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Research Article

RUIN PROBABILITY OF INSURANCE COMPANIES IN NIGERIA USING MODEL WITH ECONOMIC ENVIRONMENT: PRE AND POST CAPITALIZATION ANALYSIS

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ABSTRACT

The behavior of the reserve of insurance firms in Nigeria from 1996 to 2011 is investigated using ruin model with economic environment. The investment portfolios are classified according to the types of returns expected: Investments with fixed returns and investments with stochastic returns. Against the usual ways of monitoring the performances of insurance companies using regression and correlation, a risk reserve model in economic environment is used. The ruin probability is determined from the integro-differential equation for the model. The results show that there has been a positive growth in the reserve before recapitalization, though the rate of growth of the reserve after recapitalization is higher. The total probability of ruin shows that there is a drop in the probability of ruin after recapitalization. Also, the ruin probabilities show that companies are more liable to get ruined from investment than from claims after the recapitalization against the converse before recapitalization.

Keywords: Reserve, Stochastic returns, fixed returns, Ruin probability, recapitalization.

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1. INTRODUCTION

Nigeria insurance companies have undergone two rounds of recapitalization. Initially, the capital base was increased in line with the Insurance Act 2003 [1]. In 2005, the industry embarked on the second round of recapitalization with a view to strengthening them further. The companies were requested to increase their capital base from N150 million to N2 billion for Life Insurance businesses, N200 million to N3 billion for Non-Life Insurance and N350 million to N10 billion for the Re-Insurance companies. This resulted in a lot of liquidation and merger between some companies as the number of insurance companies reduced from over 100 to 49 at the end of the exercise in 2007. Some Insured were migrated from one company to another and some shareholders were subjected to a considerable slash in their shareholdings as a result of merger and acquisition. Many of the surviving companies whose scope where hitherto limited began to diversify their investment, yet a considerable number still rely on premium from insured for survival. Despite the recapitalization, the Nigerian insurance industry like most developing economy, presents an example of untapped and gross under-utilization of boundless potentials [2]. Yet, companies may go bankrupt if their assets and or investments are not properly managed.

In Nigeria, Just like in most countries, insurance companies constitute the next largest mobilizers of funds for investment after banks [3]. Primarily, insurance companies exist to reduce financial uncertainties as well as to provide protection against insurable risks at the insurance companies own risk [4]. The economic development of the country may be positively affected if these companies record remarkable improvements in their business activities. Fund mobilized by the companies; apart from payment of claims, salaries and other administrative cost; are basically used for investment purposes. These investments which includes real estates, loans, ordinary shares, debentures, bonds etc. [5] can generally be classified into two categories based on their return type: investments with fixed returns and investments with stochastic returns.

Basically, regression and correlation analyses are used in monitoring the performances of insurance companies with a view to forecast the profitability of the portfolios [6, 7]. Factors affecting the performances are first identified and subsequently regressed but this approach does not allow for a measure of survivability and are mostly limited to the effect of the identified factors. Several models have been proposed in risk reserve analysis which measure the ruin\survival probabilities of these companies and independent of determining factors which are not often identified entirely in most cases. These models are structured in such a way as to allow for different types of investments by the companies. A model which considers investments into risky and riskless assets was considered by Oseni and Jolayemi [8]. The risky investments was assumed to follow the geometric Brownian motion. Another model discrete time ruin model which considers Takaful (Islamic Insurance) with investment and Qard-Hasan (Benevolent Loan) activities was considered by Puspita, Kolkiewicz and Tan [9]. This paper considers the performances of the Nigeria insurance companies in the periods before and after the most recent recapitalization using the model proposed by Oseni and Jolayemi [8]

2. THE MODEL AND THEORETICAL RESULTS

Consider the basic insurance model, where the company's only source of income is the premium and claims are paid collectively at a specific epoch of time. Then the reserve process is given by [10-13],

$$Y(t) = u + \pi(t) - \sum_{i=1}^{N(t)} X_i \tag{1}$$

where u is the initial capital and $\pi(t)$ is the income process which, in its simplest form, represents the premium collected over the period (0,t]. $X_i (i = 1,..., N(t))$ are independent identically distributed random variables representing the claims sizes over the period $(0,t]$ and $N(t)$ is a Poisson process with rate λ denoting the number of claims over the period.

Since the funds mobilized by insurance companies are often channeled towards investment which could be risky or riskless, model (1) has been modified by many authors [14-17] to include the investments. If insurance company invest a fraction of its reserve on assets with fixed income and another fraction on assets with returns that follow geometric Brownian motion, then the instantaneous income process will be,

$$d\pi(t) = cdt + r(1-\varphi)Y(t)dt + \varphi Y(t)\left(adt + \sigma_R dB(t)\right) \tag{2}$$

where c is the premium rate, r is the fixed return rate, a is the drift rate, σ is the volatility, $B(t)$ is a standard Brownian motion process and φ is the fraction invested in assets with stochastic return.

Suppose, the premium and claim is affected by inflation and the level of inflation $I(t)$ is deterministic and given by the solution of the equation below [14].

$$\frac{dI(t)}{I(t)} = \delta dt; \qquad I(0) = 1 \tag{3}$$

Then, the reserve in model (1) can be modified to include both income process in equation (2) and inflation in (3) as given in the theorem below.

Theorem 1. Suppose an insurance company invest a fraction of its reserve in risky which follows geometric Brownian motion and the rest in riskless assets as given by equation (2). If the premiums and claims collected by the company is subjected to inflation, then the risk reserve of the company is

$$\begin{aligned}\tilde{Y}(t) &= e^{-\delta t}u + c\int_0^t e^{-\delta(t-s)}ds + r(1-\varphi)e^{-\delta t}\int_0^t Y(s)ds \\ &+ a\varphi e^{-\delta t}\int_0^t Y(s)ds + e^{-\delta t}\int_0^t (\sigma_p + \sigma_R \varphi Y(s))dB - \sum_{i=1}^{N(t)} e^{-\delta(t-T_i)}X_i\end{aligned} \tag{4}$$

where $\tilde{Y}(t)$ is the value of the reserve in terms of real units.

Proof: To allow for uncertainty in premiums, model (1) can be writing in the form

$$Y(t) = u + \pi(t) - \sum_{i=1}^{N(t)} X_i + \sigma_p dB \tag{5}$$

where $\sigma_p dB$ captures the possible fluctuation in premium income. Integrating equation (2) and substituting into model (5), then subjecting both the premiums and claims in the resulting equation to inflation gives

$$Y(t)=u+c\int_0^t I(s)ds+r(1-\varphi)\int_0^t Y(s)ds+a\varphi\int_0^t Y(s)ds + \int_0^t(\sigma_p+\sigma_R\varphi Y(s))dB-\sum_{i=1}^{N(t)} I(s)X_i \quad (6)$$

The reserve in terms of the real units at time t is given as [14],

$$\tilde{Y}(t)=e^{-\delta t}Y(t) \quad (7)$$

Substituting equation (6) into (7) in combination with equation (4) completes the proof.

Ruin occurs either as a result of claims exceeding the total income or as a result of loss from the investments. In both cases, the level of the reserve falls below zero. Thus the infinite-horizon ruin probability may be defined as

$$\psi(u)=P\left(\inf_{0\geq t\geq T} Y(t)<0\right)=P\left(\tau(u)<T\right) \quad (8)$$

Lemma 1. Given the risk reserve model expressed in nominal units and that expressed in real units, the survival probabilities for both models are same.

Proof: Recall the relationship between reserve expressed in nominal unit and reserve expressed in real units from equation (7) above. Also recall that the time of ruin for a reserve $Y(t)$ is defined as

$$\tau(u)=\inf\left\{t\geq 0: Y(t)<0\right\}. \quad (9)$$

Suppose $t>0$ and $Y(t)<0$ then $\tilde{Y}(t)<0$ and if $t=0$; $Y(t)=\tilde{Y}(t)=u$. Thus

$$\tau(u)=\inf\{t\geq 0: Y(t)<0\}\equiv\inf\{t\geq 0:\tilde{Y}(t)<0\} \quad (10)$$

Since the time of ruin of both models are equivalent, thus their probabilities of ruin are same.

Due to the obvious complexity and mathematical intractability of equation (4), it is preferable to work with equation (6). $Y(t)$ is a homogenous strong Markov process and can be transformed to the following integro-differential equation using Ito's formula.

$$(c+r(1-\varphi)u+a\varphi u)g'(u)+\frac{1}{2}\left(\sigma_p{}^2+(\sigma_R\varphi)^2u^2\right)g''(u)-\lambda g(u)+\lambda\int_0^\infty g(u-y)dF(y)=0 \quad (11)$$

The following theorem due to Paulsen and Gjessing [15] establishes that $g(u)$ is the ruin probability under certain conditions.

Theorem 2. Assume that $g(u)$ is bounded twice continuously differentiable on $u \geq 0$ with a bounded first derivative there and satisfy the boundary conditions

$$g(u) = 1 \quad \text{on} \quad u < 0,$$

$$\lim_{y\to\infty} g(y) = 0$$

If $g(u)$ solves the equation, $Ag(u) = 0$ on $u > 0$, then $g(u)$ is the probability of ruin.

Proof: See Paulsen and Gjessing [15].

Equation (11) can be converted to a differential equation by eliminating the integral part through differentiation. Assuming the claim size is distributed as exponential, that is

$$F(y) = 1 - e^{-\theta y}. \tag{12}$$

Thus equation (11) becomes

$$\begin{aligned}\frac{1}{2}\left(\sigma_p{}^2 + (\sigma_R\varphi)^2u^2\right)\psi'''(u) + \left(c + [(\sigma_R\varphi)^2 + \overline{r}]u + \frac{1}{2}\theta[\sigma_p{}^2 + (\sigma_R\varphi)^2u^2]\right)\psi''(u) \\ + \left(\overline{r} - \lambda + c\theta + \overline{r}\theta u\right)\psi'(u) = 0\end{aligned} \tag{13}$$

where $\overline{r} = r(1-\varphi)u + a\varphi u$, $\psi(u) = 0$ on $u < 0$, $\lim_{u\to\infty}\psi(u) = 0$ and

$\psi'(0) = \frac{\lambda}{c}\psi'(0)$.

It is noted that ruin can be caused either by claims or by oscillations, thus the equation can be decomposed into two parts by setting some parameters to zero. For ruin due to claims, $\sigma_R = \sigma_p = 0$ is set to zero and equation (13) becomes

$$(c + \overline{r})\psi''(u) + \left(\overline{r} - \lambda + c\theta + \overline{r}\theta u\right)\psi'(u) = 0 \tag{14}$$

and for ruin due to oscillation, $\sigma_R = \lambda = 0$ is set to zero in equation (14) resulting to

$$\frac{1}{2}\sigma_p{}^2\psi'''(u)+\left(c+\bar{r}u+\frac{1}{2}\theta\sigma_p{}^2\right)\psi''(u)+\left(\bar{r}-\lambda+c\theta+\bar{r}\theta u\right)\ \psi'(u)=0 \qquad (15)$$

Using theorem 2 of Paulsen and Gjessing [15], equation (15) can be solved to obtain

$$\begin{aligned}\psi(u)=&C_1\exp((-u\bar{r}+c)\theta/\bar{r})U\left(1-\frac{\lambda}{\bar{r}},1-\frac{\lambda}{\bar{r}};\frac{(u\bar{r}+c)\theta}{\bar{r}}\right)\\&+C_2\left(\frac{(u\bar{r}+c)\theta}{\bar{r}}\right)^{\lambda/\bar{r}}\exp((-u\bar{r}+c)\theta/\bar{r})M\left(1,1+\frac{\lambda}{\bar{r}};\frac{(u\bar{r}+c)\theta}{\bar{r}}\right)\end{aligned} \qquad (16)$$

where $C_1=\frac{-(c\mathrm{H}_{21}+(r-\lambda-c\theta)H_{11})e^{-c\theta/\bar{r}}}{K}$ and $C_2=\frac{(c\mathrm{H}_{22}+(r-\lambda-c\theta)H_{12})e^{-c\theta/\bar{r}}}{K}$

and

$$K=\left(cH_{22}+(r-\lambda-c\theta)H_{12}\right)U\left(1-\frac{\lambda}{\bar{r}},1-\frac{\lambda}{\bar{r}};\frac{c\theta}{\bar{r}}\right)-\left(cH_{21}+(r-\lambda-c\theta)H_{11}\right)M\left(1,1+\frac{\lambda}{\bar{r}};\frac{c\theta}{\bar{r}}\right)$$

$$H_{11}=-\theta e^{-c\theta/\bar{r}}U\left(1-\frac{\lambda}{\bar{r}},1-\frac{\lambda}{\bar{r}};\frac{c\theta}{\bar{r}}\right)-\theta\left(1-\frac{\lambda}{\bar{r}}\right)\theta e^{-c\theta/\bar{r}}U\left(2-\frac{\lambda}{\bar{r}},2-\frac{\lambda}{\bar{r}};\frac{c\theta}{\bar{r}}\right)$$

$$\begin{aligned}H_{12}=&\frac{\theta\lambda}{\bar{r}}\left(\frac{c\theta}{\bar{r}}\right)^{\lambda/\bar{r}-1}e^{-c\theta/\bar{r}}M\left(1,1+\frac{\lambda}{\bar{r}};\frac{c\theta}{\bar{r}}\right)-\theta\left(\frac{c\theta}{\bar{r}}\right)^{\lambda/\bar{r}}e^{-c\theta/\bar{r}}M\left(1,1+\frac{\lambda}{\bar{r}};\frac{c\theta}{\bar{r}}\right)\\&+\theta\left(\frac{c\theta}{\bar{r}}\right)^{\lambda/\bar{r}}\left(\frac{\lambda+\bar{r}}{\bar{r}}\right)e^{-c\theta/\bar{r}}M\left(2,2+\frac{\lambda}{\bar{r}};\frac{c\theta}{\bar{r}}\right)\end{aligned}$$

$$\begin{aligned}H_{21}=&\theta^2e^{-c\theta/\bar{r}}U\left(1-\frac{\lambda}{\bar{r}},1-\frac{\lambda}{\bar{r}};\frac{c\theta}{\bar{r}}\right)-2\theta^2\left(1-\frac{\lambda}{\bar{r}}\right)e^{-c\theta/\bar{r}}U\left(2-\frac{\lambda}{\bar{r}},2-\frac{\lambda}{\bar{r}};\frac{c\theta}{\bar{r}}\right)\\&-\theta^2\left(1-\frac{\lambda}{\bar{r}}\right)\left(2-\frac{\lambda}{\bar{r}}\right)e^{-c\theta/\bar{r}}U\left(3-\frac{\lambda}{\bar{r}},3-\frac{\lambda}{\bar{r}};\frac{c\theta}{\bar{r}}\right)\end{aligned}$$

$$\begin{aligned}H_{22}=&\frac{\theta^2\lambda}{\bar{r}}\left(\frac{\lambda}{\bar{r}}-1\right)\left(\frac{c\theta}{\bar{r}}\right)^{\lambda/\bar{r}-2}e^{-c\theta/\bar{r}}M\left(1,1+\frac{\lambda}{\bar{r}};\frac{c\theta}{\bar{r}}\right)-2\theta^2\frac{\lambda}{\bar{r}}\left(\frac{c\theta}{\bar{r}}\right)^{\lambda/\bar{r}-1}e^{-c\theta/\bar{r}}M\left(1,1+\frac{\lambda}{\bar{r}};\frac{c\theta}{\bar{r}}\right)\\&-2\theta^2\frac{\lambda}{\bar{r}}\left(\frac{c\theta}{\bar{r}}\right)^{\lambda/\bar{r}-1}\left(\frac{\lambda+\bar{r}}{\bar{r}}\right)e^{-c\theta/\bar{r}}M\left(2,2+\frac{\lambda}{\bar{r}};\frac{c\theta}{\bar{r}}\right)+\theta^2\left(\frac{c\theta}{\bar{r}}\right)^{\lambda/\bar{r}}e^{-c\theta/\bar{r}}M\left(1,1+\frac{\lambda}{\bar{r}};\frac{c\theta}{\bar{r}}\right)\\&-2\theta^2\left(\frac{c\theta}{\bar{r}}\right)^{\lambda/\bar{r}-1}\left(\frac{\lambda+\bar{r}}{\bar{r}}\right)e^{-c\theta/\bar{r}}M\left(2,2+\frac{\lambda}{\bar{r}};\frac{c\theta}{\bar{r}}\right)\\&+\theta^2\left(\frac{c\theta}{\bar{r}}\right)^{\lambda/\bar{r}}\left(\frac{\lambda+\bar{r}}{\bar{r}}\right)\left(\frac{\lambda+2\bar{r}}{2\bar{r}}\right)e^{-c\theta/\bar{r}}M\left(3,3+\frac{\lambda}{\bar{r}};\frac{c\theta}{\bar{r}}\right)\end{aligned}$$

And equation (16) can be solved to obtain

$$\psi(u)=\frac{H_2(u)\left(cH_1''(0)+\bar{r}H_1'(0)\right)-H_1(u)\left(cH_2''(0)+\bar{r}H_2'(0)\right)}{H_2(0)\left(cH_1''(0)+\bar{r}H_1'(0)\right)-H_1(0)\left(cH_2''(0)+\bar{r}H_2'(0)\right)} \quad (18)$$

where $H_1(u)=\int_u^\infty (x+\bar{a})\exp\left[-\left(\theta x+\bar{r}(x+\bar{a}^2)/\sigma_p{}^2\right)\right]M\left(\frac{1}{2},\frac{3}{2};\frac{\bar{r}(x+\bar{a})^2}{\sigma_p{}^2}\right)dx$

$$H_2(u)=\int_u^\infty \exp\left[-\left(\theta x+\bar{r}(x+\bar{a}^2)/\sigma_p{}^2\right)\right]U\left(0,\frac{1}{2};\frac{\bar{r}(x+\bar{a})^2}{\sigma_p{}^2}\right)dx$$

$$H_1'(0)=\bar{a}e^{-\bar{r}\ \bar{a}/\sigma_p{}^2}M\left(\frac{1}{2},\frac{3}{2};\frac{\bar{r}\ \bar{a}^2}{\sigma_p{}^2}\right)$$

$$H_2'(0)=e^{-\bar{r}\ \bar{a}/\sigma_p{}^2}U\left(0,\frac{1}{2};\frac{\bar{r}\ \bar{a}^2}{\sigma_p{}^2}\right)$$

$$H_1''(0)=\bar{a}e^{-\bar{r}\ \bar{a}^2/\sigma_p{}^2}M\left(\frac{1}{2},\frac{3}{2};\frac{\bar{r}\ \bar{a}^2}{\sigma_p{}^2}\right)-\frac{2\bar{r}\ \bar{a}^2}{\sigma_p{}^2}e^{-\bar{r}\ \bar{a}^2/\sigma_p{}^2}M\left(\frac{1}{2},\frac{3}{2};\frac{\bar{r}\ \bar{a}^2}{\sigma_p{}^2}\right)$$
$$+\frac{2\bar{r}\ \bar{a}^2}{3\sigma_p{}^2}e^{-\bar{r}\ \bar{a}^2/\sigma_p{}^2}M\left(\frac{3}{2},\frac{5}{2};\frac{\bar{r}\ \bar{a}^2}{\sigma_p{}^2}\right)$$

$$H_2'(0)=\frac{\theta+2\bar{r}\ \bar{a}^2}{\sigma_p{}^2}e^{-\bar{r}\ \bar{a}^2/\sigma_p{}^2}U\left(0,\frac{1}{2};\frac{\bar{r}\ \bar{a}^2}{\sigma_p{}^2}\right)$$

3. RESULTS AND DISCUSSION

3.1. Data and Source

Aggregated income and expenditure of insurance firms (Non-life) in Nigeria from 1996-2011, extracted from the work of Ubom (2014), as shown in table 1. The data contain the premium and claim collected from 1996 - 2011.

The investments captured by the data are government security, stock and bonds, real estate and mortgage, other loans, cash deposits and bill of exchange from 1996 – 2011 (all in billions) as shown in table 2. These investments can be classified into investments with fixed return and investments with stochastic returns. Based on these, "bill of exchange", "cash deposits" and "other loans" are classified as investments with fixed returns while the other investments are classified as investments with stochastic returns.

Table 1: Income and Expenditures of Insurance Companies in Nigeria from 1996-2011

	Income ('000,000,000)			Expenditure ('000,000,000)			
Year	Premium	Others	Total	Claims	Others	Total	Balance
1996	11.09133	2.05923	13.15056	1.65407	4.26207	5.91614	7.23442
1997	10.94158	5.57744	16.51902	1.67728	4.82212	6.49940	10.01962
1998	11.68825	6.15822	17.84647	1.95621	5.21807	7.17428	10.67219
1999	14.59728	0.04658	14.64386	5.92318		5.92318	8.72068
2000	22.53146		22.53146	5.62952		5.62952	16.90194
2001	28.98129		28.98129	6.11052		6.11052	22.87077
2002	37.76589		37.76589	6.85615		6.85615	30.90975
2003	43.44181	0.50289	43.94470	9.41520		9.41520	34.52950
2004	50.10083	0.39505	50.49588	12.08408		12.08408	38.41180
2005	67.46556	0.28075	67.74631	12.40240		12.40240	55.34391
2006	81.58375	0.77814	82.36189	12.77447		12.77447	69.58742
2007	89.18489	1.06518	90.25007	25.13324		25.13324	65.11683
2008	107.22120	2.16127	109.38247	37.41255		37.41255	71.96992
2009	153.12712		153.12712	61.96915		61.96915	91.15797
2010	157.33681		157.33681	53.81535		53.81535	103.52146
2011	175.75675		175.75675	66.20476		66.20476	109.55199

Table 2: Income and Expenditures of Insurance Companies in Nigeria from 1996-2011

Year	Gov. Sec.	Stock & Bond	Real Estate & Mortgage	Policy &Loans	Cash Deposits	Bill of Exchange	Total
1996	1.54616	4.04781	2.52320	0.79593	3.34706	0.11930	12.37946
1997	2.01201	4.09538	2.68350	0.84311	3.81591	0.16417	13.61308
1998	4.14588	3.63317	0.21195	2.30122	1.99319	3.37147	15.65688
1999	2.98721	4.17404	0.33265	4.12447	4.18416	5.78093	21.58346
2000	3.55895	4.99287	0.28234	5.21208	3.84437	7.30203	25.19264
2001	3.84271	6.88626	0.35933	6.70640	4.28455	10.17802	32.25727
2002	3.75208	8.35085	0.96031	7.90101	4.09540	11.88122	36.94087
2003	3.55895	4.99287	0.28234	5.21208	3.84437	7.30202	25.19263
2004	8.70800	0.00000	0.35184	8.35651	2.66858	2.59387	22.67880
2005	4.17806	61.80082	33.78815	5.59070	10.18535	6.30114	121.84422
2006	4.85810	121.81313	45.18677	7.88473	30.31417	6.30301	216.35991
2007	20.91481	222.27892	45.33191	12.94582	22.50809	5.26778	329.24733
2008	21.37490	227.16910	46.32920	13.23060	23.00390	5.38370	336.49140
2009	21.84520	232.16680	47.34850	13.52170	23.51000	5.50210	343.89420
2010	22.32580	237.27440	48.39010	13.81920	24.02720	5.62320	351.45990
2011	22.81690	242.49450	49.45470	14.12320	24.55580	5.74690	359.19220

3.2. Parameter Estimation

In other to carry out the analysis of the data, there is a need to estimate the parameters of the model. This requires that the distribution of the claim size be known. The hypothesis below is formulated to examine if the data fits exponential distribution as specified by the model.

$H_0 : F_X(x) = F_0(x)$

$H_0 : F_X(x) \neq F_0(x)$ for all x (19)

where $F_X(x)$ is the unknown distribution of the data and $F_0(x)$ is completely specified as exponential with parameter θ. Figure 1 shows the distribution fit to the data using easyfit statistical software.

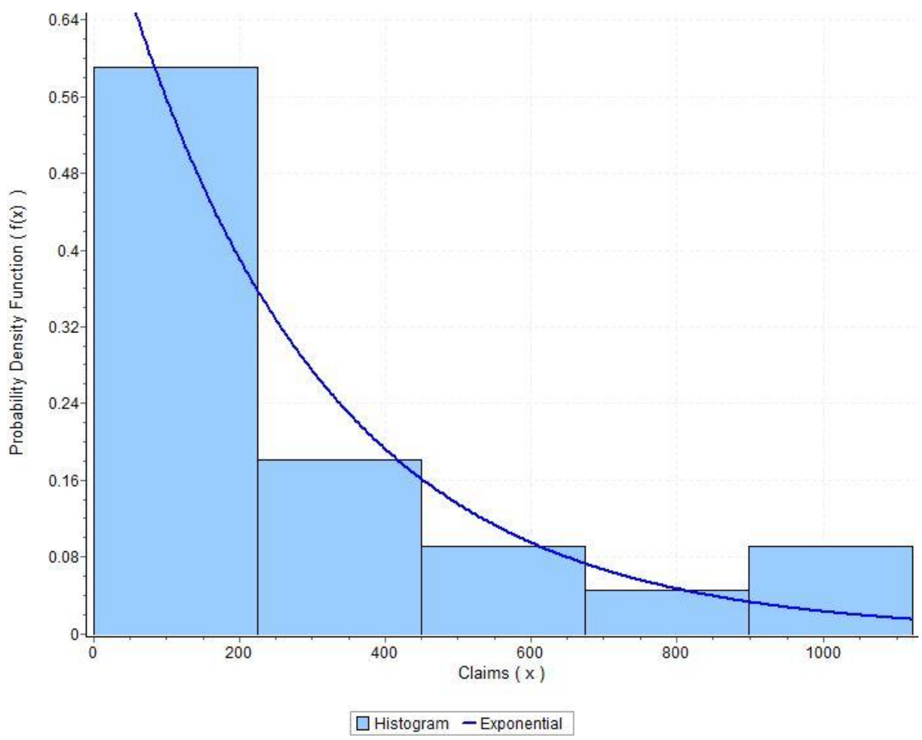

Fig.1. Distribution fit of the claim sizes

The plot show that the data can be fitted to exponential distribution with parameter $\theta = 0.6711$. A goodness of fit test on claim sizes using Kolmogorov-Smirnov test at 5% level of significance is shown in table 3.

Table 3: Kolmogorov – Smirnov goodness of fit test on claim size.

	Kolmogorov - Smirnov
Sample	
Size	22.00000
Statistic	0.24426
P-Value	0.12176

A p-value of 0.12176 implies that claim size is distributed as exponential with parameter θ = 0.6711. All the parameters of the model as estimated from the data in both tables 1 and 2 are given in table 4 below.

Table 4: Parameters of the model estimated from the dataset in table 1 and 2

Year	u	c	λ	θ	r	σ	a	φ
1996	9.2226	0.2264	4.1909	0.6711	0.1169	2.1602	0.5558	0.6557
1997	7.2344	0.2233	4.0771	0.6711	0.0480	2.4495	1.1546	0.6457
1998	10.0196	0.2385	3.7343	0.6711	0.0549	2.7386	1.6638	0.5104
1999	10.6722	0.2979	1.5403	0.6711	0.0533	3.0277	1.4269	0.3472
2000	8.7207	0.4598	2.5015	0.6711	0.0529	3.3166	1.2186	0.3507
2001	16.9019	0.5915	2.9643	0.6711	0.0549	3.6056	1.0202	0.3437
2002	22.8708	0.7707	3.4427	0.6711	0.0415	3.8944	0.8655	0.3536
2003	30.9097	0.8866	2.8838	0.6711	0.0411	4.1833	0.7915	0.3507
2004	34.5295	1.0225	2.5913	0.6711	0.0419	4.4721	0.7271	0.3995
2005	38.4118	1.3768	3.3998	0.6711	0.0383	4.7610	0.6463	0.8188
2006	55.3439	1.6650	3.9915	0.6711	0.0314	5.0498	0.5719	0.7943
2007	69.5874	1.8201	2.2178	0.6711	0.0355	5.3385	0.5191	0.8763
2008	65.1168	2.1882	1.7912	0.6711	0.0284	5.6273	0.5434	0.8763
2009	71.9699	3.1250	1.5444	0.6711	0.0268	5.9161	0.4593	0.8763
2010	91.1580	3.2110	1.8273	0.6711	0.0221	6.2048	0.3918	0.8763
2011	103.5215	3.5869	1.6592	0.6711	0.0143	6.4936	0.3451	0.8763

3.3. Results

Figure 2 show the behavior of the reserve over the years considered. From the figure, prior to year 2000, the reserve fluctuates between values smaller than 10.672 billion of 1999 (the values of the reserve for each of the years denoted as “u” are shown in table 4). An approximate linear growth in the reserve is observed between year 2000 and year 2005. From year 2005 to 2007 which is the recapitalization period, a further upwards growth is observed as insurance firms were allowed to float initial public offers of shares. After the recapitalization, merger and acquisitions of firms which were unable to meet up with the minimum share capital resulted in decrease of the reserve, before an upward trend was subsequently observed.

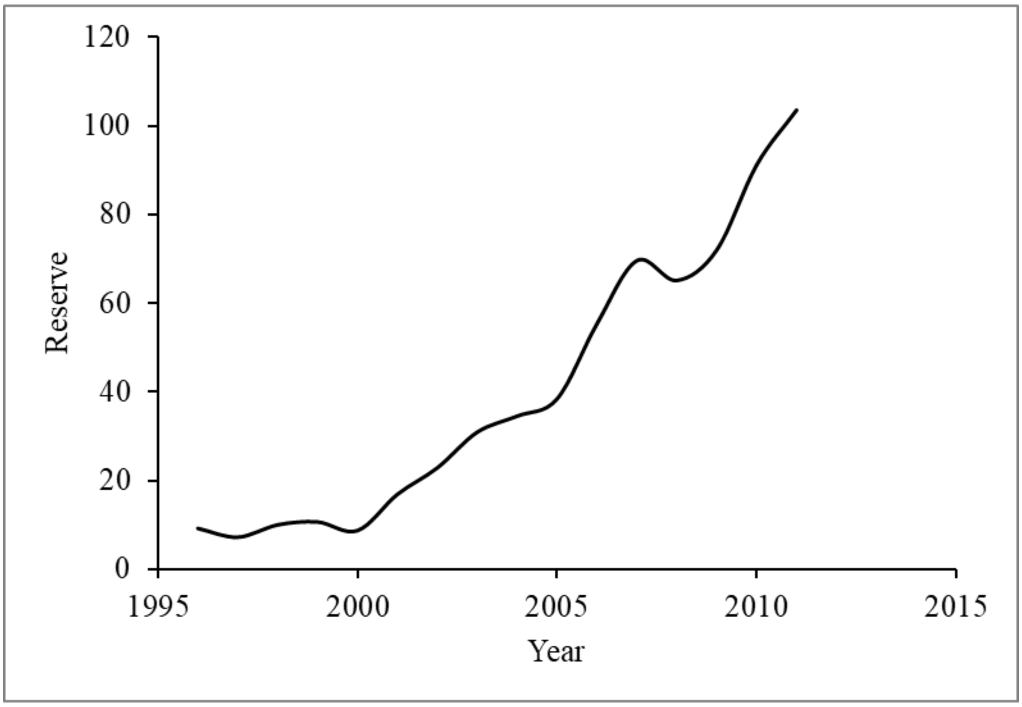

Fig.2. The reserve of Insurance firms in Nigeria from 1996-2011

The probabilities of ruin computed using the estimated parameters in table 4 are shown in table 5. Prior to the period of recapitalization, companies are more liable to be ruined from claims than investment as the total ruin probability is dominated by ruin probability attributable to claim. At the recapitalization period, ruin can almost equally result from claims and investments. Then after recapitalization, ruin due to investments dominates the total ruin probability. It is observed that heavy reliance on premium fund reduced as more companies began investing in different assets. In most cases, the ruin probability is approximately zero. This is an indication that insurance firms in Nigeria were far from being bankrupt before the recapitalization excess. Though, the sector was further strengthened by the recapitalization.

Table 5: Ruin probability for Insurance firms in Nigeria between 1996-2011

Year	ϕ_{claim}	ϕ_{Invest}	ϕ_{Total}
1996	4.227E-01	1.834E-04	4.229E-01
1997	3.347E-01	6.698E-04	3.353E-01
1998	7.789E-02	3.731E-06	7.789E-02
1999	8.114E-03	4.766E-04	8.590E-03
2000	7.330E-02	1.269E-02	8.599E-02
2001	7.699E-03	2.178E-05	7.721E-03
2002	1.538E-03	4.415E-07	1.539E-03
2003	1.181E-05	1.896E-09	1.182E-05
2004	2.958E-07	1.174E-08	3.075E-07
2005	1.244E-08	6.835E-12	1.244E-08
2006	6.517E-10	7.638E-10	1.416E-09
2007	1.590E-09	4.678E-09	6.268E-09
2008	6.542E-10	8.475E-09	9.129E-09
2009	1.095E-12	6.669E-08	6.669E-08
2010	2.473E-12	9.844E-07	9.844E-07
2011	1.127E-13	4.661E-06	4.661E-06

4. CONCLUSION

The basic model is the Crammer-Lundberg model while the investment part of the reserve was modelled using geometric Brownian motion process. The resulting integro-differential equation was converted to differential equation and spilt into two parts which captured ruin from claims and ruin from investments. The model was applied to an aggregated data of the Nigeria Insurance firm from 2006-2011. It was observed that the reserve of insurance firms in Nigeria was going through a positive growth before the recapitalization. The recapitalization exercise further increased the rate of growth of the reserve until around 2007 when it began to fall. Generally, there has been a positive growth in the reserve after recapitalization. A decrease in the ruin probability was also observed despite more funds being committed into risky investments.

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