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STRESS-STRAIN ANALYSIS OF THE SHELLS OF THE LONG OBLIQUE HELICOID FORM

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ABSTRACT

The paper discusses the issue of numeric-analytical analysis of the stress-strain state of thin elastic shell in the form of oblique helicoid. The stress-strain state of thin shells under static load is considered. The equation of surface in non-conjugate non-orthogonal coordinate system is used. The assumptions and simplifications of thin shell theory are used in two variants: for shallow and non-shallow shells. The one-dimentional problem is analyzed for quasisymmetric loading. The examples for steel and reinforced concrete shells are given. The results are compared to the results obtained by means of the finite element analysis. The results for some simple cases, obtained by the numeric-analytical method proposed, can be used as a sort of standard for controlling and comparing results, obtained by finite element analysis while designing more complex real objects.

Keywords: oblique helicoid; thin elastic shell, numeric-analytical method, stress-strain state analysis.

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1. INTRODUCTION

1.1. Introduction

The shells with middle surface of oblique helicoid form have wide application in technics[1]. The structural elements of buildings [2], elements of road transport works [3], hydraulic engineering works, machine elements can be qualified as these shells [4,5].

For the initial designing of such an objects and for verification of the results, obtained by complex numeric metodologies, the analytical approaches are very desireable. One of the most simple and transparent shell theory is the classical theory by A. Love [6] . Getting analytical solution is the strong gains for the tasks of such class. But only the small part of differential equation systems of shell theory have analytical solution, most of them can be solved only numerically.

1.2 The methodology

In the presented research the thin elastic shell theory is used in A.L. Goldenveizer's statement [7]. The basic equations system consists of three groups: strain-displacement relations, equilibrium equations and stress-strain relations (Hooke's law).

The parametric equation of the middle surface of the shell can be conceived of as:

$$x = u \cos v, \quad (1)$$

$$y = u \sin v,$$

$$z = k\,u + c\,v$$

k is generator slope ratio [8].

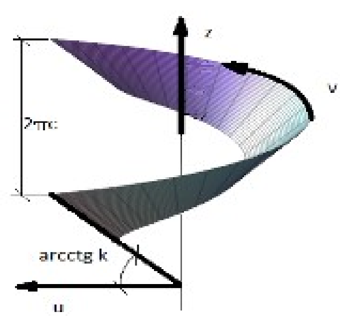

Fig.1. Oblique helicoid surface

This coordinate system is non-orthogonal and non-conjugate. The expressions of quadratic forms:

$$A=\sqrt{1+k^2},\ B=\sqrt{u^2+c^2},\ F=kc,\ L=0, M=-\frac{c}{\sqrt{A^2u^2+c^2}}, N=\frac{ku^2}{\sqrt{A^2u^2+c^2}}. \quad (2)$$

To simplify the task let us assume that the helicoid has many coils, and the stress-strain state doesn't depend on the boundary conditions along the straight borders, and, consequently, doesn't depend of v coordinate.

For the oblique helicoid case the strain-displacement relations are given by:

$$\varepsilon_u=\frac{\frac{\partial}{\partial u}u_u-\frac{k\,c\,u}{A\,(c^2+u^2)^{\frac{3}{2}}}+\frac{k\ c\ \frac{d}{du}u_v}{A\,(c^2+u^2)^{\frac{1}{2}}}}{A},\ \varepsilon_v=\frac{u\,u_u}{A\,(c^2+u^2)}+\frac{ku\sqrt{1+k^2}u_z}{\sqrt{A^2u^2+c^2}\,(c^2+u^2)},$$

$$\omega_u=\frac{\sqrt{1-\frac{k^2c^2}{A^2(c^2+u^2)}}\frac{d}{du}u_v}{A}-\frac{cu_z}{A\sqrt{u^2+c^2}\sqrt{A^2u^2+c^2}\sqrt{1-\frac{k^2c^2}{A^2(c^2+u^2)}}}+\frac{k^2c^2\,u\,u_v}{A^3(c^2+u^2)^2\sqrt{1-\frac{k^2c^2}{A^2(c^2+u^2)}}},$$

$$\omega_v=-\frac{u\,u_v}{\sqrt{u^2+c^2}\sqrt{A^2u^2+c^2}}-\frac{cu_z}{(u^2+c^2)}-\frac{c\ k\ u\ u_u}{A\,(u^2+c^2)\sqrt{A^2u^2+c^2}},$$

$$\varepsilon_{uv}=\omega=\omega_u+\omega_v,$$

$$\gamma_u=-\frac{\frac{d}{du}u_z}{A}-\frac{c\ u_v}{A\sqrt{u^2+c^2}\sqrt{A^2u^2+c^2}},$$

$$\gamma_v=-\frac{cu_u}{A\sqrt{u^2+c^2}\sqrt{A^2u^2+c^2}}+\frac{ku^2u_v}{(u^2+c^2)\sqrt{A^2u^2+c^2}},$$

$$\kappa_u=\frac{2ku^2c^2\left(\left(A^4+\frac{1}{4}\right)c^4+\frac{5}{4}A^2u^2\right)}{A^2(A^2u^2+c^2)^2(c^2+u^2)^{\frac{3}{2}}}u_u-\frac{u\ c\ \left(k^2+\frac{3}{2}\right)(A^6u^6+3A^4u^4c^2+3A^2u^2c^4+c^{10})}{(A^2u^2+c^2)^3(c^2+u^2)^2}u_v-$$

$$-\frac{c^2k^2u}{A(A^2u^2+c^2)^{\frac{3}{2}}(c^2+u^2)^{\frac{1}{2}}}\frac{\mathrm{d}}{\mathrm{du}}\mathrm{u_z}+\frac{c\left(A^2+\frac{1}{2}\right)}{A^3(c^2+u^2)}\frac{\mathrm{d}}{\mathrm{du}}\mathrm{u_v}-\frac{c^2k}{A^2(A^2u^2+c^2)(c^2+u^2)^{\frac{1}{2}}}\frac{\mathrm{d}}{\mathrm{du}}\mathrm{u_u}+$$

$$+\frac{(c^2+u^2)^{\frac{1}{2}}}{A(A^2u^2+c^2)^{\frac{1}{2}}}\frac{\mathrm{d}^2}{\mathrm{du}^2}\mathrm{u_u}+\frac{k^2u^2c^2}{2(A^2u^2+c^2)^{3/2}(c^2+u^2)^{\frac{3}{2}}}u_z,$$

$$\kappa_v=\frac{u\ \frac{d}{du}u_z}{A(A^2u^2+c^2)^{1/2}(c^2+u^2)^{\frac{1}{2}}}+\frac{c\ u\ u_v}{2A^3(c^2+u^2)^2}-\frac{u\ c^2k\ \ u_u}{2A^2(A^2u^2+c^2)(c^2+u^2)^{\frac{3}{2}}}-$$

$$-\frac{c\,\frac{d}{du}u_v}{2\,A^3(c^2+u^2)}-\frac{c^2u^2k^2u_z}{A(A^2u^2+c^2)^{3/2}(c^2+u^2)^{\frac{3}{2}}},$$

$$\kappa_{uv}=\frac{ck(A^2u^2+c^2)^{1/2}u_z}{(u^2+c^2)A^2}\quad\frac{c\,u\,u_u}{A(u^2+c^2)(A^2u^2+c^2)}+\frac{cuk(A^2u^2+c^2)^{\frac{1}{2}}\frac{d}{du}u_z}{(u^2+c^2)A^2}$$

$$-\frac{ku\left(-A^3u^8-3A^6u^6c^2-3A^2u^2c^4-A^2u^2c^6+\left(A^2+\frac{1}{2}\right)c^{12}-\left(A^2+\frac{1}{2}\right)c^8\right)u_v}{A^4(u^2+c^2)^{\frac{5}{2}}(A^2u^2+c^2)^3}+$$

$$+\frac{c\frac{d}{du}u_u}{(u^2+c^2)A^3}-\frac{u^2k\frac{d}{du}u_v}{(u^2+c^2)^{\frac{3}{2}}A^2}.$$

The stress-strain relations: (4)

$$N_u=\frac{Eh}{(1-\nu^2)}\left(\frac{\varepsilon_u-\varepsilon_{uv}ctg\chi+\nu\varepsilon_v}{sin\chi}\right),N_v=\frac{Eh}{(1-\nu^2)}\left(\frac{\varepsilon_v-\varepsilon_{uv}ctg\chi+\nu\varepsilon_u}{sin\chi}\right),$$

$$S_u=-S_v=\frac{Eh}{2(1-\nu^2)}\left(\left(\frac{1+cos^2\chi}{sin^2\chi}\varepsilon_{uv}-(\varepsilon_u-\varepsilon_v)ctg\chi\right)-\frac{Eh}{2(1-\nu^2)}\nu((\varepsilon_{uv}-(\varepsilon_u-\varepsilon_v)ctg\chi).$$

$$M_u=-\frac{Eh^3}{12(1-\nu^2)}\left(\frac{\kappa_u+\nu\kappa_v}{sin\chi}\right),M_v=-\frac{Eh^3}{12(1-\nu^2)}\left(\frac{\kappa_v+\nu\kappa_u}{sin\chi}\right),$$

$$M_{uv}=\frac{Eh^3}{12(1+\nu)}\left(\frac{\kappa_{uv}-cos\chi\,\kappa_v}{sin\chi}\right),M_{vu}=-\frac{Eh^3}{12(1+\nu)}\left(\frac{\kappa_{uv}+cos\chi\,\kappa_u}{sin\chi}\right).$$

(4)

The equilibrium equations in non-conjugate non-orthogonal coordinate system:

$$\frac{1}{sin\chi}\frac{\partial}{\partial u}(B(N_u+cos\chi S_u))-\frac{B^2}{A}\Gamma_{11}^2 sin\chi S_u-\frac{1}{sin\chi}\frac{\partial}{\partial v}(A(S_v-cos\chi N_v))-B\Gamma_{12}^2 sin\chi N_v-$$
$$-\frac{AB}{sin\chi}\left(\frac{Q_u}{R_u}-\frac{Q_v}{R_{uv}}\right)+AB(X+cos\chi Y)=0,$$

$$\frac{1}{sin\chi}\frac{\partial}{\partial u}(B(S_u+cos\chi N_u))-A\Gamma_{12}^1 sin\chi N_u+\frac{1}{sin\chi}\frac{\partial}{\partial v}(A(N_v-cos\chi S_v))+\frac{A^2}{B}\Gamma_{22}^1 sin\chi S_v-$$
$$-\frac{AB}{sin\chi}\left(\frac{Q_v}{R_v}-\frac{Q_u}{R_{uv}}\right)+AB(Y+cos\chi X)=0,$$

$$AB\left(\frac{N_u}{R_u}+\frac{N_v}{R_v}+\frac{S_v-S_u}{R_{uv}}\right)+\frac{\partial}{\partial u}(BQ_u)+\frac{\partial}{\partial v}(AQ_v)+ABsin\chi Z=0,$$

$$\frac{1}{sin\chi}\frac{\partial}{\partial u}(B(M_{uv} + \ cos\chi M_u)) - \frac{B^2}{A}\Gamma_{11}^2 sin\chi M_u - \frac{1}{sin\chi}\frac{\partial}{\partial v}(A(M_v - cos\chi M_{uv})) -$$

$$-B\Gamma_{12}^2 sin\chi M_{vu} + ABQ_v = 0,$$

$$\frac{1}{sin\chi}\frac{\partial}{\partial u}(B(M_u + \ cos\chi M_{uv})) - A\Gamma_{12}^1 sin\chi M_{uv} + \frac{1}{sin\chi}\frac{\partial}{\partial v}(A(M_{vu} - cos\chi M_v)) +$$

$$+\frac{A^2}{B}\Gamma_{22}^1 sin\chi M_v - ABQ_u = 0,$$

$$sin\chi(S_u + S_v) + \frac{M_{uv}}{R_u} + \frac{M_{vu}}{R_v} + \frac{M_v - M_u}{Ru_v} = 0, \quad (5)$$

Γ_{ij}^k - Christoffel's koefficients.

The outer forces vector resolves into components along the axis of loose component trihedral:

$$P = X\frac{\overline{r_u}}{A} + Y\frac{\overline{r_v}}{B} + Z\overline{n}, \quad (6)$$

Let us express the forces Q_u, Q_v from the fourths and fifths equilibrium equations and plug them into the first three equilibrium equations. So we can get three equations of the form:

$$k1_{u_u}u_u + k1_{u_v}u_v + k1_{u_z}u_z + k1_{du_u}\frac{d}{du}u_u + k1_{du_v}\frac{d}{du}u_v + k1_{du_z}\frac{d}{du}u_z +$$

$$+k1_{d2u_u}\frac{d^2}{du^2}u_u + k1_{d2u_v}\frac{d^2}{du^2}u_v + k1_{d2u_z}\frac{d^2}{du^2}u_z + k1_{d3u_z}\frac{d^3}{du^3}u_z = 0,$$

$$k2_{u_u}u_u + k2_{u_v}u_v + k2_{u_z}u_z + k2_{du_u}\frac{d}{du}u_u + k2_{du_v}\frac{d}{du}u_v + k2_{du_z}\frac{d}{du}u_z +$$

$$+k2_{d2u_u}\frac{d^2}{du^2}u_u + k2_{d2u_v}\frac{d^2}{du^2}u_v + k2_{d2u_z}\frac{d^2}{du^2}u_z + k2_{d3u_z}\frac{d^3}{du^3}u_z = 0,$$

$$k3_{u_u}u_u + k3_{u_v}u_v + k3_{u_z}u_z + k3_{du_u}\frac{d}{du}u_u + k3_{du_v}\frac{d}{du}u_v + k3_{du_z}\frac{d}{du}u_z +$$

$$+k3_{d2u_u}\frac{d^2}{du^2}u_u + k3_{d2u_v}\frac{d^2}{du^2}u_v + k3_{d2u_z}\frac{d^2}{du^2}u_z + k3_{d3u_u}\frac{d^3}{du^3}u_u +$$

$$+k3_{d3u_v}\frac{d^3}{du^3}u_v + k3_{d3u_z}\frac{d^3}{du^3}u_z + k3_{d4u_z}\frac{d^4}{du^4}u_z = 0. \quad (7)$$

The coefficients are not listed because of the huge volume.

It is necessary to differentiate two first equations to express $\frac{d^3}{du^3}u_v \frac{d^3}{du^3}u_u$, $\frac{d^2}{du^2}u_u$, $\frac{d^2}{du^2}u_v$ and plug into the third one.

The elementary transformations of equations and their coefficients were carried out by means of symbolic computation systems, such as Maple 16.

Finally we get the 8-order system of three equations, which can be transformed into normal first-order system of 8 equations:

$$y = \begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \end{bmatrix} = \begin{bmatrix} u_u \\ (u_u)' \\ u_v \\ (u_v)' \\ u_z \\ (u_z)' \\ (u_z)'' \\ (u_z)''' \end{bmatrix}$$

$$f(u, y_i) = \begin{bmatrix} f_0 \\ f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \\ f_7 \end{bmatrix} = \begin{bmatrix} y_1 \\ f_1 \\ y_3 \\ f_3 \\ y_5 \\ y_6 \\ y_7 \\ f_7 \end{bmatrix} = \begin{bmatrix} (u_u)' \\ (u_u)'' \\ (u_v)' \\ (u_v)'' \\ (u_z)' \\ (u_z)'' \\ (u_z)''' \\ (u_z)'''' \end{bmatrix} \quad (8)$$

$$f_1 = k_{10}y_0 + k_{11}y_1 + k_{12}y_2 + k_{13}y_3 + k_{14}y_4 + k_{15}y_5 + k_{16}y_6 + k_{17}y_7,$$

$$f_3 = k_{30}y_0 + k_{31}y_1 + k_{32}y_2 + k_{33}y_3 + k_{34}y_4 + k_{35}y_5 + k_{36}y_6 + k_{37}y_7,$$

$$f_7 = k_{70}y_0 + k_{71}y_1 + k_{72}y_2 + k_{73}y_3 + k_{74}y_4 + k_{75}y_5 + k_{76}y_6 + k_{77}y_7.$$

All operations with huge expressions were computerized using symbolic computation systems to exclude the human mistakes while data manipulation.

The system can be solved numerically by means of sweep method.

After calculating the deformations and their derivatives we can calculate internal axial and shear forces and bending and twisting moments.

2. RESULTS AND DISCUSSION

The examples have been calculated by the numeric-analytical methodology and also by the finite element method. The comparison of the results can be used as verification for both methodologies.

The finite element analysis was carried out by ANSYS APDL 15, using the shell 181 finite element. The coordinate systems are not identical to each other in the finite element and numeric-analytical method (fig. 2,3), but for the shallow shells with small generator angle the curvilinear coordinated system can be approximated by the cylindrical one and also the forces and moments have the analogues in the other method.

For the non-shallow shells the most appropriate quantity to compare is sum displacement vector modulus.

The numeric-analytical method coordinate system is more convenient for the analysis because it operates with the forces and moments more common in structural mechanics.

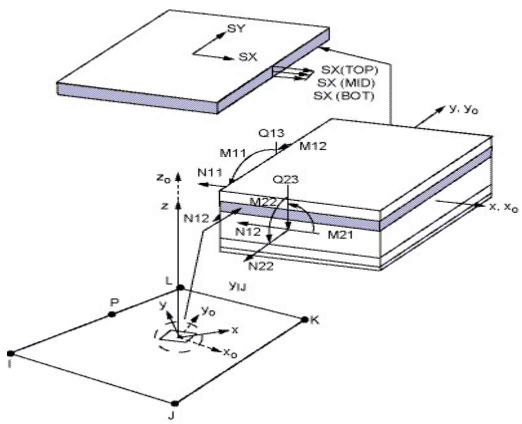

Fig.2. Coordinates and inner forces and moments in FE analysis (from ANSYS guideline)

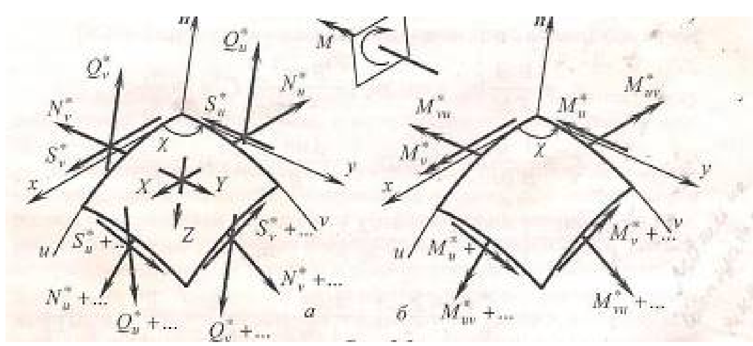

Fig.3. Coordinates and interior forces and moments in numeric-analytical analysis

Example 1

The shell of reinforced concrete material is loaded by pressure, intensity is 10 KPa. Both curved edges are fixed, the straight edges don’t affect the stress-strain state. The generators obliquity angle φ=3°, contour radiuses R1=1m, R2=2m; thickness – 10 cm, pitch – 0.01·2π. The material characteristics: E=32500 MPa, ν=0.17. The analysis was performed for the interval of 45°. Alexandrov et al.[9] proved that stress distribution in the middle section would be correct for any long helicoid (fig. 3).

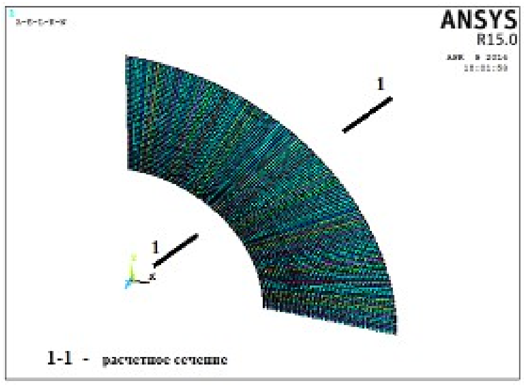

Fig.3. The model and the design section

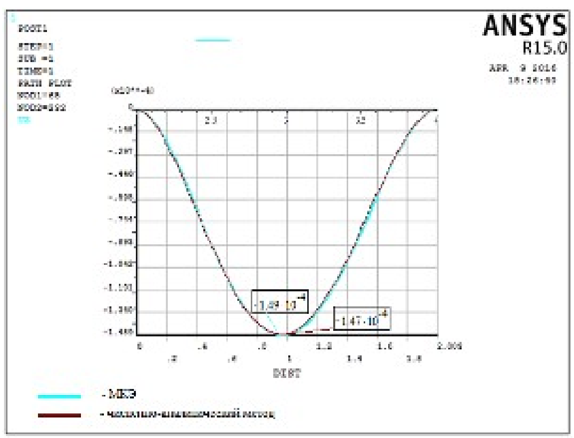

Fig.4. The deflection uz diagram, m.

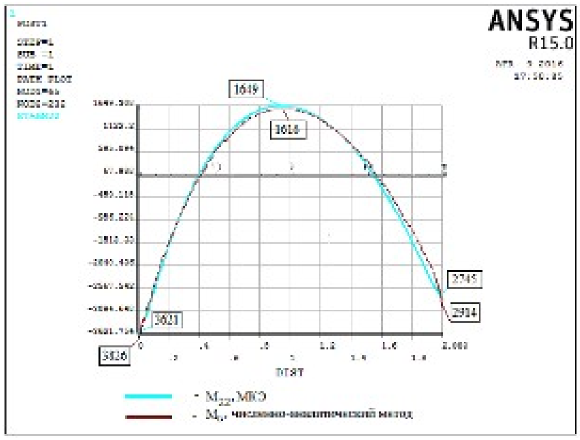

Fig.5. The bending moment Mu diagram, N m.

The diagrams of forces N_u, N_v, S_u are not represented because of their small quantities comparatively to bending moments and shear forces. Shear force Q_{13} in the finite element analysis, corresponding to shear force Qv in numeric-analytical analysis, shows instable behavior and small quantities, as well as twisting moments M_{12}, corresponding to adjusted Muv= Mvu.

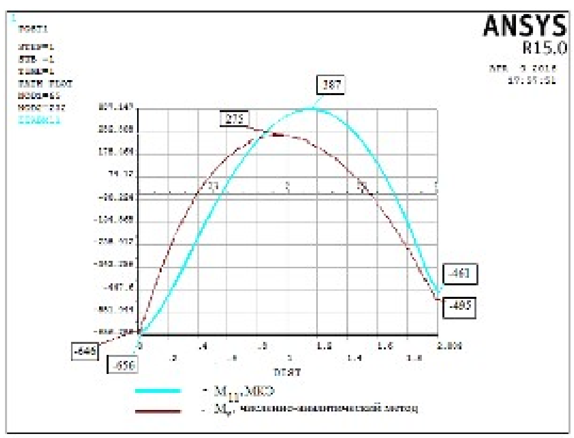

Fig.6. The bending moment Mv diagram, N m

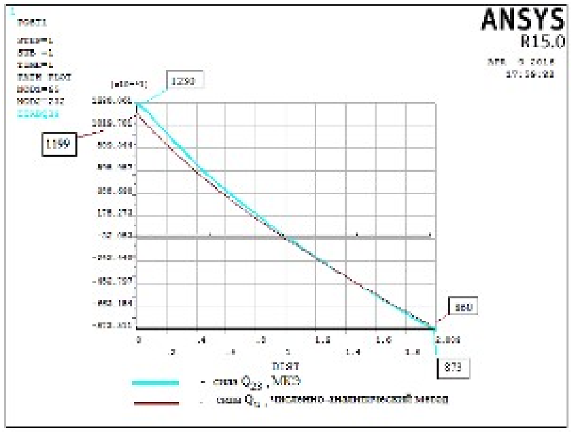

Fig.7. The shear force Qu diagram, N

Example 2

The shell of steel is loaded by equally distributed vertical load, intensity is 10 KPa. Both curved edges are fixed, the straight edges don't affect the stress-strain state. The generators obliquity angle φ=45°, contour radiuses R1=2m, R2=4m; thickness – 2 cm, pitch – 0.01·2π. The material characteristics: E=200000 MPa, ν=0.3.

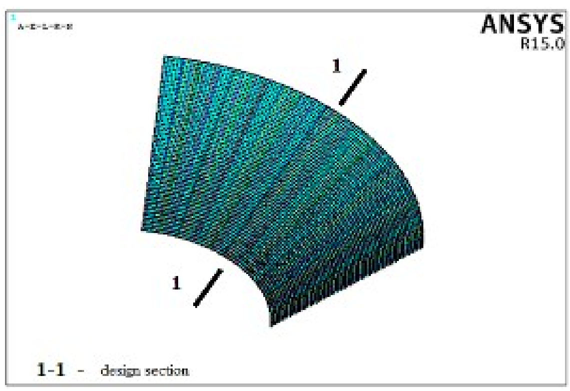

Fig.8. The model and the design section

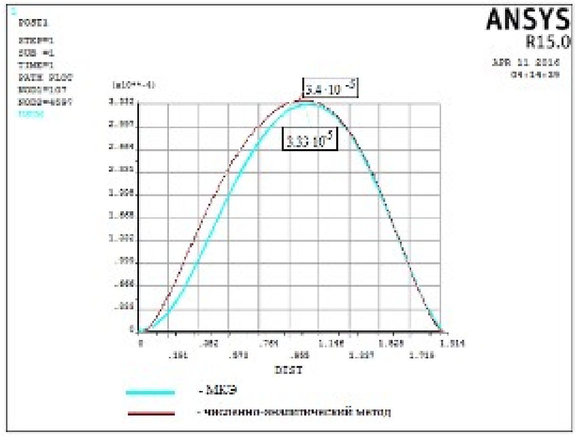

Fig.9. The sum displacement diagram

4. CONCLUSION

The formulas for bending thin elastic shell theory were established for the case of oblique helicoid in non-orthogonal non-conjugate coordinate system. The methodology for numeric-analytical analysis of the oblique helicoid shells is developed. The examples for shallow and non-shallow shells are given, the results compared to the finite element solution. The presented methodology is recommended for preliminary design and calculations, for analyzing the simple cases as a standart and for comparison and testing results, got by more complex finite element models. The similar approach can be taken for stress-strain analysis of other thin elastic shells of complex geometry. The article is of interest for the researchers, dealing with thin elastic helicoidal shells in linear statement, like [10-15].

5. LIST OF SYMBOLS

The list of symbols comes after the acknowledgment and before references. The English symbols come first followed by the Greek symbols. Both must be typed in alphabetical order and separated.

E	Modulus of elasticity
k	Generator slope
N_u	Axial force along u
N_v	Axial force along v

S_u	Axial force along v
S_v	Axial force along v
M_u	Bendind moment
M_v	Bendind moment
M_{uv}	Twisting moment
M_{vu}	Twisting moment
Qu	Shear force
Qv	Shear force
Greek symbols	
ε_u, ε_v, ε_{uv}	Linear strain
γ_u, γ_v	Angle strain
ω_u, ω_v	Angle strain
$\kappa_u,\kappa_v,\kappa_{uv}$	Torsion strain
φ	Obliquity angle
ν	Poisson's ratio
Γ_{ij}^{n}	Christoffel's symbols

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