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CASE STUDY

Comparison between the azimuths of astronomical and geodetic coordinates in Ghana

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Abstract

Astronomic azimuths are bearings, which are referenced to the true north direction whereas geodetic azimuths are those referenced to a grid north direction. The difference between these two directions can be expressed through the deflection of the vertical. This deflection of the vertical refers to the difference between the normal to the geoid (Astronomic) and the normal to the ellipsoid (geodetic). The deflection of the vertical components are used to convert between astronomic and geodetic coordinates and azimuths using Laplace equations. This paper compares astronomic and geodetically derived coordinates at eight Laplace stations constituting twenty-eight baselines for which both geodetic and astronomic azimuths were derived. The comparison of azimuths shows that differences of 000°00'05.95" up to 000°23'06.93" exist between the two different versions of azimuths. These differences are not accounted for in application of the simplified Laplace equations under the assumptions made in their derivation. It is, thence, recommended that an alternative didactic approach based on rotations of three-dimensional right-handed local frames as used in Helmert's coordinate transformations be explored for transformation between astronomic and geodetic coordinates.

Keywords: Deflection of Vertical, Astronomical Coordinate, Geodetic Coordinate, Laplace Correction

Introduction

An important use of geodetic astronomy is the determination of astronomical longitude Λ and latitude Φ at points on the Earth's surface and also used to control azimuths. These quantities represent the orientation of the local gravity vector g in space and hence the direction of the local plumbline. The local gravity vector, g coupled by the gravity acceleration, g (Torge and Muller, 2012) is given by:

$$g = -\,g\begin{pmatrix}\cos\Phi\cos\Lambda\\ \cos\Phi\sin\Lambda\\ \sin\Phi\end{pmatrix} \qquad (1)$$

Astrogeodetic methods are being used to determine the basic longitude and latitude related to the global terrestrial coordinate system and upon which the geodetic coordinate frameworks are realised (Torge and Muller, 2012). Another important objective is the determination of astronomical azimuths of terrestrial points by combining direction measurements to terrestrial and celestial targets (Hofmann-Wellenhof *et al.*, 2008. The astronomical azimuths provide orientation for all survey works and are thus used to control the orientations (bearings) of long traverse lines.

With the evolution of Global Navigation Satellite Systems (GNSS) based positioning Techniques, currently, an essential part of traditional astrogeodetic tasks had become readily obtainable using GNSS (Hirt et. al., 2010). Presently, Geodetic coordinates in latitude and longitudes (ϕ, λ), related directly to the reference ellipsoid of the GNSS instrument are obtainable using differential positioning techniques to accuracies on the order of centimetres.

This notwithstanding, surveyors are required to apply various independent measuring techniques so as to introduce check measurements that could check computations in their work. For instance, by merely repeating measurements using the same equipment means increasing only the inner accuracy, but not the external accuracy or the security of results. More reliable results are obtainable using two or more completely independent ways of determination. Thus, in spite of the use of GNSS techniques, astronomical method of azimuth determination still provides such an alternative check (Bilich and Mader, 2010). For instance, obtaining an azimuth for a traverse starting leg from beacon coordinates, and then rotating the entire traverse to fit the ending coordinates, would sometimes include angular blunders in the network that would not be detected. However, when azimuths of long traverses are controlled after a few segments between these two ends, these errors become apparent.

The Federal Geodetic Control Standards (FGCS- United States of America) specifications for conventional control surveys, stated that traverses shall be controlled by an astronomic azimuth at each end of the traverse line and at not more than every six (6) segments along the line for primary traverses and not more than 12 segments for secondary traverses, and that such astronomic azimuths shall have a standard deviation of 1.5 to 2 seconds (Bossler, 1984).

It is worthy to note however that, astronomical coordinates are based on the physical properties of the earth instead of on an ellipsoid whereas Geodetic coordinates are based on and oriented to an ellipsoid. This makes Astronomical coordinates necessarily different from their corresponding Geodetic coordinates. In the ideal case of parallelism between the two sets of systems, transformation between the astronomic and geodetic coordinates is established through the astrogeodetic deflection of the vertical components. For comparison of astronomical and geodetic azimuths, the Laplace condition provides a means of conversion between them. This paper reports a comparison between Astronomical and Geodetic coordinates and their azimuths derived for different locations and baselines in Ghana.

Geodetic Coordinate Systems

Cartesian coordinates

The three-dimensional Cartesian coordinates (X, Y, Z) are known as Earth-Centered-Earth-Fixed (ECEF) coordinates (Szaboya and Duchon, 2016). It is a right-handed orthogonal system that rotates with and is attached to the Earth. It is defined with respect to an ellipsoid as follows (Figure 1):

1. The Origin is at the center of the reference ellipsoid used for defining the datum in question.

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2. The Z-axis coincides with the semi-minor axis b of the reference ellipsoid or the ellipsoid's polar axis.
3. The X-axis is the horizontal axis from the centre of the ellipsoid in the equatorial plane that intersects with the Prime Meridian.
4. Y-axis forms a right-handed triad with the Z- and X-axes.

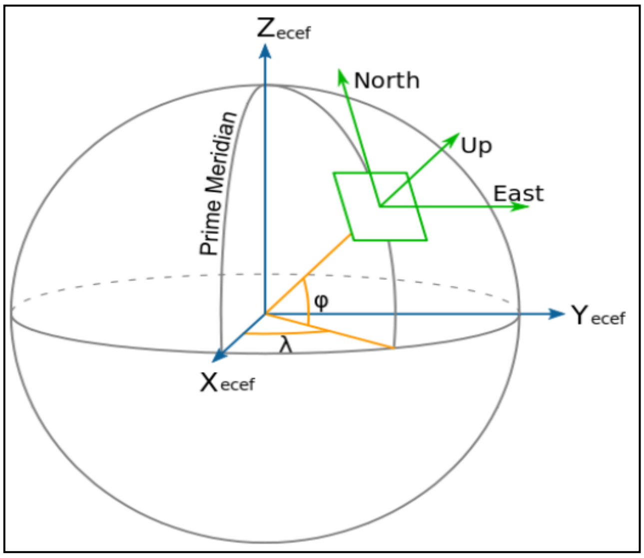

Figure 1 Geodetic coordinate systems

The cartesian coordinate is not used directly in mapping on the grid but provides a model for transforming between different ellipsoids. Its values can also readily be converted to geodetic coordinates (Soler *et al*., 2014).

Curvilinear or geodetic coordinates

The best mathematical representation of the earth used as a reference surface for defining coordinates is the ellipsoid. For this, geodetic coordinates are defined in terms of geodetic latitude (ϕ) geodetic longitude (λ) and geodetic height (h). The Geodetic longitude of a point is the angle measured in the equatorial plane between the prime meridian and the geodetic meridian passing through the point measured positive toward the east. The Geodetic latitude is the angle between the normal to the ellipsoid at a point and the plane of the equator. The Geodetic height is the distance along the normal to the reference ellipsoid between a point and the surface of this ellipsoid (Figure 1) (Soler *et al*., 2012).

Natural (or astronomic) coordinates

Natural coordinates are strictly based on the physical properties of the earth instead of on an ellipsoid and are unique at each point. They include the Astronomic longitude and latitude directly determined from observations of celestial bodies and so are referred to the instantaneous earth rotation axis and equator. The normal to the ellipsoid in this case is replaced by the direction of the plumb line. In addition, an Orthometric Height (H) is loosely referred to as elevation or mean sea level height and is derived from levelling observations and gravity data. The height is the distance along the plumb line from P to the geoid. This third natural coordinate may be replaced by the value of the geopotential at P. The geoid is the equipotential surface of the earth gravity field that best approximates mean sea level. Figure 2 shows difference between astronomic and geodetic latitudes (Soler *et al*., 2014).

Projected or grid coordinates

A projected coordinate system is a flat, two-dimensional representation of the Earth based on linear units of measure for coordinates. The latitude and longitude coordinates are converted to Easting and Northing coordinates on the flat projection. The intersection of the Easting and Northing axes is the origin. Usually, the Northing axis coincides with the central meridian. Mathematical formulas are used to convert the three-dimensional geographic coordinates to the two-dimensional flat projected coordinate system. This transformation is referred to as a map projection.

Sometimes large positive values are added to the origin coordinates to avoid negative values for the coordinates. These are called false origin values. The origin is on the earth's surface instead of earth centre so this system is topo-centric (Figure 1) (Maling, 2013).

Theoretical relationship between astronomic coordinates and geodetic coordinates

Pierre Simon, Marquis de Laplace (1749-1827), has defined a mathematical relationship between astronomic and geodetic coordinates as (Ayer *et al*., 2022; Soler *et al*., 2014; Featherstone and Rüeger, 2000):

$$\sin\eta = \sin(\Lambda_a - \lambda_G)\cos\phi_{G.} \qquad (2)$$

which simplifies for small angles to:

$$\eta = (\Lambda_a - \lambda_G)\cos\phi_G \qquad (3)$$

Also:

$$\sin\Phi_G = \cos\eta\sin(\Phi_A - \xi) \qquad (4)$$

which simplifies to:

$$\sin\phi_G = \sin(\phi_A - \xi) \qquad (5)$$

From equations (3) and (4) the following relationships between astronomic and geodetic latitude and astronomic and geodetic longitude respectively are:

$$\phi_G = \Phi_A - \xi \qquad (6)$$

$$\lambda_G = \Lambda_a - \eta\sec\phi_G \qquad (7)$$

The quantity gives ξ the north/south component of the deflection-of-the-vertical and η is an east/west component of the deflection of the vertical. The usefulness of these verti-

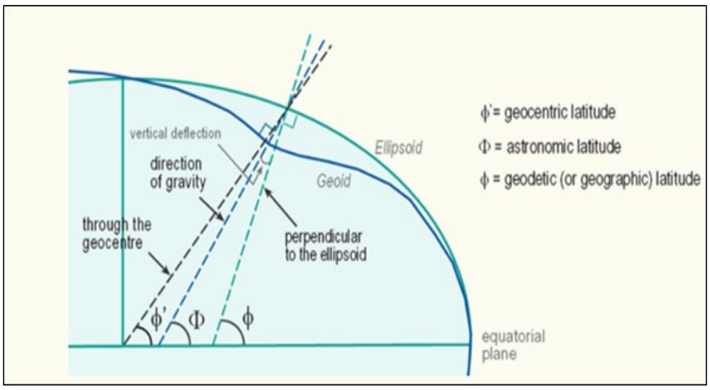

Figure 2 Three different latitudes: the geodetic, the astronomic and the geocentric latitudes (Soler *et al*., 2014)

cal deflections is in several areas of applicability such as:

i. Transforming geodetic coordinates into astronomical coordinates and vice versa,
ii. Determining local geoids,
iii. Transforming ellipsoid heights to orthometric heights, and
iv. Gravity anomaly studies.

Theoretical relationship between astronomic and geodetic azimuths

The difference between the astronomic and geodetic azimuths is given by Soler *et al*., (2014) and Tomoiaga *et al*., (2008) as:

$$\alpha_A - \alpha_G = \eta \tan \phi_A \qquad (8)$$

But

$$\eta = (\Lambda - \lambda) \cdot cos\phi_G \qquad (9)$$

Therefore, on substitution, it gives:

$$\alpha_A - \alpha_G = (\lambda_A - \lambda_G) \cos \phi_G \frac{\sin \phi_A}{\cos \phi_A} \qquad (10)$$

Since , $\cos \emptyset_G \approx \cos \emptyset_A$ it may be written as:

$$\alpha_A - \alpha_G = (\lambda_A - \lambda_G) \sin \phi_A \qquad (11)$$

Therefore, the basic relationship for astronomic to geodetic coordinate and azimuth given by Soler *et al*. (2014) as:

$$\begin{bmatrix} \eta \\ \xi \\ \alpha \end{bmatrix} = \begin{Bmatrix} (\Lambda - \lambda) \cos \phi_G \\ \Phi_A - \phi_G \\ \mathrm{A} - (\Lambda - \lambda) \sin \phi_G \end{Bmatrix} \qquad (12)$$

Methodology

The Ghana coordinate framework is already supplemented by eight key Laplace Stations. These have both astronomic and geodetic coordinates determined for them (Table 1) (Annan *et al*., 2016). The Laplace stations are situated in the southern sector of the country. Laplace stations are geodetic stations at which astronomic observations have been made to be used for orienting geodetic networks. The geodetic coordinates based on the World Geodetic System 1984 (WGS84) ellipsoid were obtained through GPS observations. The astronomic latitudes and longitudes were obtained by taking altitude observations to east and west pairs of stars at the stations. The astronomic azimuths were computed from the astronomic coordinates whereas the geodetic azimuths were obtained from the geodetic coordinates. Equations 11 and 12 were used to compute deflection of the vertical components from the astronomic and geodetic coordinates and azimuths from several conjugate point pairs forming different baselines.

Results and Analysis

Table 2 shows the differences between astronomic coordinates and geodetic coordinates. The absolute differences range from 0.20 seconds up to 16.60 seconds in latitudes and from 0.27 seconds up to 12.98 seconds in the longitudes. The latitude differences constitute the North/South component (ξ) of the deflection of the vertical (Equation 6). The East/West component of the deflection of the vertical (η) (Equation 7) is also shown in table 2 as well as the resultant deflection of vertical (ϴ). The resultant deflection of vertical values reflects the non-parallelism of the astronomic and geodetic axis and hence the fact that, a transformation is needed to the astronomical coordinates before they can be used interchangeably with geodetic values for azimuth and coordinate determinations.

The simple Laplace equation for azimuths, equation 11 and equation 12 is used to compute astronomic to geodetic azimuth correction factors. The results shown in Table 3 reveals that, due to the very low latitudes for Ghana, the corrections as given by equations 11 and 12 are very insignificant ranging from 0° 0′ 0.03" to 0° 0′ 1.19". Perhaps it is for this reason that this correction is not applied to the astronomical values when they are used to control coordinates in Ghana. Nevertheless, as revealed by the comparison of the astronomic and geodetic azimuths (Table 3), there are significant differences between the geodetic and astronomical azimuths constituting an orientation error. These errors range from absolute values of 5.95" to 00° 23′ 06.93". which is not modelled by the Laplace azimuth correction (Equation 11).

The Ghana Coordinate System has been based on a single astronomic position used to orient the datum at Accra Governors lodge. This could mean the geoid and ellipsoid are coincidence only at that point. This type of orientation would make deflections to occur at other positions of the network as re-

Table 1 Test point astronomic and geodetic coordinates

Stn Name	Astronomic Coordinates		Geodetic (WGS84) Coordinates	
	Latitude	Longitude	Latitude	Longitude
ACCRA (GOV, LODGE)	005 34 37.43	000 10 27.45W	005 34 47.52	000 10 28.10 W
AKUSE WEST END BASE (CFP215)	006 07 39.30	000 00 26.00 E	006 07 39.84	000 00 30.62 E
KUMASI PILLAR E4	006 42 04.50	001 37 20.70 W	006 42 05.14	001 37 24.48 W
OBUASI NORTH END BASE CFP193)	006 12 13.20	001 41 33.80 W	006 12 13.40	001 41 29.39 W
APAM (GCS102)	005 16 41.10	000 44 00.60 W	005 16 57.70	000 44 03.85 W
ODA NTS2	005 55 20.40	000 59 43.90 W	005 55 23.43	000 59 43.63 W
NSUTA (CFP242)	005 16 22.50	001 58 35.70 W	005 16 29.01	001 58 22.72 W
LEGON (GCS 121)	005 38 54.39	000 11 52.65 W	005 39 02.52	000 11 45.05 W

Source: Ghana Survey and Mapping Records (legacy data) vealed in Table 3. As evident in the Table 3, these orientation errors are dependent not only in the distance from the Accra Base but could probably be the results of magnetic and gravimetric anomalies.

In order to reduce the orientation errors throughout the network, it is required to make a correction to the initial coordinate at the origin such that the sum of the squares of the Astro-geodetic deflections at all other points is reduced to a minimum through a least square adjustment process. This can be achieved by using one of the Laplace stations in the network as an arbitrary origin. Then, instead of a zero deflection at the origin as with the single astronomic position orientation, there is a deflection at the origin. However, this correction was not done to the Ghanaian coordinate network.

Perhaps as suggested in Soler *et al*. (2014), the transformation between astronomic and geodetic coordinates may be better achieved through the application of Helmert's 3D model. This would have included a geoid-ellipsoid separation at the origin. Also, Abbey and Featherstone (2020) made a comparative study on the various models that could be of help when using the Helmert's 3D model. This is recommended, however, for a future investigation.

Conclusion

Astronomic and Geodetic Azimuths compared for twenty-eight different baselines between Laplace stations in Ghana show differences of 000°00'05.95" up to 000°23'06.93". These differences are not accountable in the error budget for the astronomical observations. Again because of the low latitudes of observation, Laplace corrections are negligible and cannot accommodate the observed differences. This finding confirms that the Laplace stations have not been involved in a least square solution to re-orient the Ghana datum ellipsoid. Consequently, errors in azimuth and position are introduced as evident from our findings.

This study thus revealed significant differences between geodetic and astronomical azimuths resulting to orientation error. It is therefore recommended that, the Laplace equations be introduced into the triangulation adjustments to control azimuths and orient the Ghana ellipsoid. It is further recommended that the transformation between astronomic and geodetic coordinates should be further investigated through the application of a Helmert's transformation model. The results would then reveal the effect on the astronomic and geodetic coordinates in Ghana.

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Conflict of Interest Declarations

The authors have no conflict of interest to declare.

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Table 2 Latitude and longitude differences between Astronomic and Geodetic Coordinates

Station Name	Astronomic		Geodetic (WGS84)		Latitude Differences (Astro-Geod)= ξ	Longitude differences (Astro-Geod)	η	Resultant deflection of the vertical Θ
	Latitude	Longitude	Latitude	Longitude	(Seconds)	(Seconds)	(seconds)	(seconds)
ACCRA (GOV LODGE)	005 34 37.43	000 10 27.45 W	005 34 47.52	000 10 28.10 W	-10.09	+0.65	0.65	10.11
AKUSE WEST END BASE (CFP215)	006 07 39.30	000 00 26.00 E	006 07 39.84	000 00 30.62 E	-0.54	-4.62	-4.59	4.62
KUMASI PILLAR E4	006 42 04.50	001 37 20.70 W	006 42 05.14	001 37 24.48 W	-0.64	+3.78	3.75	3.80
OBUASI NORTH END BASE(CFP193)	006 12 13.20	001 41 33.80 W	006 12 13.40	001 41 29.39 W	-0.20	-4.41	-4.38	4.38
APAM (GCS102)	005 16 41.10	000 44 00.60 W	005 16 57.70	000 44 03.85 W	-16.60	+3.25	3.24	16.91
ODA NTS2	005 55 20.40	000 59 43.90 W	005 55 23.43	000 59 43.63 W	-3.03	-0.27	-0.27	3.04
NSUTA (CFP242)	005 16 22.50	001 58 35.70 W	005 16 29.01	001 58 22.72 W	-6.51	-12.98	-12.92	14.47
LEGON (GCS 121)	005 38 54.39	000 11 52.65 W	005 39 02.52	000 11 45.05 W	-8.13	-7.60	-7.56	11.10

Table 3 Astronomic and Geodetic Azimuth Conversions

From	To	Raw Astro-nomic Azimuth	(Λ-λ) Sinϕ	Corrected Astro Azimuth	Geodetic Azimuth	Orientation error
ACCRA (GOVERNOR'S LODGE T.P)	AKUSE WEST END BASE (CFP 215)	18°15'47.40"	0° 0′ 0.06"	18°15'47.34"	018°29'00.64"	000°13'13.24"
ACCRA (GOVERNOR'S LODGE T.P)	KUMASI PILLAR E4	307°52'21.08"	0° 0′ 0.06"	307°52'21.02"	307°47'30.60"	000°04'50.48"
ACCRA (GOVERNOR'S LODGE T.P)	OBUASI NORTH END BASE	292°28'24.73"	0° 0′ 0.06"	292°28'24.67"	292°25'05.90"	000°03'18.83"
ACCRA (GOVERNOR'S LODGE T.P)	APAM (GCS102)	241°56' 49.2"	0° 0′ 0.06"	241°56' 49.14"	242°07'18.08"	-00°10'28.88"
ACCRA (GOVERNOR'S LODGE T.P)	ODA (NTS 2)	292°48'39.87"	0° 0′ 0.06"	292°48'39.81"	292°42'04.04"	000°06'35.83"
ACCRA (GOVERNOR'S LODGE T.P)	NSUTA (CFP242)	260°31'39.38"	0° 0′ 0.06"	260°31'39.32"	260°28'37.13"	000°03'02.25"
AKUSE WEST END BASE (CFP 215)	KUMASI PILLAR E4	289°28'32.38"	-0° 0′ 0.49"	289°28'32.87"	289°27'07.21"	000°01' 25.17"
AKUSE WEST END BASE (CFP 215)	OBUASI NORTH END BASE	272°39'05.77"	-0° 0′ 0.49"	272°39'06.26"	272°38'59.82"	000°00'05.95"
AKUSE WEST END BASE (CFP 215)	APAM (GCS102)	221°10'20.02"	-0° 0′ 0.49"	221°10'20.51"	221°24'19.90"	-00°13' 59.88"
AKUSE WEST END BASE (CFP 215)	ODA (NTS 2)	258°29' 50.72"	-0° 0′ 0.49"	258°29'51.21"	258°32'56.16"	-00°03'05.44"
AKUSE WEST END BASE (CFP 215)	NSUTA (CFP242)	246°49'43.04"	-0° 0′ 0.49"	246°49'43.53"	246°50'40.24"	-00°00'57.20"
AKUSE WEST END BASE (CFP 215)	LEGON	203°13'14.09"	-0° 0′ 0.49"	203°13'14.58"	203°13'32.40"	-00°00'18.31"
KUMASI PILLAR E4	OBUASI NORTH END BASE	188° 02'56.18"	0° 0′ 0.44"	188°02'55.74"	187°39'49.25"	000°23'06.93"
KUMASI PILLAR E5	APAM (GCS102)	147°55'57.83"	0° 0′ 0.44"	147°55'57.39"	147°50'55.25"	000°05'02.58"
KUMASI PILLAR E6	ODA (NTS 2)	141°07'14.28"	0° 0′ 0.44"	141°07'13.84"	141°02'51.34"	000°04'22.94"
KUMASI PILLAR E7	NSUTA (CFP242)	193°57'39.30"	0° 0′ 0.44"	193°57'38.86"	193°47'57.33"	000°09'41.97"
KUMASI PILLAR E8	LEGON (GCS 121)	126°21'58.56"	0° 0′ 0.44"	126°21'58.12"	126°15'12.57"	000°06'45.99"
OBUASI NORTH END BASE	APAM (GCS102)	133°52'55.72"	-0° 0′ 0.48"	133°52'56.20"	133°50'16.97"	000°02'38.75"
OBUASI NORTH END BASE	ODA (NTS 2)	111°55'03.17"	-0° 0′ 0.48"	111°55'03.65"	111°55'36.25"	-00°00'33.08"
OBUASI NORTH END BASE	NSUTA (CFP242)	197°00'04.05"	-0° 0′ 0.48"	197°00'04.53"	196°57'34.26"	000°02'29.79"
OBUASI NORTH END BASE	LEGON (GCS 121)	110°16'29.92"	-0° 0′ 0.48"	110°16'30.40"	110°12'19.75"	000°04'10.17"
APAM (GCS102)	ODA (NTS 2)	337°50'34.14"	0° 0′ 0.30"	337°50'33.84"	337°48'00.86"	000° 02'33.28"
APAM (GCS102)	NSUTA (CFP242)	269°49'10.54"	0° 0′ 0.30"	269°49'10.24"	269°41'21.39"	000°07'49.15"
APAM (GCS102)	LEGON (GCS 121)	055°21'53.94"	0° 0′ 0.30"	055°21'53.64"	055°41'20.52"	-00°19'26.58"
ODA (NTS 2)	NSUTA (CFP242)	236°35'42.60"	-0° 0′ 0.03"	236°35'42.63"	236°32'21.39"	000°03'20.71"
ODA (NTS 2)	LEGON (GCS 121)	108°53'06.89"	-0° 0′ 0.03"	108°53'06.92"	108°45'10.91"	000°07'55.98"
NSUTA (CFP242)	LEGON (GCS 121)	078°01'07.71"	-0° 0′ 1.19"	078°01'08.90"	077°59'48.86"	000°01'18.85"

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