission through this study was to determine a road sub-network of minimum cost from the road network of all the neighborhoods of this city, which these services must plow and salt it, so that any neighborhood can communicate with everyone else. It is therefore a question of determining the spanning tree of minimum weight which connects all the districts of the city for a minimum overall cost.

**5.1. Sanding, salting and snow removal:**

**5.1.1. Sanding:** Sanding consists of spreading abrasive aggregates such as a mixture of sand and gravel or pozzolan on the roads in order to help melt the film of ice or snow and to avoid the constraints of snow and iceand restore grip to the slippery road surface. In fact, this spreading allows better grip for used vehicles and pedestrians on snowy and icy roads.

**5.1.2. Salting:** In the road sector, salting is the action of spreading a road melt on snowy or icy roads and roads in order to melt the film of ice or snow compacted on them. The usual flux is sodium chloride type salt, different from table salt in its particle size and hygroscopicity. Calcium chloride and magnesium chloride are also used individually or mixed with sodium chloride. To be more effective more quickly the salt can be pre-diluted in water, resulting in brine. This brine can also be mixed again when spreading with solid salt, resulting in salt slurry. Salt can be spread either for preventive or curative purposes:

1. **For preventive purposes:** in this case the salt must be spread before the formation of the ice film to prevent its formation. The objective of preventive treatment is the search for both the greatest safety and the greatest effectiveness. To do this, preventive salting should not be carried out too long before the risk of ice appearing, hence the need to monitor changes in weather conditions. Preventive treatment does not exempt from network monitoring. Indeed, salt is a hygroscopic material, that is to say which has the capacity to absorb water in the form of vapor present in the air which thus passes into the liquid state by depositing on the surface of pavement. The salt content of the saline solution present on the road therefore tends to decrease more quickly as the air is more humid. It may therefore become insufficient to prevent the formation of ice on the roadway.
2. **For curative purposes:** in this case the salt must be spread after the appearance of hardened ice or snow. When frost or black ice forms at daybreak or in the event of fog or the presence of water on the roadway with a very marked drop in temperature, curative intervention becomes necessary.

**5.1.3. Snow removal:** Snow removal is the work of sweeping snow. This work mainly involves removing snow on and around homes, roadways and sidewalks. Snow removal from roads is generally done by pushing the snow along the edge of the road, then carrying it away in trucks and finally carrying out salting of the roads. Snow removal from streets and roads is done using specialized vehicles by municipal, regional and national public road services or by private companies hired by the authorities. Private roads and accesses are, for their part, left to the care of the owners who do the snow removal personally or contract it out to specialized firms. Snow removal is thus part of the context of winter viability, for the maintenance of safety conditions and the continuity of economic activities.

### 5.2. Presentation of the problem of snow removal and salting of the road network in the city of Tiaret:

Through this section we will present the problem of salting the road network which connects the different districts of the city of Tiaret by the services of the general resources department of the municipality of this city. The latter faces the following problem:

The climate of the city of Tiaret is characterized by a rainy and cold winter with snowfall, which leads to the formation of ice on the road network connecting the different districts of this city, which paralyzes the circulation of users (pedestrians and vehicle owners) between these different neighborhoods.

Each winter, municipal authorities activate their winter intervention plan to secure the road network and allow users to travel in better conditions. This intervention plan consists of carrying out salting and snow removal operations on this road network.

Previously, the sandblasting operation of the road network in the town of Tiaret was carried out with gravel from different quarries located in the wilaya. These gravels are characterized by a very high hardness and a very high percentage of fine clay elements. Sanding snow-covered roads with this gravel initially proves effective, especially with regard to roughness. However, once the snow melts, these gravels, which are very hard and clayey, constitute a danger for the safety of road users because they increase the slipperiness of vehicles and the fine clay elements contained in these gravels form in the presence of water and mud on the surface of the roadway. Because of its negative effects and for road safety, the use of these gravels has been abandoned, and these gravels are replaced by salting which uses salt such as sodium chloride, calcium chloride and magnesium chloride.

Each year the municipal popular assembly devotes a very large budget to this winter maintenance program, which has exhausted the municipal budget, for this reason the service managers of the department of general resources of the municipality are no longer interested in snow removal and maintenance. salting of all roads in this city's road network, but they are only interested in salting and plowing certain roads in this network so that all areas of the city are accessible.

The problem of salting the road network in the town of Tiaret can be reformulated and summarized as follows:

> Given a certain number of neighborhoods distributed in a dispersed manner over the entire territory of the city of Tiaret and linked together by a road network, Due to the insufficiency of credits entered in the municipal budget and for economic reasons, the municipal authorities and municipal services which manage the salting and snow removal operation of the city's road network want to seek and determine a partial road network (sub-road network) of this initial road network which is of minimal cost and which connects these different neighborhoods, to

salt and clear snow so that any neighborhood can communicate with all the other neighborhoods of the city, in other words that these authorities and these services want to eliminate as many connections as possible, while maintaining the possibility of connecting between them all neighborhoods. Furthermore, they want the subnetwork to remain as efficient as possible, that is to say that the sum of the distances of the remaining links is as small as possible: this is the problem of finding the spanning tree of minimum weight

In the language of graph theory, the above problem involves determining the minimum weight spanning tree which must connect the different districts of the city of Tiaret for a minimum overall cost so that any district can communicate with all the other districts, that is to say passes through all the districts of the city.

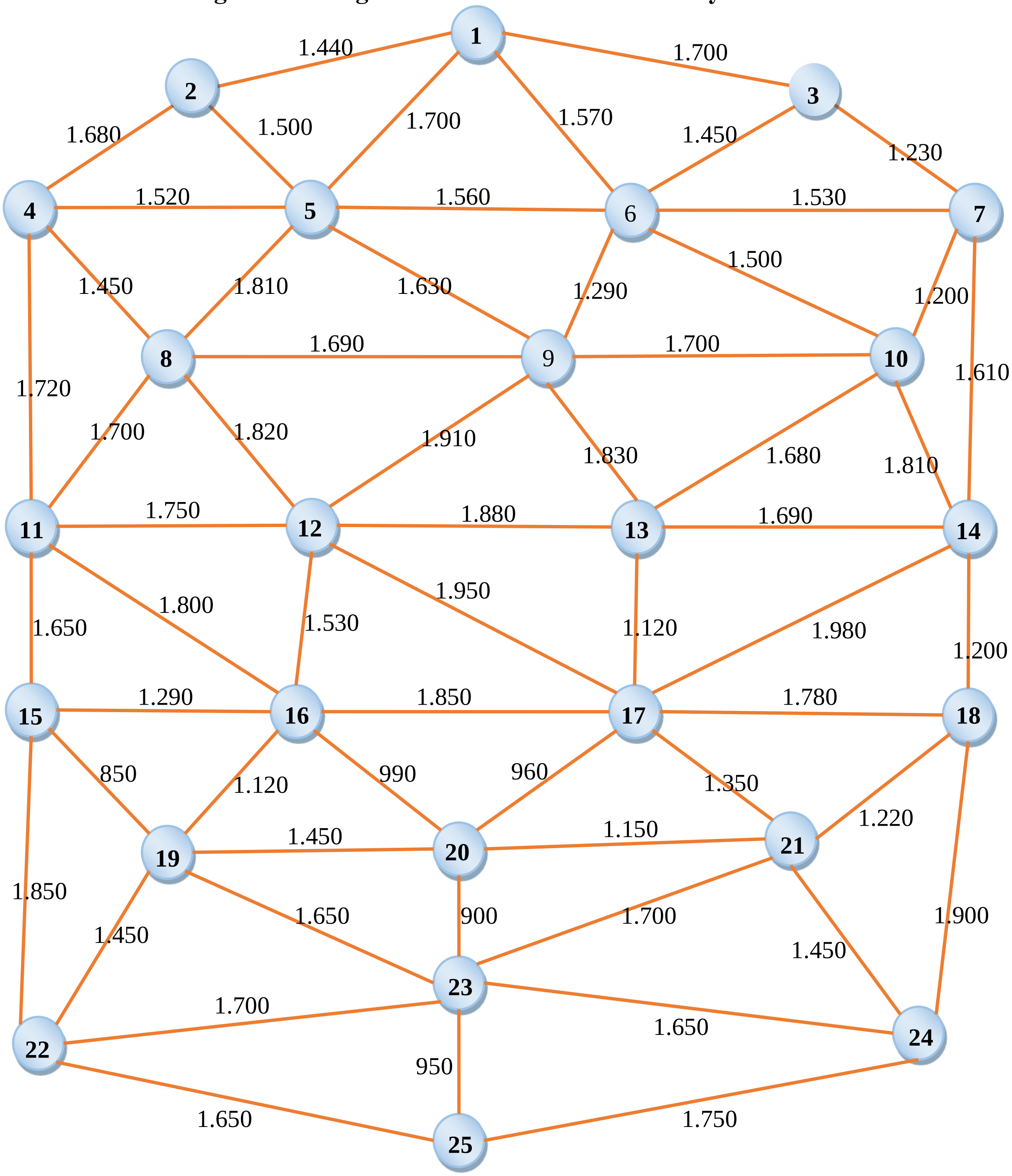
**5.3. Resolution of the salting and snow removal problem from the road network in the city of Tiaret:**The main objective of this study is to search for the minimum weight cover tree which allows municipal authorities and municipal services to determine the road sub-network to be salted and snow cleared. To achieve this objective, we proceed to model the problem in a first phase and apply the resolution algorithms in a second phase.

**5.3.a. Modeling the problem of salting and snow removal from the road network of the city of Tiaret:**Directed or undirected graphs are irreplaceable tools for modeling and solving many concrete problems, they make it possible to represent sets of elements which are connected in a certain way. Graphs are therefore a basic mathematical structure which makes it possible to model real-life problems.

The road network of a city can be represented by a graph whose vertices are the districts. If we consider that all roads are two-way, we will use an undirected graph and we will connect by an edge any pair of vertices corresponding to two neighborhoods connected by a road, if we consider, however, that certain roads are one-way, we will use a directed graph. These edges can be evaluated by the length of the corresponding roads.

Figure 3 below represents the modeling by an undirected graph of order 25 of the problem of salting and snow removal from the road network of the town of Tiaret.

**Fig.3. Modeling of the road network of the city of Tiaret**




***Source:*** *Established by researchers*

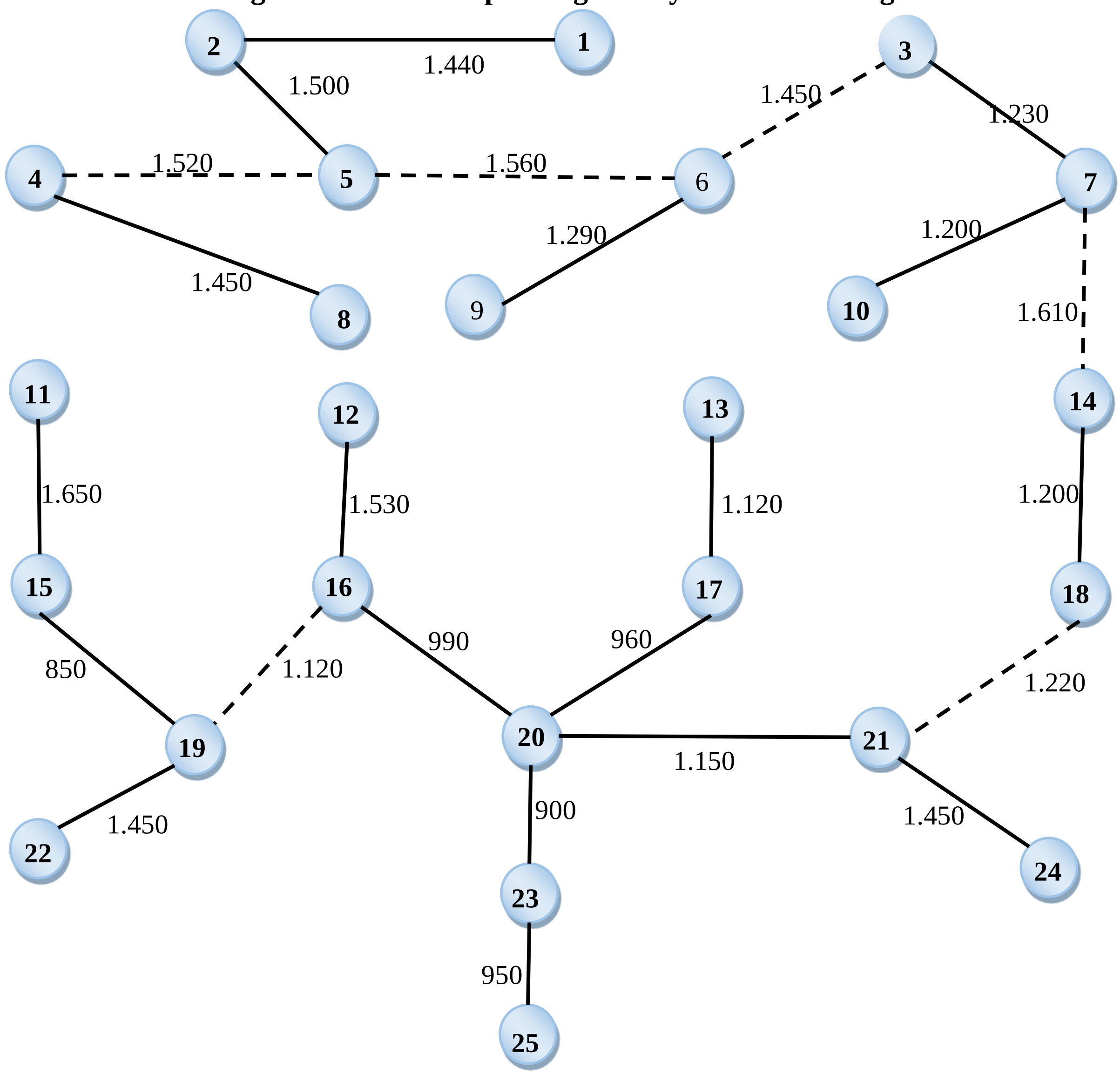
Such as :

- The verticesof the graph represent the different districts of the city of Tiaret, the initial verticenumber 1 represents Zaaroura district and the final vertice 25 represents riding stabledistrict.
- The Ridges represent the different roads that connect the different districts.
- The numbers next to the Edges are called Edge Weights and represent the distance measured in.

**5.3.b. Application of resolution algorithms:** Through this section, we apply the three most well-known search algorithms for the minimal weight spanning tree: those of Borùvka, Kruskal and Prim.

**5.3.b.1.Application of the Borùvka algorithm:**The minimum weight spanning tree following this algorithm is reached in two phases, the first consists in visiting all the vertices of the graph, vertice by vertice and at each vertice we choose the minimum cost edge connected to it, the application of this phase allows us to obtain a number of disjoint sets in the graph. The second phase consists in considering each disjoint set as a single vertice and reapplying the same logic from the first phase. In other words, in this second phase we are interested in the least expensive edges which will connect the groups of vertices together. This phase is repeated until all our vertices are only in a single group which forms the MST.

**Fig.4. The minimal spanning tree by the Borùvka algorithm**

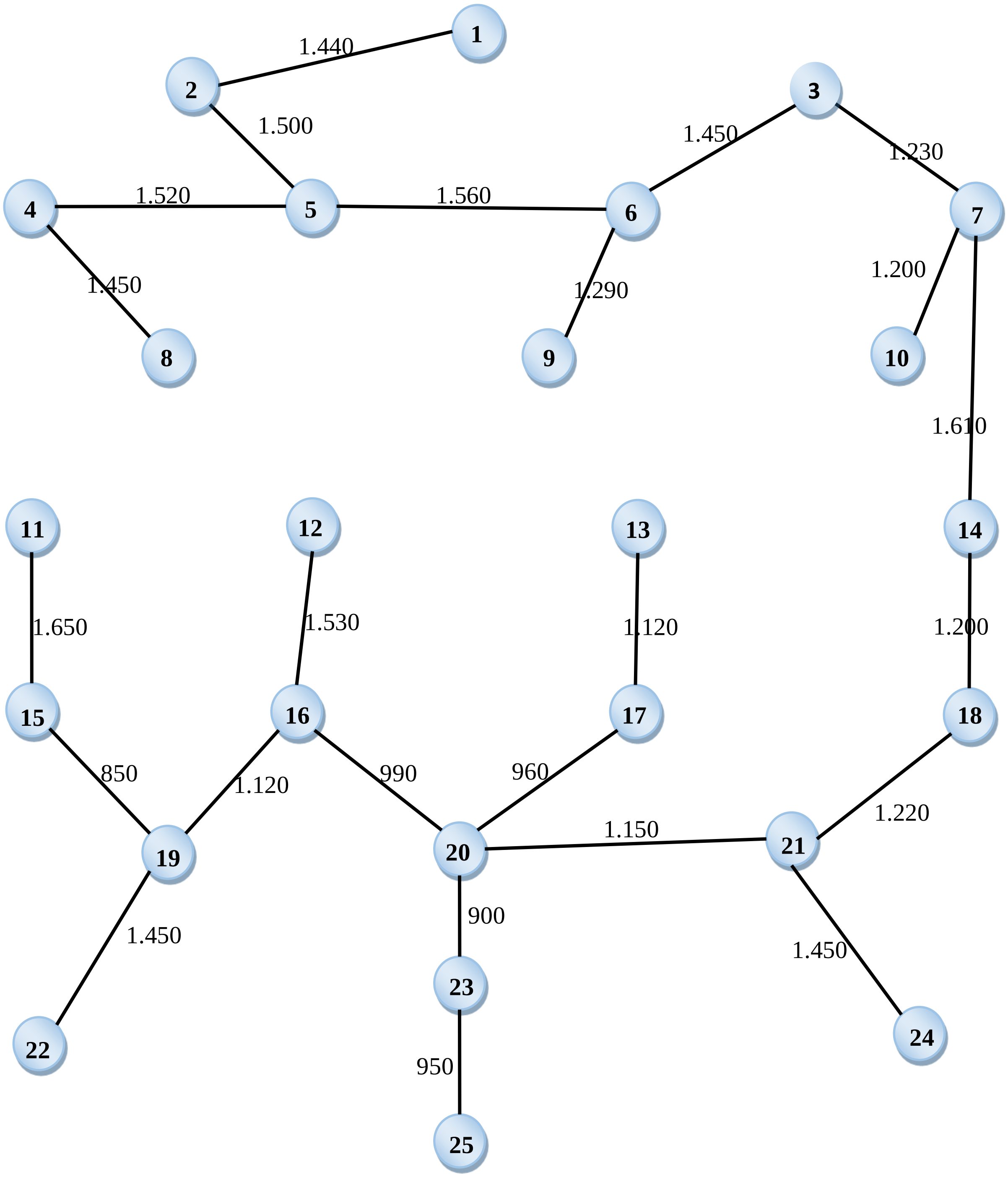



***Source:*** *Established by researchers*

**5.3.b.2.Application of the Kruskal algorithm:**

According to this algorithm the minimum weight spanning tree (MWT) is reached in two phases, the first consists of arranging and sorting the edges of the graph in order of increasing weight, the second phase consists of constructing the spanning tree of minimum weight by removing the edges one by one in this order and adding them to the MWT sought as long as this addition does not cause a cycle to appear in the MWT.

**Fig.5. The minimal spanning tree by the Kruskal algorithm**

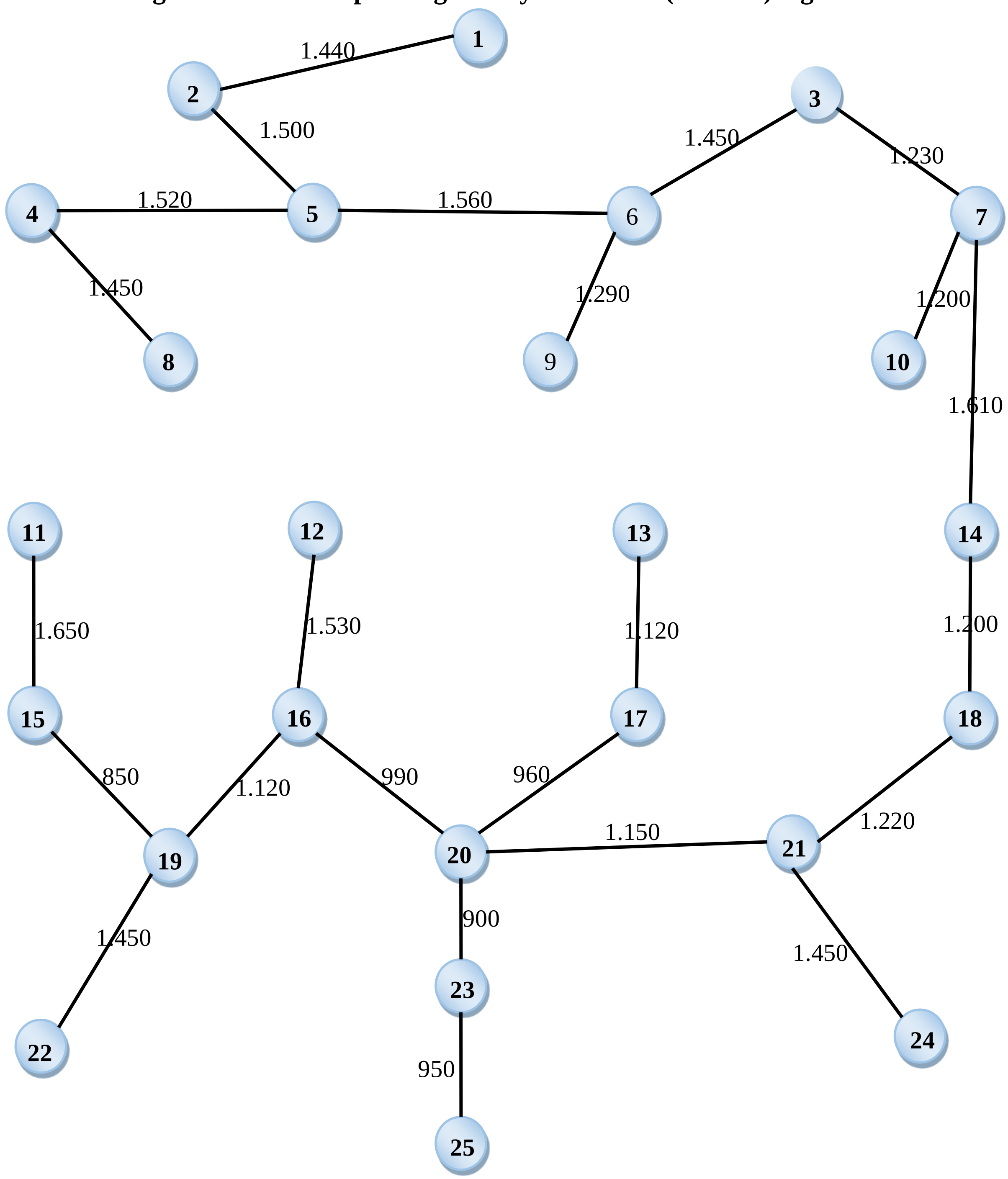



***Source:*** *Established by researchers*

### 5.3.b.3.Application of the Jarnik (or Prim) algorithm

The minimum spanning tree following this algorithm is reached in two steps, the first consists of randomly and arbitrarily choosing a vertice among $S = 25$ the vertices of the graph, thisvertice represents the initial starting vertice of the minimum spanning tree. The second step consists of starting from the initial vertice chosen in the previous step and choosing the least expensive edge connected to this vertice to go to the next vertice until covering all the vertices of the graph, knowing that this step must be repeated $(S-1) = 24$ times to find all edges of the MST.

**Fig.6. the minimal spanning tree by the Jarnik (or Prim) algorithm**




***Source:*** *Established by researchers*

## 6. RESULTS AND DISCUSSION

According to the above and After the application of the techniques and notions of graph theory in particular the minimum spanning tree, and in order to solve the problem of determining the optimal road sub-network to be salted and cleared of snow and ice by the municipal authorities and the municipal services of the city of Tiaret, let's start in a first phase with the graph modeling of the city's road network, and in a second phase with the use of the main methods and algorithms for solving graph problems, namely the Borûvka algorithm, Kruskal algorithm and Prim algorithm. A set of results can be presented, these results are listed as follows:

- The application of the three search algorithms for the minimum spanning tree resulted in the same optimal subnetwork which contains the main roads of the main road network of the city of Tiaret, to be salted and cleared of snow and ice at the lowest possible cost.
- Indeed we have succeeded in reaching the optimal sub-road, which the municipal authorities and municipal services must salt and clear of snow and ice
- The length of the optimal road sub-network is 30.790 meters. While the length of the initial road network is 92.390 meters
- The municipal authorities and municipal services of the city of Tiaret ; the general resources department ; will salt, clear snow and de-ice a distance of 30.790 meters instead of 92.390 meters.
- The credits planned and registered in the municipal budget devoted to the winter viability program of the city of Tiaret will decline and decrease in a remarkable manner.

## 7. CONCLUSION

The main problem of this article on which we focused is very well known in graph theory, it was to determine the minimum weight covering tree which allows municipal authorities and municipal services to determine the road sub-network to be salted and clear snow so that any district can communicate with all the other districts of the city, moreover they want the subnetwork to remain as efficient as possible, that is to say that the sum of the distances of the remaining connections is as small as possible. To answer this problem we used the notions and techniques of graph theory, namely, graph modeling, the main methods and resolution algorithms those of Borûvka, Kruskal and Prim. Indeed, through this study we were able to respond to this problem by achieving the minimum spanning tree, which makes it possible to determine the partial road network of the city's road network to be salted and cleared of snow, which is of minimal cost and which connects these different neighborhoods. , so that any neighborhood can communicate with every other neighborhood in the city.

The conclusions of this study are numerous, we limit ourselves only to those relating and linked to the experimental part.

- The main and most important conclusion of this study is that the municipal authorities and municipal services of the town of Tiaret will salt and clear snow from a sub-road network of distance equal to 30.790meters instead of salting and clearing snow from 92,390 meters which represents the overall distance of all the roads in this city's road network.
- The gap and difference between the overall distance of the initial road network and that of the sub-network allows municipal authorities and municipal services to reduce the volume of the budgetary envelope entered in the municipal budget.

- The road sub-network to be salted and cleared of snow by the municipal authorities and municipal services of the city of Tiaret allows better grip for used vehicles and pedestrians on the snowy and icy roads which connect the different districts of this city.

We invite our readers to fully understand the algorithms and implementations presented in this article in order to best adapt them according to their needs.

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## 9. Appendices

**Appendice 1 : vertices and districts**

| vertices | districts |
|---|---|
| 01 | ZAAROURA |
| 02 | HAY ESSALEM |
| 03 | LES AMANDIERS |
| 04 | GUERDJOU |
| 05 | BADR |
| 06 | LA CADAT |
| 07 | CHAIB |
| 08 | EPLF |
| 09 | MOHAMED DJAHLEN |
| 10 | FRIGO |
| 11 | SONATIBA |
| 12 | EL ABDIA |
| 13 | BENSEGHIR |
| 14 | PINS |
| 15 | TEFFAH |
| 16 | TITANIC |
| 17 | MED SERRIR |
| 18 | BENACEUR |
| 19 | LEPLEY |
| 20 | REGINA |
| 21 | LA CIA |
| 22 | LES 405 |
| 23 | CITE AHMED KAID |
| 24 | LA GLACIERE |
| 25 | RIDING STABLE |