

2-STEP INTEGRAL BACKSTEPPING CONTROL OF THE TWO-ROTOR AERO-DYNAMICAL SYSTEM (TRAS)

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ABSTRACT

This work proposes a simplified nonlinear backstepping control method for fast trajectory tracking of a 2-DOF laboratory helicopter called the Two Rotor Aero-dynamical System (TRAS). The relative degree 3 system is decomposed into the yaw subsystem (YS) and the pitch subsystem (PS) and an integral backstepping controller (IBC) is designed for each subsystem with the coupling effects considered as uncertainties. The control design considers the system as having relative degree 2, resulting in a much less complex 2-step backstepping control law requiring partial state feedback. Real time experiments show good tracking ability of the proposed method to different input waveforms within a wide operating range.

Keywords: integral; backstepping; MIMO; nonlinear.

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1. INTRODUCTION

The Two Rotor Aero-dynamical System (TRAS) otherwise known as the Twin Rotor MIMO System (TRMS) is a laboratory setup designed for control experiments and in many ways

resembles a helicopter albeit with less degrees of freedom. It typifies an under-actuated, high-order, nonlinear system with significant cross couplings. The control objective is to make the beam of the TRAS track a predetermined trajectory quickly and accurately under the cross couplings effects.

Over the years, several methods have been suggested to achieve this control objective. Optimised PID controllers have been proposed for the system in [1-3] while in [4] a robust PID based deadbeat control technique has been suggested. Optimal control methods like LQR [5], LQR with integral action [6], LQG [7] and intelligent controllers based on fuzzy logic [8-9] and neural networks [10] have also been proposed.

Application of nonlinear control methods including feedback linearization [11], Sliding Mode Control [12-14] have also been reported. In [15] a Sliding Mode Controller with an extended Kalman filter for state estimation has been proposed, while in [16] a backstepping controller with command filtered compensation is suggested for the system. Other nonlinear controllers proposed for the system include nonlinear H-infinity [17], nonlinear model predictive control [18] and Linear Parameter Varying (LPV) control [19].

This paper proposes a simplified integral backstepping control method for the yaw (horizontal) and pitch (vertical) subsystems of the TRAS respectively. The design considers the thrust produced by the rotors as inputs to the system leading to a much less complex 2-step backstepping control law for the relative degree 3 system. The final control law (the input voltage to the TRAS' motors) is then realised using partial state feedback and the reverse static characteristics of the rotors. Experimental results show the effectiveness and fast trajectory tracking of the proposed control method to constant and time varying input waveforms. The remainder of this paper is organised as follows. Section 2 gives a model description of the TRAS. In section 3 the integral backstepping controllers are designed for each of the decoupled subsystems. Section 4 presents experimental results and concluding remarks are drawn in section 5.

2. MODEL DESCRIPTION OF THE TRAS

The TRAS shown in Fig. 1 consists of a beam pivoted at its base. The articulated joint allows the beam to rotate in such a way that its ends move on the yaw and pitch planes. The main and

tail rotors are driven by DC motors and attached to the ends of the beam. A counterbalance arm with a weight at its end is fixed to the beam at the pivot. The system is balanced in such a way that when the motors are switched off, the main rotor end of the beam is lowered. Unlike conventional helicopters where aerodynamic thrust is generated by changing the angles of attack of the propellers, aerodynamic thrust in the TRAS is generated by increasing the rotation speed of the rotors. The result is a complex, high order nonlinear system with significant cross couplings i.e. each rotor affects both position angles.

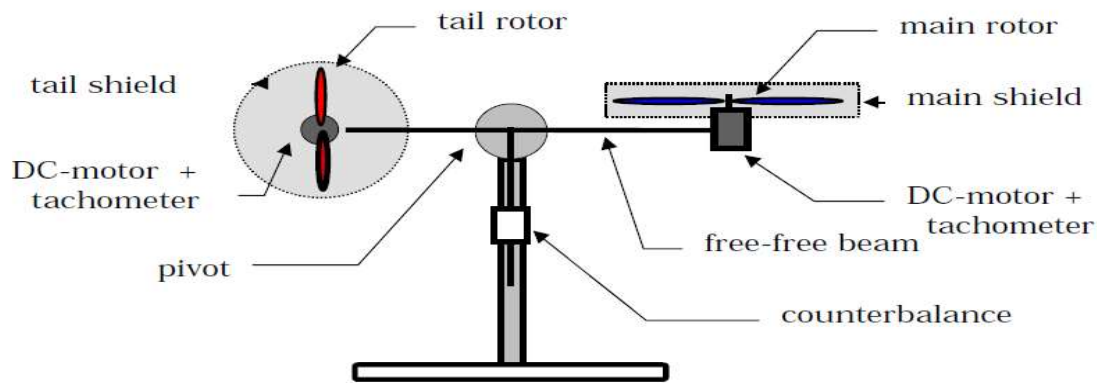


Fig.1. TRAS component parts [1]

2.1. TRAS Dynamic Model

An approximate Newtonian mathematical model of the TRAS is obtained using Newton's second law of motion and converted into the state-space form [1]. The state equations describing the motion of the system are given in Equation (1) and described in Table 1. Values of the physical parameters are provided in Table 2.

$$\begin{aligned}
 \dot{x}_1 &= x_3 \\
 \dot{x}_2 &= x_4 \\
 \dot{x}_3 &= \frac{1}{J_h} [l_t F_h(x_5) \cos x_2 - k_h x_3 - a_2 x_3 |x_5| + u_v k_{vh}] \\
 \dot{x}_4 &= \frac{1}{J_v} \left[l_m F_v(x_6) - k_v x_4 - a_1 x_4 |x_6| \right. \\
 &\quad \left. + g((A - B) \cos x_2 - C \sin x_2) \right. \\
 &\quad \left. - \frac{1}{2} x_3^2 (A + B + C) \sin(2x_2) u_h k_{hv} + u_h k_{hv} \right] \\
 \dot{x}_5 &= \frac{1}{I_h} (u_h - H_h^{-1}(x_5))
 \end{aligned} \tag{1}$$

$$\dot{x}_6 = \frac{1}{I_v} (u_v - H_v^{-1}(x_6))$$

Table 1. Model description of the TRAS

Symbol	Description
x_1, x_2	Horizontal and vertical positions of TRAS beam (rad)
x_3, x_4	Horizontal and vertical angular velocities of TRAS beam (rad/s)
x_5, x_6	Rotational speeds of the tail and main rotors (RPM)
J_h, J_v	Nonlinear functions of moments of inertia relative to the vertical and horizontal axis
l_t, l_m	Effective arms of aerodynamic forces from the tail and main rotors
F_h, F_v	Nonlinear aerodynamic forces from the tail and main rotors
H_h, H_v	Differential equations of the tail and main rotors
u_h, u_v	Tail and main rotor DC-motor PWM control inputs
k_{hv}, k_{vh}	Vertical and horizontal angular momentums of the tail and main rotors
k_h, k_v	Moments of friction forces in the vertical and horizontal axes
A, B, C	Mechanical constants
a_1, a_2	Mechanical constants
g	Gravitational acceleration

Table 2. TRAS physical parameters

Symbol	Value	Symbol	Value
l_m	0.202 m	k_v	1.271×10^{-2} Nm
l_t	0.216 m	A	0.0652
I_h	2.703×10^{-5} kgm ²	B	0.0707
I_v	1.639×10^{-4} kgm ²	C	0.0046
k_{hv}	4.200×10^{-3} Nms	a_1	9.280×10^{-6}
k_{vh}	-1.78×10^{-2} Nms	a_2	3.300×10^{-6}
k_h	5.900×10^{-3} Nm	g	9.810 ms ⁻²

The sum of moments of inertia relative to the horizontal axis J_v is constant and given as 3.07×10^{-2} kgm² while that relative to the vertical axis J_h is dependent on the pitch position (x_2) of the beam and expressed as:

$$J_h = 0.0013\sin^2 x_2 + 0.0279\cos^2 x_2 + 0.0021 \quad (2)$$

2.2. Motor Static Characteristics

The input voltage signal to the motors is normalised and changes in the range -1 to +1 which corresponds to a voltage range of ± 12 V. The nonlinear relationships between the input voltage and speed as well as the speed and thrust generated by both rotors are determined experimentally and approximated in Equations (3)-(6).

$$x_5 \approx -2.2 \times 10^3 u_h^5 - 1.7 \times 10^2 u_h^4 - 4.5 \times 10^3 u_h^3 + 3 \times 10^2 u_h^2 + 9.8 \times 10^3 u_h - 9.2 \quad (3)$$

$$F_h \approx -2.6 \times 10^{-20} x_5^5 + 4.1 \times 10^{-17} x_5^4 + 3.2 \times 10^{-12} x_5^3 - 7.3 \times 10^{-9} x_5^2 + 2.1 \times 10^{-5} x_5 + 0.0091 \quad (4)$$

$$x_6 \approx -5.2 \times 10^3 u_v^7 - 1.1 \times 10^2 u_v^6 + 1.1 \times 10^4 u_v^5 + 1.3 \times 10^2 u_v^4 - 9.2 \times 10^3 u_v^3 - 31 u_v^2 + 6.1 \times 10^3 u_v - 4.5 \quad (5)$$

$$F_v \approx -1.8 \times 10^{-18} x_6^5 - 7.8 \times 10^{-16} x_6^4 + 4.1 \times 10^{-11} x_6^3 + 2.7 \times 10^{-8} x_6^2 + 3.5 \times 10^{-5} x_6 - 0.014 \quad (6)$$

3. INTEGRAL BACKSTEPPING CONTROL DESIGN

Backstepping is a nonlinear control scheme based on the Lyapunov stability theorem. The technique can be used to synthesize a control law to force a system follow a desired trajectory. The control law is designed by iteratively selecting some appropriate state variables as virtual inputs for lower dimension subsystems of the overall system. Lyapunov functions are then designed for each stable virtual controller. Therefore, the final designed actual control law guarantees the asymptotic stability of the total control system. The integral of the error is added to the first stabilizing function in the backstepping design to eliminate steady state error and improve disturbance rejection. A detailed description of the backstepping design method can be found in [20].

3.1. Yaw Subsystem Integral Backstepping

The model for the yaw (horizontal) subsystem of the decoupled TRAS can be obtained as:

$$\begin{aligned} \dot{x}_1 &= x_3 \\ \dot{x}_3 &= \frac{1}{J_h} [l_t F_h(\omega_h) \cos x_2 - k_h x_3 - a_2 x_3 |\omega_h|] \end{aligned} \quad (7)$$

$$\dot{x}_5 = \frac{1}{I_h} (u_h - H_h^{-1}(x_5))$$

Let:

$$z_1 := x_1 - x_{1d} \quad (8)$$

$$z_3 := x_3 - \alpha_1 \quad (9)$$

$$z_5 := F_h(x_5) - \alpha_3 \quad (10)$$

where x_{1d} is the desired yaw angle and α_1 and α_3 are stabilizing functions.

Step 1:

Assuming $x_{1d} = 0$ without loss of generality, then $z_1 = x_1$.

Select a Lyapunov function candidate as:

$$V_1 = \frac{\lambda_1}{2} Q_1^2 + \frac{1}{2} z_1^2 \quad (11)$$

where $Q_1 = \int_0^t z_1(\tau) d\tau$ and λ_1 is a positive constant.

Therefore,

$$\begin{aligned} \dot{V}_1 &= \lambda_1 Q_1 \dot{Q}_1 + z_1 \dot{z}_1 = \lambda_1 Q_1 z_1 + z_1 \dot{x}_1 \\ &= \lambda_1 Q_1 z_1 + z_1 x_3 \end{aligned} \quad (12)$$

By taking x_3 as the virtual control, a stabilizing function $\alpha_1(x_1)$ is designed as:

$$\alpha_1(x_1) = -c_1 z_1 - \lambda_1 Q_1, \quad c_1, \lambda_1 > 0 \quad (13)$$

Hence,

$$\dot{V}_1 = -c_1 z_1^2 \quad (14)$$

Step 2:

Design a stabilising function (α_3) for z_3 by taking $F_h(x_5)$ as the virtual control.

$$\dot{z}_1 = \dot{x}_1 = x_3 \quad (15)$$

$$= x_3 - \alpha_1 + \alpha_1$$

$$\dot{z}_1 = z_3 - c_1 z_1 - \lambda_1 Q_1 \quad (16)$$

Select a Lyapunov function candidate:

$$V_3 = \frac{\lambda_1}{2} Q_1^2 + \frac{1}{2} z_1^2 + \frac{1}{2} z_3^2 \quad (17)$$

$$\begin{aligned} \dot{V}_3 &= \lambda_1 Q_1 \dot{Q}_1 + z_1 \dot{z}_1 + z_3 \dot{z}_3 \\ &= \lambda_1 Q_1 z_1 + z_1 (z_3 - c_1 z_1 - \lambda_1 Q_1) + z_3 \dot{z}_3 \end{aligned} \quad (18)$$

But, $\dot{z}_3 = \dot{x}_3 - \dot{\alpha}_1(x_1)$

Hence,

$$\dot{z}_3 = \frac{1}{J_h} [l_t F_h(\omega_h) \cos x_2 - k_h x_3 - a_2 x_3 |x_5|] - \frac{\partial \alpha_1}{\partial x_1} \dot{x}_1 \quad (19)$$

Therefore,

$$\dot{V}_3 = -c_1 z_1^2 + z_3 \left[z_1 + \frac{l_t}{J_h} (F_h(\omega_h) \cos x_2) - \frac{1}{J_h} (k_h x_3 + a_2 x_3 |x_5|) - \frac{\partial \alpha_1}{\partial x_1} \dot{x}_1 \right] \quad (20)$$

To cancel out the nonlinear terms and stabilize the dynamics of z_3 design α_3 as:

$$\alpha_3 = \frac{J_h}{l_t \cos x_2} \left[-z_1 + \frac{1}{J_h} (k_h x_3 + a_2 x_3 |x_5|) + \frac{\partial \alpha_1}{\partial x_1} \dot{x}_1 - c_3 z_3 \right] \quad (21)$$

So that,

$$\dot{V}_3 = -c_1 z_1^2 - c_3 z_3^2, \quad c_1, c_3 > 0 \quad (22)$$

3.2. Pitch Subsystem Integral Backstepping

The decoupled model for the pitch subsystem of the TRAS is obtained as:

$$\begin{aligned} \dot{x}_2 &= x_4 \\ \dot{x}_4 &= \frac{1}{J_v} [l_m F_v(x_6) - k_v x_4 + G(x_2) - a_1 x_4 |x_6|] \\ \dot{x}_6 &= \frac{1}{I_v} (u_v - H_v^{-1}(x_6)) \end{aligned} \quad (23)$$

where $G(x_2) = g((A - B) \cos x_2 - C \sin x_2)$

Let:

$$z_2 := x_2 - x_{2d} \quad (24)$$

$$z_4 := x_4 - \alpha_2 \quad (25)$$

$$z_6 := F_v(x_6) - \alpha_4 \quad (26)$$

where x_{2d} is the desired pitch angle and α_2 and α_4 are stabilizing functions.

Step 1:

Design a stabilising function $\alpha_2(x_2)$ for the error state z_2 .

Assuming $x_{2d} = 0$ then $z_2 = x_2$.

Select a Lyapunov function candidate as:

$$V_2 = \frac{\lambda_2}{2} Q_2^2 + \frac{1}{2} Z_2^2 \quad (27)$$

Therefore,

$$\dot{V}_2 = z_2 \dot{z}_2 = z_2 \dot{x}_2 = z_2 x_4 \quad (28)$$

Design the stabilizing function $\alpha_2(x_2)$ by taking x_4 as the virtual control.

$$\alpha_2(x_2) = -c_2 z_2 - \lambda_2 Q_2, \quad c_2, \quad \lambda_2 > 0 \quad (29)$$

where $Q_2 = \int_0^t z_2 dt$

Hence,

$$\dot{V}_2 = -c_2 z_2^2 \quad (30)$$

Step 2:

Design a stabilising function (α_4) for z_4 by taking $F_v(x_6)$ as the virtual control.

$$\dot{z}_2 = \dot{x}_2 = x_4 \quad (31)$$

$$= x_4 - \alpha_2 + \alpha_2$$

$$\dot{z}_2 = z_4 - c_2 z_2 - \lambda_2 Q_2 \quad (32)$$

Select a Lyapunov function candidate as:

$$V_4 = \frac{\lambda_2}{2} Q_2^2 + \frac{1}{2} z_2^2 + \frac{1}{2} z_4^2 \quad (33)$$

$$\dot{V}_4 = \lambda_2 Q_2 \dot{Q}_2 + z_2 \dot{z}_2 + z_4 \dot{z}_4 \quad (34)$$

$$= \lambda_2 Q_2 z_2 + z_2 (z_4 - c_2 z_2 - \lambda_2 Q_2) + z_4 \dot{z}_4$$

But, $\dot{z}_4 = \dot{x}_4 - \dot{\alpha}_2$

$$\dot{z}_4 = \frac{1}{J_v} [l_m F_v(x_6) - G(x_2) - k_v x_4 - a_1 x_4 |x_6|] - \frac{\partial \alpha_2}{\partial x_2} \dot{x}_2 \quad (35)$$

Therefore,

$$\begin{aligned} \dot{V}_4 = & -c_2 z_2^2 + z_4 \left[z_2 + \frac{l_m}{J_v} F_v(x_6) \right. \\ & \left. - \frac{1}{J_v} (G(x_2) + k_v x_4 + a_1 x_4 |x_6|) - \frac{\partial \alpha_2}{\partial x_2} \dot{x}_2 \right] \end{aligned} \quad (36)$$

To cancel out the nonlinear terms and stabilise the dynamics of z_4 , $\alpha_4(x_2, x_4)$ is designed as:

$$\alpha_4 = \frac{J_v}{l_m} \left(-z_2 + \frac{1}{J_v} (G(x_2) + k_v x_4 + a_1 x_4 |x_6|) + \frac{\partial \alpha_2}{\partial x_2} \dot{x}_2 - c_4 z_4 \right) \quad (37)$$

Such that,

$$\dot{V}_4 = -c_2 z_2^2 - c_4 z_4^2, \quad c_2, c_4 > 0 \quad (38)$$

The final MIMO control law is realised as shown in the control structure in Fig. 2 where IBC represents the 2-step integral backstepping controller and $x_5^{-1}(x_6^{-1})$ and $F_h^{-1}(F_v^{-1})$ are the reverse static characteristics of the tail (main) rotors in Equations (3)-(6).

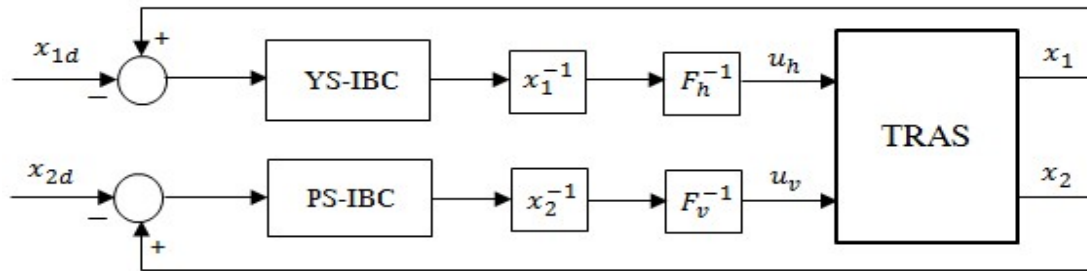


Fig.2. TRAS 2-Step IBC control structure

4. IMPLEMENTATION AND RESULTS

Implementation of the proposed control method and real time results for step and sine-wave inputs are given in this section.

4.1. Controller Parameters

The IBC parameters for the yaw subsystem are set at $c_1 = 0.8, c_3 = 11, \lambda_1 = 0.3$ and $c_2 = 1.05, c_4 = 4.2, \lambda_2 = 1.7$ for the yaw subsystem. To improve tracking of sinusoidal references however, the integral gains λ_1 and λ_2 are increased to 1.1 and 1.8 respectively.

4.2.State Estimation

The 2-step IBC requires feedback of the immeasurable velocity states (x_3) and (x_4) of the beam in the yaw and pitch planes respectively. The beam has a low speed range and as such, the Enhanced Differentiator in [21] designed for systems with low speeds is used to provide estimates of x_3 and x_4 from only measurements of their respective positions.

4.3.Experimental Results

Fig. 3 shows the step response of the system with $x_{1d} = 1.0$ rad and $x_{2d} = 0.4$ rad. It is observed that the system is able to track the setpoints with minimal overshoot. In Fig. 4, the operating points are changed ($x_{1d} = 1.5, x_{2d} = 0.0$) and the controller is able to track the references with the same settings of the control parameters, indicating its nonlinear handling capability. The response to a 0.025 Hz sine-wave input is shown in Fig. 5. It can be seen that the controller shows good tracking ability. The slight variation of the control signal over the time range shows the degree of precision required in the control of the TRAS.

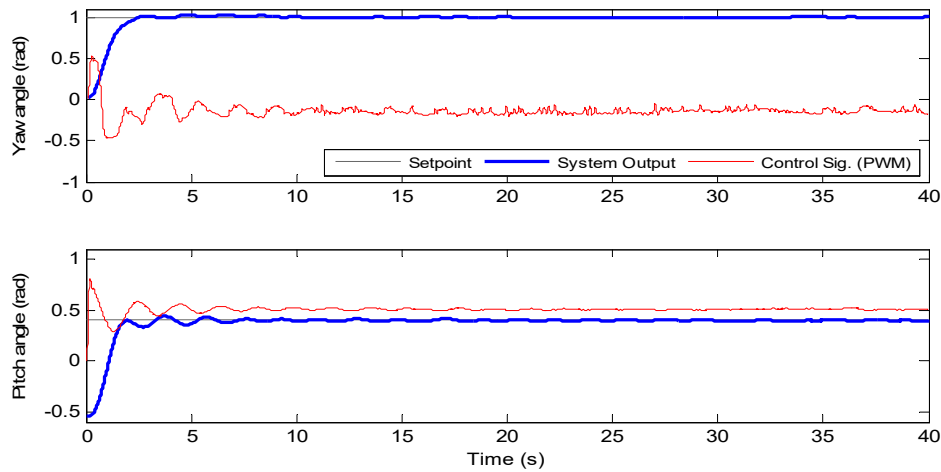


Fig.3. Step response and control signal with $x_{1d} = 1.0$ rad and $x_{2d} = 0.4$ rad

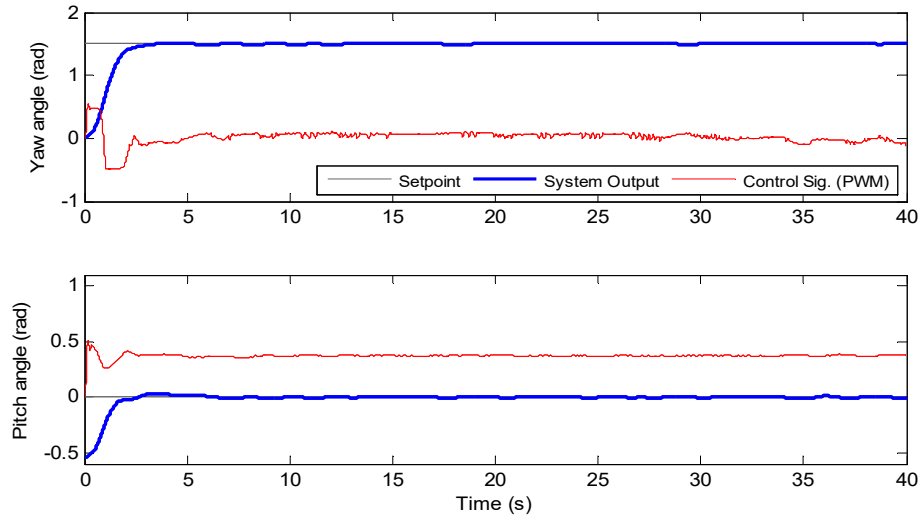


Fig.4. Step response and control signal with $x_{1d} = 1.5$ rad and $x_{2d} = 0.0$ rad

5. CONCLUSION

A simplified 2-step integral backstepping controller has been designed and implemented on the TRAS in real time using only position measurements. Experimental results show the proposed control method successfully counters the cross coupling effects between the main and tail rotors and handles the nonlinearities in the system by accurately tracking constant and time varying wave forms at different operating points.

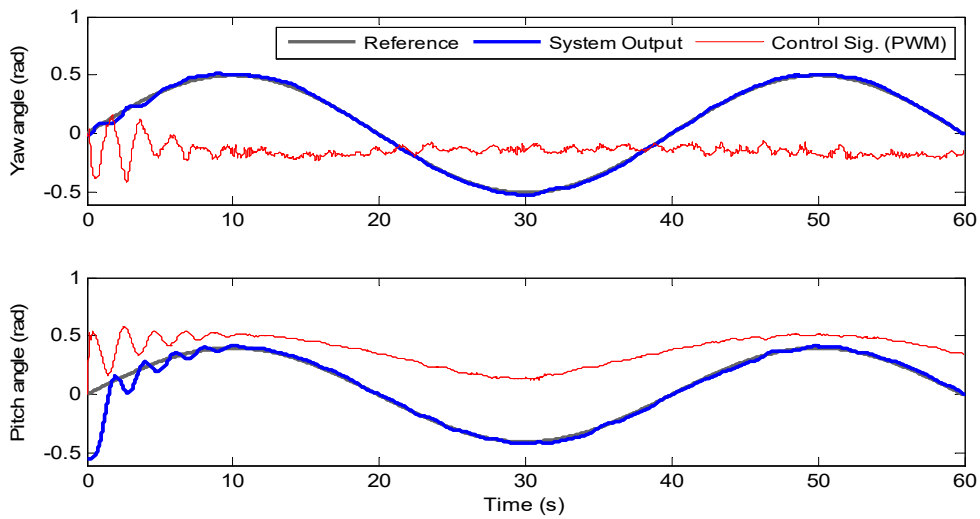


Fig.5. Sine-wave tracking response and control signal with $x_{1d} = 0.5 \sin 0.05\pi t$ rad and $x_{2d} = 0.4 \sin 0.05\pi t$

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