Research Article

Modeling and Predicting Exchange Rate Volatility: Application of Symmetric GARCH and Asymmetric EGARCH and GJR-GARCH Models

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Abstract

Modeling and predicting volatility has become increasingly important in recent times given that an understanding of future volatility can help investors and various stakeholders to minimize their losses. This paper applies univariate time series analysis in the modeling and prediction of the volatility of the exchange rates between Cameroon's FCFA (XAF) and the US Dollar (USD) and between Cameroon's FCFA and the Chinese Yuan (CNY). Using daily closing prices from 01 January 2017 to 30 September 2022, both symmetric Generalized Autoregressive Conditional Heteroscedasticity (GARCH) and asymmetric Exponential GARCH (EGARCH) and Glosten-Jagannathan-Runkle GARCH (GJR-GARCH) models are used to capture stylized facts about exchange rate returns. The in-sample and out-of-sample data sets contain data from 01 January 2017 to 31 December 2021 and from 01 January 2022 to 30 September 2022 respectively. The residuals are assumed to follow the normal, student's t and generalized error distributions along with their skewed counterparts. Considering the model with the lowest Akaike Information Criteria (AIC), the paper finds ARMA(0,1) + GJR-GARCH(1,1) - SGED[1] and ARMA(1,1)+GJR-GARCH(2,2) - SGED as the most appropriate models to estimate the volatility of the USD/XAF and CNY/XAF exchange rate returns respectively. Equally, ARMA(0,1)+GARCH(1,1) - SGED and ARMA(1,1)+GJR-GARCH(2,2)-SGED are the best out-of-sample predictive models for the volatility of the USD/XAF and CNY/XAF exchange rate returns respectively using Mean Absolute Error (MAE) and Root Mean Square Error (RMSE). Leverage effects are found to characterize the CNY/XAF exchange rate but absent in the USD/XAF exchange rate data. The results show that conditional heteroscedastic models can be effectively used to model and predict the conditional volatility of exchange rate series. This research recommends that, in the design of appropriate exchange rate policies, Cameroon's monetary authorities and BEAC should take into consideration the fact that the exchange rate market is very volatile and reacts differently to both good and bad news.

Keywords: ARCH, GARCH, EGARCH, GJR-GARCH, Volatility, Modeling and Prediction.

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Résume

La modélisation et la prévision de la volatilité sont devenues de plus en plus importantes ces derniers temps étant donné qu'une compréhension de la volatilité future peut aider les investisseurs et les diverses

[1] SGED:Skewed Generalized Error Distribution

parties prenantes à minimiser leurs pertes. Cet article applique l'analyse de séries chronologiques univariées dans la modélisation et la prédiction de la volatilité des taux de change entre le FCFA Camerounais (XAF) et le Dollar Américain (USD) et entre le FCFA Camerounais et le Yuan Chinois (CNY). En utilisant les prix de clôture quotidiens du 1er Janvier 2017 au 30 Septembre 2022, les modèles d'hétéroscédasticité conditionnelle autorégressive généralisée symétrique (GARCH) et asymétrique GARCH exponentiel (EGARCH) et Glosten-Jagannathan-Runkle GARCH (GJR-GARCH) sont utilisés pour capturer des faits stylisés sur l'échange rendements des taux. Les ensembles de données dans l'échantillon et hors échantillon contiennent des données du 1er Janvier 2017 au 31 Décembre 2021 et du 1er Janvier 2022 au 30 Septembre 2022 respectivement. Les résidus sont supposés suivre les distributions normale, t et d'erreur généralisée avec leurs homologues asymétriques. En considérant le modèle avec les critères d'information d'Akaike (AIC) les plus bas, l'article trouve ARMA (0,1) + GJR-GARCH (1,1) - SGED et ARMA(1,1)+GJR-GARCH(2,2) - SGED comme les modèles les plus appropriés pour décrire la volatilité des rendements des taux de change USD/XAF et CNY/XAF respectivement. De même, ARMA(0,1)+GARCH(1,1) - SGED et ARMA(1,1)+GJR-GARCH(2,2)-SGED sont les meilleurs modèles prédictifs hors échantillon pour la volatilité de taux de change USD/XAF et CNY/XAF utilisent respectivement l'erreur absolue moyenne (MAE) et l'erreur quadratique moyenne (RMSE). Les effets de levier caractérisent le taux de change CNY/XAF mais sont absents des données sur le taux de change USD/XAF. Les résultats montrent que les modèles hétéroscédastiques conditionnels peuvent être utilisés efficacement pour modéliser et prédire la volatilité conditionnelle des séries de taux de change. Cette recherche recommande que, dans la conception de politiques de taux de change appropriées, les autorités monétaires Camerounaises et la BEAC prennent en considération le fait que le marché des taux de change est très volatil et réagit différemment aux bonnes comme aux mauvaises nouvelles.

Mots clés: ARCH, GARCH, GARCH-E, GJR-GARCH, Volatilité, Modélisation et Prédiction

1. Introduction

A key feature of financial time series is the element of uncertainty or risk. Volatility is considered by investors as a measure of risk (Zivot, 2016) and investors always want an extra benefit for investing in risky assets (Anderson *et al.*, 2009). The modeling and prediction of volatility is important in the management of risks, pricing of options and asset allocations since an understanding of future volatility can help investors and policy makers to minimize loss.

In the classical method of modeling time series, it is usually assumed that the conditional variance of the one-step ahead prediction is time-invariant (homoscedastic) (Tsay, 1987). However, in practice, the conditional variance or volatility of financial time series returns tends to change over time (heteroscedastic) rendering conventional time series and econometric models unattractive to the modeling of financial time series. A stationary non-linear model for returns called the Auto regressive Conditional Heteroscedasticity (ARCH) model was first proposed by Engle (Engle, 1982) to model the volatility of heteroscedastic time series. Bollerslev (Bollerslev, 1986) and Taylor (Taylor, 1986) independently improved on this model to obtain a more realistic and parsimonious Generalized ARCH (GARCH) model. However, the ARCH and GARCH models fail to capture leverage effects commonly exhibited by financial data due to its asymmetry. Asymmetric variants of the GARCH model such as the Exponential GARCH (EGARCH) model proposed by Nelson (Nelson, 1991), the Glosten-Jagannathan-Runkle GARCH (GJR-GARCH) model of Glosten, Jagannathan and Runkle (Glosten *et al.* 1993), the Threshold GARCH (TGARCH) model proposed by Zakoian (1994) and the Asymmetric Power ARCH model of Ding, Granger and Engle (Ding *et al.,* 1993) were also developed to handle issue of asymmetry.

Since the fall of the Bretton Woods system of fixed exchange rate in 1973, exchange rate movement and fluctuation has received a great deal of interest from various stakeholders as they seek to design policies to mitigate the adverse effects of exchange rate volatility on important economic and financial indicators (Yang, nd). Cameroon trades and carries out several financial transactions with countries abroad especially the United States of America and China. As of 2020, 29% of Cameroon's public debt was in US Dollars and 6% in Chinese Yuan (Owen and Estevao, 2020). International businesses today grapple with a volatile and complex foreign exchange environment as currency exchange rates are constantly changing in response to sometimes tiny changes in global economic conditions. This volatility of foreign exchange rate affects the Cameroonian economy, thus the need for accurate modeling and prediction of the volatility of the exchange rates. Morina *et al.* (2020) notes that high volatility of exchange rate has a significant negative effect on real economic growth.

The importance of predicting the volatility of foreign exchange rate returns cannot be overemphasized. XAF is pegged to the Euro at a fixed parity and whenever the value of the Euro changes with respect to other currencies, the value of XAF is equally affected. As a result of the war between Russia and Ukraine, and the surge in the price of oil on the international market, the value of the Euro fell compared to the US Dollar. This also led to a drop in the value of XAF compared to the US Dollar as seen on Fig 1. These fluctuations led to an increase in Cameroon's debt of 420 billion francs CFA (+11.2%) between June 2021 and June 2022 giving an average monthly increase of 35 billion francs CFA within that period (Mbodiam, 2022). If these fluctuations could be predicted and appropriate actions taken, a lot of money could have been saved by the government of Cameroon.

The rest of the paper is structured as follows: Section 2 presents a review of existing literature, some stylized facts on financial returns are reviewed in Section 3 while Section 4 describes the approach used in model construction. Empirical analysis and results are presented in Section 5 while the conclusion and policy implications of the paper will wrap up the paper in Section 6. All the computations in this paper were carried out using the R software.

2. Literature Review

Academicians and researchers have been attracted to time series modeling and prediction during the last few decades. Dhamija and Bhalla (2010) used the ARCH, GARCH, GARCH-M, TGARCH, EGARCH and IGARCH models to estimate the mean and variance equations of NIFTY daily log returns and to estimate the volatility of the exchange rates between the British Pounds, German Mark, Japanese Yen, Indian Rupees and the Euro versus the US Dollar. The results show that conditional heteroscedastic models can be effectively used to model and predict exchange rates. A comparative analysis showed that IGARCH and TGARCH models perform better than other models in forecasting exchange rates. The Naira exchange rate versus the US Dollar, British Pound, European Union's Euro and the Japanese Yen was equally examined by David *et al.* (2016) using GARCH (1,1) model and its asymmetric variants and 3 years of weekly data. Heteroscedasticity was seen in three of the four return series and the fitted models indicated significant parameters with persistent volatility. Different impacts for both negative and positive shocks could be seen in the asymmetric models which demonstrated superior forecasting performance to the asymmetric GARCH models. Vee et *al.* (2011) in a similar paper evaluated volatility forecasts for USD/Mauritian Rupee Exchange rate using a GARCH (1,1) model, the Generalized

Error Distribution (GED), the Student's-t Distribution and daily data from 30/06/2003 to 31/03/2008. They assess the forecasting ability of the models using the Mean Absolute Error (MAE) and the Root Mean Square Error (RMSE) loss functions. The results show that both distributions may forecast quite well with GARCH (1,1) - GED having a slight advantage for out-of-sample forecasts. Symmetric GARCH-M and asymmetric EGARCH and TGARCH were equally employed by Kamal *et al.* (2012) to forecast the volatility of Pakistani foreign exchange (FOREX) market using daily data from January 2001 to December 2009. Results show first order autoregressive behaviour in GARCH-M and EGARCH and the GARCH-M model supported the fact that previous days FOREX rate affected that of the current day. Epaphra (2017) also applied GARCH and EGARCH models to analyze daily exchange rates between the Tanzanian Shillings (TZS) and the US Dollars. He concluded by the asymmetric volatility estimate that positive shocks implied a higher next period conditional variance than negative shocks of the same sign. This usually happens when the coefficient for the asymmetric volatility θ is positive.

Symmetric and asymmetric GARCH models were used by Abdalla (2012) to model exchange rate volatility in 19 Arab countries between the year 2000 and 2011. The models captured stylized facts about exchange rate returns such as volatility clustering and leverage effects. The GARCH (1,1) model showed volatility persistence in 9 and an explosive process in 10 of the 19 countries. EGARCH(1,1) gave evidence of leverage effects for majority of the currencies showing that negative shocks imply a higher next period volatility than positive shocks. While the results obtained by Okyere *et al.*(2013) in the modeling and forecasting of the exchange rate between Ghana Cedi and US Dollars using GARCH models equally showed volatility persistence, the estimated coefficient in TGARCH was negative suggesting that positive shocks imply a higher next period conditional variance than negative shocks of the same sign. Monthly exchange rate return series from 1985:1 to 2011:7 for Naira/US Dollar return and from 2004:1 to 2011:7 for Naira/British Pounds and Naira/Euro returns rates with exogenously determined break points were equally examined by Bala and Asemota (2013). Results reveal the presence of volatility in the three currencies and equally indicate that most of the asymmetric models rejected the existence of a leverage effect except for models with volatility break. Evaluating the models through standard information criteria, volatility persistence and the log likelihood statistic, showed that results improved with estimation of volatility models with breaks as against those of GARCH models without volatility breaks and that the introduction of volatility breaks reduces the level of persistence in most of the models. In the GARCH model of Bollerslev (1986) and Taylor (1986), it is assumed that the residuals are Gaussian. However, data analysis has often shown that this is not the case as investigated by Abdullah *et al.*(2017) who addressed the issue of error distribution assumption in modeling and forecasting exchange rates between the Bangladeshi Taka (BDT) and the US Dollar using GARCH, APARCH, EGARCH, TGARCH and IGARCH. Results showed that the student's-t distribution errors improved forecasting accuracy with AR(2) - GARCH(1,1) as best prediction model.

As far as parameter estimation is concerned, the Maximum Likelihood Estimate (MLE) using the Marquardt Algorithm was used by Dritsaki (2017) to estimate the parameters of GARCH(1,1), EGARCH(1,1) and GJR-GARCH(1,1) volatility models in the analysis of daily stock returns from the Stockholm stock Exchange. The results showed that negative shocks have a large impact than positive shocks of the same magnitude in this market. In addition to the models used by Dritsaki (2017), Almisshal and Emir (2021) included PGARCH(1,1) in their modeling of the US Dollar and EURO against the new Turkish Lira (TRY). The results showed that GARCH(1,1) and GJR-GARCH(1,1) were the most appropriate models for estimating the volatility of USD/TRY exchange rate and PGARCH(1,1) for EUR/TRY exchange rate. The loss functions RMSE, MAE and MAPE showed that GJR-GARCH(1,1) is the best model in predicting the future pattern for both USD and EURO. As for the Chinese emerging markets, Wang *et al.* (2022) used the Shanghai Composite Index and Shenzhou Component Index returns to conduct empirical analysis based GARCH-type model. Results using loss functions indicate that ARMA(4,4)-GARCH (1,1) under student-t distribution outperform the other models when forecasting the Shanghai Composite Index returns series and ARMA(1,1)-TGARCH(1,1) displayed the best forecasting model for Shenzhou Composite Index. Although extensive empirical research has been carried out on modeling the volatility of exchange rate using different currency pairs in the financial markets of several developed countries, little attention has been given to the economies of developing countries in general and Cameroon in particular except for the work of Ayuk (2018). He used both symmetric (ARCH and GARCH) and asymmetric (APARCH, GJR-GARCH, EGARCH) family of models to model the volatility of daily exchange rate return series between CEMAC XAF and US Dollars from January 01, 2010 to January 04, 2018. All the models were estimated using MLE under the assumption of the normal, student's-t and skew student's-t distributions. The results showed that the XAF/USD exchange rate return series exhibit some of the stylized facts of financial returns such as volatility clustering and leverage effects. Using AIC and BIC, EGARCH(1,1) under student's-t distribution was found to be the best fitted model to model conditional volatility of the exchange rate returns.

To the best of our knowledge, little or no empirical research has been carried out in modeling and predicting the volatility of the exchange rates between XAF and the USD and between XAF and CNY using the GARCH model and its variants. This paper contributes to the existing literature in several ways. Firstly, it explains the process of volatility modeling and prediction using daily returns of exchange rates using recent data especially during the onset of the COVID – 19 pandemic and the Russian – Ukraine war. Secondly, it employs both symmetric (GARCH) and asymmetric (EGARCH and GJR-GARCH) volatility models to capture symmetry, asymmetry and leverage effects in the data. Thirdly, this research assumes not only the Gaussian distribution for the residuals of the returns but also non – Gaussian distributions such as the Student's t and Generalized Error Distributions along with their skewed counterparts to capture leptokursis prevalent in the distribution of financial returns. The main objective of this paper is to carry out a pairwise modeling and prediction of the volatility of the USD/XAF and CNY/XAF exchange rates using different residual distributions.

3. Stylized Facts on Financial Returns and Exchange Rate Regimes

3.1 Stylized Facts on Financial Returns

Most studies on finance use the returns instead of asset prices because returns are scale free and easier to handle than prices.

Definition 1. (Return Series). Let P_t and P_{t-1} be the current and previous day's currency exchange rate or asset prices respectively. The log returns y_t is defined as;

$$y_t = \log\left(\frac{P_t}{P_{t-1}}\right) \tag{1}$$

Definition 2. (Volatility). Exchange rate volatility is a statistical measure of the dispersion of exchange rate returns. It is measured as the sample standard deviation of the returns given as;

$$\sigma_t = \sqrt{\frac{1}{T-1}\sum_{t=1}^{T}(y_t-\mu)^2} \quad (2)$$

where y_t is the log returns at time t and μ is the average returns over the time T. Sometimes the variance σ_t^2 is also used as a measure of volatility.

Stylized facts are the results of many independent empirical studies on the statistical properties of financial log returns series that have been proven to be common across financial markets (Bollerslev *et al.* (1992); Cont (2001) and Chao (2020)). Some of these stylized facts according to Cont (2001) and Mandelbrot (1963) include;

3.1.1 Clustering Volatility

Volatility clustering occurs when large and small values of returns tend to occur in clusters: large changes tend to be followed by large changes of either signs and small changes tend to be followed by small changes (Neusser, 2016). Intuitively, the market becomes volatile whenever big news comes and it may take several periods for the market to fully digest the news. Volatility clustering suggest the presence of time-dependent variance that may be forecastable (Cho, 2020). A stochastic process Y_t whose standard deviation varies with time is said to be **heteroscedastic**. Otherwise, the process is said to be **homoscedastic**. Risk management, options pricing and portfolio selection can be possibly improved upon if predictability can be captured in volatility models.

3.1.2 Leverage Effects

First identified by Black (1976), leverage effects occur when negative shocks (bad news) tend to have a larger impact on volatility than positive shocks (good news). As a result, volatility tends to be higher in a falling market than in a rising market (Anderson *et al.*2009). This effect was attributed by Black (1976) to the fact that negative shocks tend to drive down the stock price thus increasing its debt - equity ratio. This will as a result increase the uncertainty of future events and consequently the volatility. Leverage effects usualy lead to an asymmetric news impact curve.

3.1.3 Heavy Tails and Non-Gaussianity

Fat or heavy tails and strong deviations from the normal distribution are usually observed when the unconditional distribution of the residuals of log returns is compared with that of the normal distribution. The degree of asymmetry and tail thickness is measured in statistics using skewness and kurtosis respectively defined as;

$$S(y) = E\left[\left(\frac{Y-\mu_y}{\sigma_y}\right)^3\right] \text{ and } K(y) = E\left[\left(\frac{Y-\mu_y}{\sigma_y}\right)^4\right] \quad (3)$$

For a normal distribution, $S(y)=0$ and $K(y)=3$. A distribution having heavy tails is said to have positive excess kurtosis $(K(y)-3)$ or leptokurtic and usually indicates the presence of more extreme values than in the normal distribution. The meaning of this is that the number and magnitude of crashes and booms is underestimated each time the normal distribution is used to model returns in financial time series (Jondeau *et al.* 2000). When the skewness of a distribution is different from zero, the distribution is asymmetric with a tail that extends more towards the right (positive or right skewness) or a tail that extends more towards the left (negative or left skewness). Thin tailed distributions such as the normal distribution have tails which decline **exponentially** fast while heavy tailed distribution such as the student's t and generalized error distributions have tails which decline by a **power** (Epaphra, 2017).

The Jarque-Bera (JB) test provides a formal method of testing how much the skewness and kurtosis of a distribution deviates from 0 and 3 respectively by testing the null hypothesis $H_0: Y_t \sim N(0,\sigma^2)$. It is given by;

$$JB = \frac{T}{6}\left[S(y) + \frac{\left(K(y)-3\right)^2}{4} \right] \sim \chi_2^2 \qquad (4)$$

where T is the sample size.

3.1.1 Uncorrelated Returns but Correlated Squared and Absolute Returns

Typically, and with the exception of the first few lags, log returns series are uncorrelated but not independent with a mean close to zero. The absolute and squared values of the return series are positively correlated, even for very large lags, and the correlation dies out very slowly (Zivot, 2016). It is the auto-correlation of squared returns that enables returns and volatility to be predictable.

3.2 Exchange Rate Regimes

The manner in which the value of the domestic currency is determined in terms of foreign currencies is referred to as exchange rate regime. Exchange rate regimes can be grouped into two main categories: floating and fixed regimes. When the value of the domestic currency is held constant with respect to a foreign and mostly more widespread currency such as US Dollars, Euro, Pound Sterling or a basket of currencies, through government intervention in the foreign exchange market, the regime is said to be fixed. A floating exchange rate regime is one in which the forces of demand and supply in the foreign exchange market determine the value of one currency with respect to another currency. The government and/or the central bank indirectly influence the exchange rate by managing the level of domestic and foreign currencies in the banking system. Between the fixed and floating exchange rate regimes are intermediate exchange rate regimes in which the central bank monitors to ensure that the exchange rate does not deviate too far from a specific target value.

Cameroon's XAF is fixed pegged to the EUR through the CEMAC monetary union at a parity rate of 1 Euro for 655.96 XAF francs (Gilde, A. M. and Tsangarides, C. G., 2008). On the other hand, USD and CNY enjoy a floating exchange rate regime. However, a currency fixed to another reserve currency continues to float with respect to other currencies (Suranovic, S., 2012). This means even though XAF is pegged to the EUR, it continues to float with the EUR with respect to the USD and CNY (Linge, I., 2022). Therefore, even though XAF exists in a fixed exchange rate regime with respect to the EUR, the fixed exchange rate regime cannot be applied to the USD/XAF and CNY/XAF exchange rates.

4 Model Construction

Many important models have been proposed in literature with the goal of improving the accuracy of time series modeling and prediction. Some of these models include: the Autoregressive models of order p written as AR (p) and defined as;

$$y_t = c + \sum_{i=1}^{p} a_i y_{t-i} + \varepsilon_t \qquad (5)$$

where y_t is the variable observed at time t, c is a constant, a_i are the autoregressive coefficients and $\varepsilon_t \sim WN(0,\sigma_t^2)$; the Moving Average model of order q written as MA(q) and defined as;

$$y_t = \varepsilon_t + \sum_{j=1}^{q} b_i \varepsilon_{t-j} \qquad (6)$$

where b_i are the moving average coefficients and the Autoregressive Moving Average model with orders p and q written as ARMA(p,q) and defined as;

$$y_t = c + \sum_{i=1}^{p} a_i y_{t-i} + \varepsilon_t + \sum_{j=1}^{q} b_i \varepsilon_{t-j} \qquad (7)$$

While the ARMA(p,q) model can be used to model and predict time series, it has the limitation that it only performs well on stationary time series (Wang *et al.* 2022).

Let F_{t-1} be the information available at time t-1. Then the conditional mean μ_t and the conditional variance σ_t^2 of the log returns y_t at time t is given by;

$$\mu_t = E\left(y_t / F_{t-1}\right) \text{ and } \sigma_t^2 = Var\left(y_t / F_{t-1}\right) = E\left[\left(y_t - \mu_t\right)^2 / F_{t-1}\right] \tag{8}$$

Given that the log returns y_t is usually either serially uncorrelated or weakly correlated (Dhamija and Bhalla, 2010), can be assumed to follow a simple time series model such as the stationary ARMA(p,q) model as defined in equation (7).

4.1 **Volatility Models**

The prices of financial assets are non-linear, dynamic, and chaotic since they are often shaken by large and time-varying shocks rendering their modeling and prediction difficult (Henrique *et al.*, 2019*)*. One of the major assumptions behind the Box and Jenkins methodology of time series analysis is homoscedasticity. If this assumption is violated, the estimated parameters via least squares regression will not be minimum variance estimates (Dhamija and Bhalla, 2010). Given that the volatility of log returns is heteroscedastic, models that model volatility tend to correct the deficiencies of the least square models.

4.1.1 **Symmetric Volatility Models**

The ARCH (p) Model

The Autoregressive Conditional Heteroscedasticity (ARCH) model of Engle (Engle, 1982) is one of the first prominent tools that emerged to characterize changing variance in time series. Engle described the conditional variance by a simple quadratic function of its own lagged values as defined in equations (9) and (10) referred to as the mean and variance equations respectively.

$$y_t = \mu_t + \varepsilon_t \quad \text{where} \quad \varepsilon_t = \sigma_t z_t \quad \text{and} \quad z_t \sim D(0,1) \tag{9}$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 \tag{10}$$

Heteroscedastic characteristics such as leptokursis and volatility clustering are not demonstrated by the mean equation (9). This necessitated the introduction by Engle (1982) of the variance equation (10). σ_t^2 and σ_t are the conditional variance and volatility of the residuals ε_t respectively and the standardized residuals z_t is white noise, assumed to be independent of and follows a distribution with probability density function (pdf) D(0, 1). To ensure that remains positive at all times, and . One of the drawbacks of the ARCH model is that large lag values are required to determine the optimal lag length in the modeling of financial data. This necessitated the use of many parameters which oftentimes could lead to over parametrization (Rydberg, 2000).

The GARCH(p,q) Model

The Generalized ARCH (GARCH) was subsequently introduced independently by Bollerslev (1986) and Taylor (1986) by extending the basic ARCH model with a view to achieving parsimony. The main idea is that the conditional variance at time t has an autoregressive structure and is positively correlated to its own recent past and to the recent past values of the square of the shocks. The mean equation for this model is same as equation (9) and the variance equation is defined by equation (11) where $\omega > 0$, $\alpha_i \geq 0$ and $\beta_j \geq 0$.

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \tag{11}$$

Its unconditional variance is

$$\sigma^2 = \frac{w}{1 - \sum_{i=1}^{p} \alpha_i - \sum_{j=1}^{q} \beta_j} \text{ and if } \sum_{i=1}^{p} \alpha_i + \sum_{j=1}^{q} \beta_j < 1,$$

the GARCH(p,q) model is stationary (Poon, 2005). This model has the advantage that it reflects the heteroscedastic characteristic of financial returns even though it still fails to capture the asymmetric characteristic of financial data (Wang *et al.* 2022).

4.1.2 Asymmetric GARCH Models

One of the weaknesses of the ARCH and GARCH models is the assumption that good and bad news have the same effect on volatility meanwhile it has been shown that volatility sometimes responds differently to good news (positive shocks) and bad news (negative shocks) of the same magnitude (Lim and Sek, 2013). In order to take into consideration this asymmetric characteristic of volatility, various extensions of the GARCH model were developed, some of which include:

The EGARCH (p,q) Model

The Exponential GARCH (EGARCH) model developed by Nelson (1991) captures the asymmetric response of the time varying volatility to shocks in the GARCH model by including an asymmetric parameter in the model. This model ensures that the conditional variance is always positive by modeling the natural logarithm of the variance and does not place any restrictions on the sign of the parameters of the model. While the mean equation is similar to equation (9), the variance equation is defined by equation (12).

$$\log\left(\sigma_t^2\right)=\omega+\sum_{i=1}^{p}\left[\alpha_i\left(\frac{|\varepsilon_{t-i}|}{\sigma_{t-i}}-\sqrt{\frac{2}{\pi}}\right)+\gamma_i\left(\frac{\varepsilon_{t-i}}{\sigma_{t-i}}\right)\right]+\sum_{j=1}^{q}\beta_j\log\left(\sigma_{t-j}^2\right) \qquad (12)$$

α_i measures the magnitude of the shock $\frac{\varepsilon_{t-1}}{\sigma_{t-1}}=z_{t-1}$, β_j measures the persistence in conditional volatility of the shocks to the market while γ_i is the asymmetric parameter measuring the leverage effect. In the case of a crisis in the market, the volatility will take a long time to die out for relatively large values of β_j (Alexander, 2009). The model is asymmetric when $\gamma_i \neq 0$. If $\gamma_i < 0$ it is expected that bad news $(z_{t-1}<0)$ would have a higher impact on volatility than good news $(z_{t-1}>0)$ known as the leverage effect (Atoi, 2014 and Epaphra, 2017). This is because when, γ_i<0 the shock effect $(\alpha_i+\gamma_i)z_{t-i}$ for $z_{t-i}<0$ and $(\alpha_i-\gamma_i)z_{t-i}$ for $z_{t-i}>0$

According to Poon (2005), this model is stationary if $\sum_{j=1}^{q}\beta_j<1$

The GJR-GARCH (p,q) Model

The Glosten-Jahannathan-Runkle GARCH (GJR-GARCH) model of Glosten, Jahannathan and Runkle (Glosten *et al. 1993)* is a simple extension of the GARCH model with an additional term to account for possible asymmetry in volatility modeling (Brooks, 2008). The variance equation is defined as;

$$\sigma_t^2=\omega+\sum_{i=1}^{p}\left(\alpha_i+\gamma_iI_{t-i}\right)\varepsilon_{t-i}^2+\sum_{j=1}^{q}\beta_j\sigma_{t-j}^2 \qquad (13)$$

where the indicator function $I_{t-i}=0$ if $\varepsilon_{t-i}\geq 0$ (good news) and 1 otherwise. Thus, good news and bad news in the market have different effects on the model (Wang *et al.,* 2022). The conditional volatility is positive when $\omega>0$, $\alpha_i\geq 0$, $\beta_j\geq 0$ and $\alpha_i+\gamma_i\geq 0$. Leverage effect occurs when $\gamma_i>0$. This model is stationary if $\sum_{i=1}^{p}(\alpha_i+\frac{1}{2}\gamma_i)+\sum_{j=1}^{q}\beta_j<1$ (Poon, 2005).

4.2 Parameter Estimations and Distributional Assumptions

Ordinary Least Squares (OLS) Regression methods are not able to estimate the parameters in GARCH volatility models and that of its variants since these models are non-linear in nature. The Maximum Likelihood Estimate (MLE) technique proposed by Bollerslev and Wooldridge (1992) is one of the most appropriate and widely used procedures for estimating the parameters of GARCH models. This is done by finding the most likely values of the parameters given the actual data (Epaphra, 2017). A log-likelihhod function is computed and the the values of the parameters that maximize the function are calculated (Brooks, 2008). This method requires the specification of the distribution of the innovation process ε_t. Details of the general procedure for Maximum Likelihood Estimation can be seen in (Zivot, 2016).

Engle (1982) and Bollerslev (1986) proposed the normal error distribution to estimate ARCH and GARCH volatility model parameters such that $z_t \sim iidN(0,1)$. Let T be the sample size, then the normal distribution has distribution function as defined in equation (14) and the log-likelihood function as defined in equation (15).

$$f(z_t) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z_t^2}, -\infty < z_t < \infty \tag{14}$$

$$L(z_t) = -\frac{1}{2}\left[T\log(2\pi) + \sum_{t=1}^{T} z_t^2\right] \tag{15}$$

Even though it was initially assumed that the innovation ε_t is Gaussian, significant empirical evidence suggests that financial time series is usually leptokurtic such that the unconditional distribution of usually display heavier tails and a more pronounced peak than can be captured by the Gaussian distribution (Bollerslev *et al.* 1992). This led Bollerslev (1987) to suggest replacing the Gaussian distribution with that of a conditional student's t distribution which has some characteristics of "heavy tailness" and produces a better fit for most asset return series (Sheppard, 2021). He defines the standardized student's t distribution of the standardized residuals and its log-likelihood function as given in equations (16) and (17) respectively.

$$f(z_t, v) = \frac{\Gamma\left(\frac{v+1}{2}\right)}{\Gamma\left(\frac{v}{2}\right)\sqrt{\pi(v-2)}\left(1+\frac{z_t^2}{v-2}\right)^{\frac{v+1}{2}}} \tag{16}$$

$$L(z_t, v) = \frac{1}{2}\left[T\log\left(\frac{\Gamma\left(\frac{v+1}{2}\right)^2}{\pi(v-2)\Gamma\left(\frac{v}{2}\right)^2}\right) - (v+1)\sum_{t=1}^{T}\log\left(1+\frac{z_t^2}{v-2}\right)\right] \tag{17}$$

The random variable in the student's t distribution is taken to a power rather than to an exponential as in the normal distribution. This gives higher values for $f(z_t, v)$ when z_t is far from zero (fatter tails). v is the number of degrees of freedom (df) which measures the degree of fat or heavy tails in the density: $v > 2$. The student's t distribution converges to the normal distribution as $v \to \infty$.

Nelson (1991) also proposed the Generalized Error Distribution (GED) in the modeling of financial time series with heavy tails. This distribution has a probability function and log-likelihood function as given in equations (18) and (19).

$$f(z_t, v) = \frac{v e^{-\frac{1}{2}\left|\frac{z_t}{\kappa}\right|^v}}{\kappa . 2^{\left(\frac{v+1}{v}\right)}\Gamma\left(\frac{1}{v}\right)} \quad \text{where} \quad \kappa = \left[\frac{2^{-\frac{2}{v}}\Gamma\left(\frac{1}{v}\right)}{\Gamma\left(\frac{3}{v}\right)}\right]^{\frac{1}{2}} \tag{18}$$

$$L(z_t, v) = -\frac{1}{2}\log\left[\frac{\Gamma\left(\frac{1}{v}\right)^3}{\Gamma\left(\frac{3}{v}\right)\left(\frac{v}{2}\right)^2}\right] - \frac{1}{2}\log\sigma_t^2 - \left[\frac{\Gamma\left(\frac{3}{v}\right)z_t^2}{\Gamma\left(\frac{1}{v}\right)}\right]^{\frac{v}{2}} \tag{19}$$

When $v = 2$, $f(z_t, v)$ becomes the normal distribution. When $v < 2$, has thicker tails than the normal density and when, has thinner tails than the normal density. The value of v is estimated during Maximum Likelihood Estimation. This paper uses the normal, student's t and generalized error distributions together with their skewed versions to estimate conditional volatility.

3 Empirical Analysis and Results

3.1 Data

This research uses the daily closing USD/XAF and CNY/XAF exchange rates. Exchange rate refers to the number of units of one currency that exchanges for a unit of another currency (Suranovic, S., 2012). The datasets were downloaded from www.investing.com between 01 January 2017 and 30 September 2022. Each of the data sets contains 1499 data points out of which 1304 (from 01 January 2017 to 31 December 2021) is used for in-sample model estimation and 195 (from 01 January 2022 to 30 September 2022) for out-of-sample model validation and prediction.

The graph of the exchange rates is shown in Fig 1 which is clearly non-stationary. The first three months of 2018 witnessed one of the worst drops in the USD/XAF exchange rate. One of the major observations from Fig 1 is the sudden drop in the value of other currencies compared to XAF at the onset of the COVID-19 pandemic. Countries that experienced so many deaths resulting from the Covid-19 pandemic such as the United States of America and China equally experienced a weakening in the value of their currencies (Jamal and Bhat, 2022). The years 2019 and 2020 witnessed a marked drop in the CNY/XAF exchange rate. Around the beginning of 2021, the USD/XAF and CNY/XAF exchange rates started rising steadily.

Equation (1) is used to calculate the daily exchange rate returns denoted and their graphs shown in Fig 2. The graphs depict clustering volatility suggesting the presence of forecastable time dependent variance or heteroscedasticity. High volatility can be noticed in both exchange rates around March 2018 and March 2020. These periods correspond to the announcement by President Trump that the US will soon impose tariffs on goods imported from China and when the COVID-19 pandemic triggered a marked decline in global trade respectively. The CNY/XAF exchange rates, which used to be relatively stable, has been highly volatile since the onset of the COVID-19 pandemic. This is indicative of great uncertainty in the Chinese economy. It can also be noticed from Fig 2 that the exchange rate returns have a mean of virtually zero but with non-constant variance.

Fig 1. USD/XAF and CNY/XAF Daily Exchange Rate.

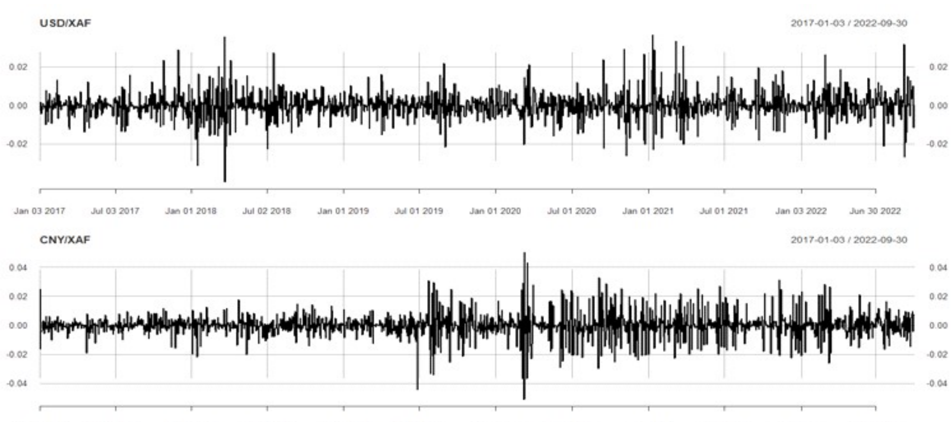


Fig 2. USD/XAF and CNY/XAF Daily Exchange Rate.

3.1 In-Sample Descriptive Statistics for Daily Exchange Rate Returns.

Table 1 contains summary statistics for the USD/XAF and CNY/XAF in-sample daily exchange rates returns from 01 January 2017 to 31 December 2021. The mean for box exchange rates are close to zero. USD/XAF is positively skewed which means the distribution has a long right tail and as a result, the return series rises more often than it drops while CNY/XAF is negatively skewed meaning the distribution has a long-left tail or there are more negative than positive outlying returns. Excess kurtosis for both returns indicates heavy tailness or leptokursis.

This departure from normality is further confirmed by the Jarque-Bera (JB) test which rejects the null hypothesis that the data is normally distributed given the JB value and corresponding small p-value (<0.05).

Ljung-Box and ARCH LM Test

The modified Ljung-Box (LB) test is used to test up to a specific lag, the null hypothesis of no autocorrelation while Engle's Lagrange Multiplier (LM) test tests the null hypothesis of no remaining ARCH effects in the residuals of the mean equation (Dhamija and Bhalla, 2010). The LB test for returns and squared returns show that both the are auto correlated at 20 lags and 5% level of significance. This is shown in Table 2 and Fig A1-A3.

The Augmented Dickey Fuller (ADF) test is used to confirm the stationarity of the exchange rate returns. The ADF test has as null hypothesis that the exchange rate returns have a unit root. From Table 2, the p-value for all the exchange rate returns is 0.01 which is less than 0.05. The null hypothesis is therefore rejected, and the conclusion can be made that all the exchange rate returns are stationary.

Table 1: **In-Sample Descriptive Statistics for USD/XAF and CNY/XAF Daily Exchange Rate**

	Mean	Median	Maximum	Minimum	Standard Deviation	Skewness	Kurtosis	Sample Size
USD/XAF	-4.5E-05	-3.4E-05	0.03651	-0.03986	6.653E-03	0.38333	5.26988	1304
CNY/XAF	5.3E-05	0.00000	0.05026	-0.05082	8.398E-03	-0.06938	5.42551	1303

Table 2: **JB, LB and ADF tests for USD/XAF and CNY/XAF Daily Exchange Rate**

	JB test (p-value)	LB test 20 lags (p-value)	LB test squared 20 lags (p-value)	ADF test (p-value)
USD/XAF	123.83 (< 2.2e-16)	123.83 (< 2.2e-16)	159.34 (< 2.2e-16)	-11.894 (0.01)
CNY/XAF	1606.8 (< 2.2e-16)	216.22 (< 2.2e-16)	529.94 (2.2e-16)	-12.273 (0.01)

JB=Jarque-Bera, LB=Ljung-Box, ADF=Augmented Dickey Fuller, p-values in brackets

Heteroscedasticity Test

The ARMA(p,q) model is used as a flexible and parsimonious approximation to the conditional mean given the autocorrelation function of the returns and squared returns as shown in Fig A1 to A3. Fig A1 shows no autocorrelation in the returns (except for the first few lags) while Fig A2 and Fig A3 show autocorrelation in the squared returns. The Automatic Model Selection for Autoregressive Fractionally Integrated Moving Average (autoarfima) model in r is used to select the best fitting ARMA model based on Information criteria. The Akaike Information Criteria (AIC) is used in this paper and measures the trade-off between model fit and complexity and is given by;

$$AIC = -2\ln(L) + 2k \tag{20}$$

where L is the log-likelihood estimate and k is the number of estimated parameters. The model with the least AIC is the most parsimonious model as shown on Table 3.

Table 3: Conditional Mean ARMA model and ARCH LM test for In-Sample Exchange Rate

Returns	AIC	Model	Conditional Mean Equation	ARCH LM test
USD/XAF	-7.273109	ARMA(0,1)	$y_t = c + \varepsilon_t + b_1\varepsilon_{t-1}$	χ^2 (18) = 40.353 (p-value = 0.001868)
CNY/XAF	-6.891973	ARMA(1,1)	$y_t = a_1 y_{-1} + \varepsilon_t + b_1\varepsilon_{t-1}$	χ^2 (27)=100.98 (p-value = 1.778e-10)

Before applying GARCH methodology to modeling the exchange rate returns, the residuals e_t from the ordinary least squares regression of the conditional mean equation must first be examined for heteroscedasticity (Abdalla, 2012). e_t^2 is regressed on r lags of the residuals and a constant as shown in equation (21).

$$e_t^2 = \delta_0 + \sum_{i=1}^{r} \delta_i e_{t-i} + \varepsilon_t \tag{21}$$

where ε_t is the error term. This test is known as Engle's Lagrange Multiplier (LM) Test and tests the null hypothesis that there is no ARCH effect $H_0 : \delta_i = 0$ for $i = 1, 2, ..., r$ against the alternative hypothesis $H_1 : \delta_i \neq 0$ for at least one i. The test statistic is $TR^2 \sim \chi^2(r)$ where T is the sample size and R^2 is the coefficient of determination (Neusser, 2016). A plot of the ACF of e_t^2 indicates the number of lags. Results of the ARCH-LM test is shown in Table 3 where the null hypothesis is rejected at the 5% level of significance showing the presence of ARCH effects in the residual series of all the exchange rate returns. Hence, GARCH models can be used to model the volatility of the returns of the USD/XAF and CNY/XAF exchange rates.

5.3 Result of the Estimated Volatility Models

The Quantile-Quantile (QQ) plot is used to compare the empirical distribution of the returns to those from a normal distribution (Neusser, 2016). Fig 3 indicates that the residuals are not normally distributed but exhibit heavy tails at both ends as can be further seen in the kernel distribution of Fig 4. This means that, compared to the normal distribution, the probability of obtaining large returns is bigger. For the residuals of USD/XAF, the right tail is heavier than the left tail indicating positive skewness while that for CNY/XAF has negative skewness with the left tail heavier than the right tail. In addition to the normal distributional assumption, this paper will make use of the student's t and generalized error distributions in modeling the volatility of the exchange rate returns. GARCH, EGARCH and GJR-GARCH models will also be used in volatility modeling of the residuals in order that the symmetric and asymmetric aspects of the distributions should be taken into consideration.

The results of volatility modeling for the in-sample data sets for the USD/XAF and CNY/XAF Daily Exchange Rate Returns are shown in Tables A1and A2 of the Appendix. The Akaike Information Criteria (AIC) is used to select the best model for each of the exchange rate returns. This paper uses AIC since it is good for prediction as it approximately minimizes the prediction error and is asymptotically equivalent to cross-validation (Stone, 1977).

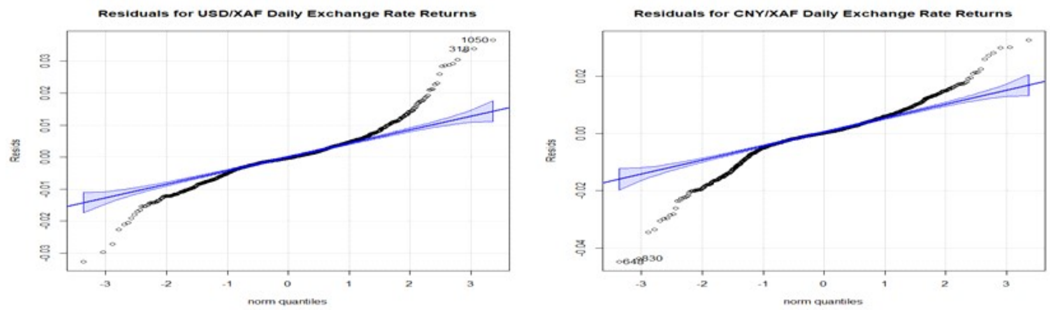
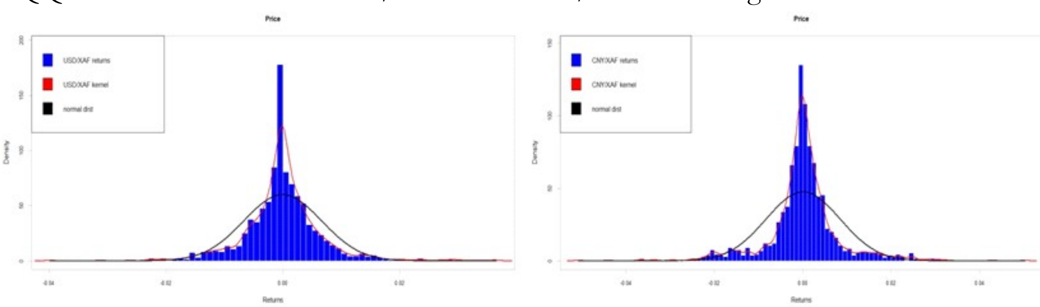


Fig 3. QQ Plot of Residuals for USD/XAF and CNY/XAF Exchange Rate



Fid 4. Kernel of USD/XAF and CNY/XAF compared to the Normal Distribution

Estimation Results for volatility of USD/XAF Exchange Rate Returns

From Table A1, it can be seen that the volatility of the USD/XAF is best described by the Skewed Generalized Error Distribution (SGED). While ARMA(0,1)+GARCH(1,1) and ARMA(0,1)+EGARCH(1,1) have very low AIC values, the model with the least AIC is ARMA(0,1)+GJR-GARCH(1,1) and is the most parsimonious model to describe the volatility of the USD/XAF daily exchange rate returns. An estimate of the parameters for ARMA(0,1)+GJR-GARCH(1,1)-SGED is shown on Table 4.

From Table 4, all estimated parameters are statistically significant at 1% level of significance. The model is stationary since $\alpha_1 + \frac{1}{2}\gamma_1 + \beta_1 < 1$. $\beta_1 = 0.608172$ shows some persistence in the volatility. This means that if a crisis hits the market, the volatility takes some time to die out (Epaphra, 2017). The model is equally asymmetric with no leverage effects since $\gamma_1 < 0$. This suggests that good news causes the USD/XAF exchange rate returns to be more volatile than bad news of the same magnitude as can be seen on the News Impact Curve (NIC) of Fig 5. The Ljung-Box and ARCH LM Test on standardized residuals of USD/XAF exchange rate are shown in Table 5. Since all the p-values at the given lags are greater than the significance level of 0.05, we fail to reject the null hypothesis at the 5% level of significance and conclude that our model adequately fits the data and there is no evidence of correlation in the residuals and no dependence in conditional variance. There are equally no more ARCH effects in the residuals.

Table 4: Parameter Estimates for ARMA(0,1)+GJR-GARCH(1,1)-SGED model

Parameter	Estimate	Standard Error	p - value
μ	0.000115	0.000019	0.000000
MA(1)	- 0.111067	0.006819	0.000000
ω	0.000008	0.000000	0.000000
α_1	0.323768	0.027912	0.000000
β_1	0.608172	0.014200	0.000000
γ_1	-0.181002	0.030358	0.000000
skew	1.054796	0.015802	0.000000
shape	0.825225	0.037935	0.000000

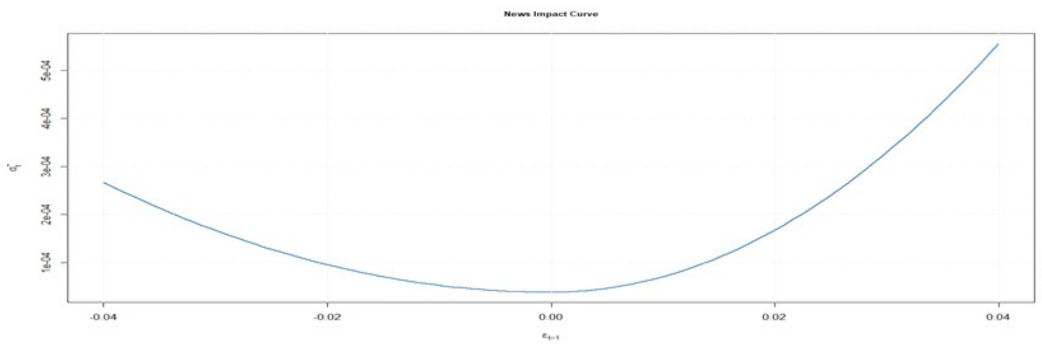


Fig 5. News Impact Curve (NIC) for the volatility of USD/XAF Exchange Rate

Table 5: Ljung-Box and ARCH LM Test on standardized residuals of USD/XAF Exchange Rate

Ljung-Box Test			ARCH LM Test		
lag	statistic	p-value	lag	statistic	p-value
1	0.7375	0.3905	3	1.061	0.3030
5	1.6100	0.7178	5	1.316	0.6420
9	1.9891	0.9072	7	1.364	0.8481

Estimation Results for volatility of CNY/XAF Exchange Rate Returns.

The volatility of the CNY/XAF equally seems to follow the SGED with ARMA(1,1)+ GARCH(1,2) and ARMA(1,1)+EGARCH(2,1) having low values for the AIC while ARMA(1,1)+GJR-GARCH(2,2) has the lowest AIC value and will be used to model the volatility of the CNY/XAF exchange rate returns. An estimate of the parameters is shown in Table 6. From the table, all parameters except β_2 are statistically significant at 1% level of significance. The value of $\sum_{i=1}^{2}\left(\alpha_i+\frac{1}{2}\gamma_i+\beta_i\right)<1$ which makes the model stationary. $\beta_1 = 0.848319$ indicates persistent volatility. The value of $\gamma_1 > 0$ indicates the presence of asymmetry and leverage effects. This can be seen on the NIC in Fig 6. From Table 7, there is no evidence of correlation in the residuals and no ARCH effects.

Table 6: Parameter Estimates for ARMA(1,1)+ GJR-GARCH(2,2)-SGED model for CNY/XAF Exchange Rate

Parameter	Estimate	Standard Error	p - value
μ	-0.000285	0.000017	0.000000
AR(1)	0.273863	0.009433	0.000000
MA(1)	-0.448543	0.010421	0.000000
ω	0.000002	0.000000	0.000000
α_1	0.046030	0.000911	0.000000
α_2	0.008717	0.000599	0.000000
β_1	0.848319	0.010858	0.000000
β_2	0.001979	0.001217	0.1038
γ_1	0.435751	0.022138	0.000000
γ_2	-0.300671	0.001633	0.000000
skew	0.937585	0.010496	0.000000
shape	0.828022	0.033997	0.000000

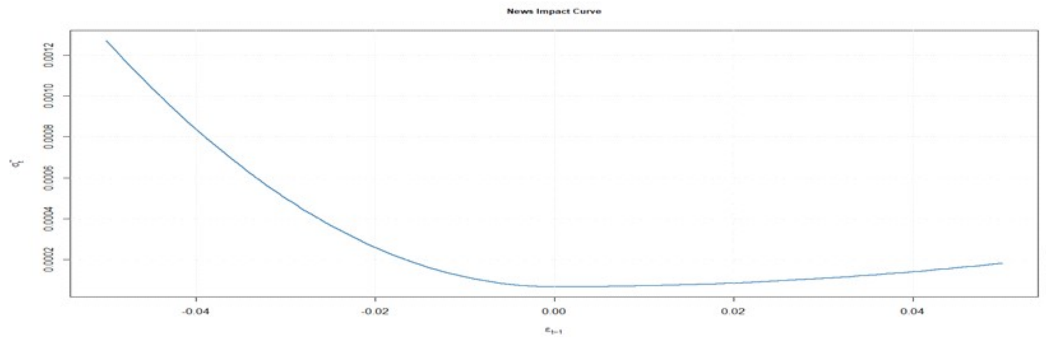


Fig 6. News Impact Curve (NIC) of the volatility of CNY/XAF Exchange Rate

Ljung-Box Test			ARCH LM Test		
lag	statistic	p-value	lag	statistic	p-value
1	1.607	0.2049	5	0.2152	0.6427
11	5.323	0.5321	7	1.3415	0.6640
19	10.272	0.4222	9	2.1147	0.7362

A major result from this analysis is the fact that the volatility of all the returns are better described by non-Gaussian Distributions and in particular, the Skewed Generalized Error distribution. This confirms previous findings that GARCH models with Gaussian residuals do not fully capture some of the stylized facts such as leptokursis found in financial data (Bollerslev *et al.* 1992).

5.4 **Predicting Conditional Volatility from GARCH Models**

Accurate prediction of the future values of the conditional volatility of financial time series returns is very important in the modeling of financial time series. Risk management, option pricing, portfolio allocation, trading strategies and model evaluation all make use of volatility prediction. Once the parameters in a given GARCH model has been estimated, the model can be used for prediction. This paper uses 195 trading days of out-of-sample prediction of the conditional volatility of exchange rate returns using loss functions.

Loss Functions

Among the three GARCH models with the lowest AIC that describe each of the exchange rate returns as shown in Tables A1 and A2, the best model for out-of-sample prediction will be the one with the lowest loss function. A loss function summarizes the errors due to prediction. In this paper, the Mean Absolute Error (MAE) and Root Mean Square Error (RMSE) will be used to determine the best model for prediction. Given their simple mathematical forms, MAE and RMSE are among the most popular statistical loss functions for evaluating the forecasting power of a model (Vee *et al.* 2011). MAE and RMSE are defined as;

$$MAE = \frac{1}{k}\sum_{t=T+1}^{T+k}\left|\sigma_t^2 - \hat{\sigma}_t^2\right| \quad (22)$$

$$RMSE = \sqrt{\frac{1}{k}\sum_{t=T+1}^{T+k}\left(\sigma_t^2 - \hat{\sigma}_t^2\right)^2} \quad (23)$$

where σ_t^2 and $\hat{\sigma}_t^2$ are the actual and predicted volatilities of the exchange rate returns at time t. k predictions are carried out from t=T+1 to t=T+k. The results are shown in Table 8.

From Table 8, the best predictive model for the conditional volatility of the USD/XAF and CNY/XAF exchange rate returns are ARMA(0,1)+GARCH(1,1)-SGED and ARMA(1,1)+GJR-GARCH(2,2)-SGED respectively since they have the smallest values for MAE and RMSE. The plots of the predicted returns and volatility are shown in red on Fig 7 - 8.

Table 8: Values for Mean Absolute Error (MAE) and Root Mean Square Error (RMSE)

Returns	Model	Distribution	MAE	RMSE
USD/XAF	**ARMA(0,1)+GARCH(1,1)**	**SGED**	**0.006561**	**0.006803**
	ARMA(0,1)+EGARCH(1,1)	SGED	0.006628	0.006909
	ARMA(0,1)+GJR-GARCH(1,1)	SGED	0.006632	0.006960
CNY/XAF	ARMA(1,1)+GARCH(1,2)	SGED	0.008457	0.008899
	ARMA(1,1)+EGARCH(2,1)	SGED	0.008591	0.008839
	ARMA(1,1)+GJR-GARCH(2,2)	**SGED**	**0.008409**	**0.008704**

6. Conclusion and Policy Implications

This paper models and predicts the volatility of the USD/XAF and CNY/XAF exchange rates between 01 January 2017 and 30 September 2022. While GARCH model is used to capture heteroscedasticity, the EGARCH and GJR-GARCH models are employed to capture asymmetry and leverage effects in the data. The results suggest the presence of conditional heteroscedastcity and persistent volatility in both exchange returns where shocks are felt further in the future. Thus, by the Akaike Information Criteria (AIC), the USD/XAF exchange rate volatility can be adequately estimated by the ARMA(0,1)+GJR-GARCH(1,1)-SGED model and that for the CNY/XAF exchange rate by the ARMA(1,1)+GJR-GARCH(2,2)-SGED model. The results of the Mean Absolute Error (MAE) and Root Mean Square Error (RMSE) out-of-sample predicted volatility show that ARMA(0,1)+GARCH(1,1)-SGED and ARMA(1,1)+GJR-GARCH(2,2)-SGED have the best predictive power for the USD/XAF and CNY/XAF exchange rates respectively.

Volatility is persistent in both exchange rate returns though the persistence in the CNY/XAF exchange rate returns ($\beta_1 = 0.848319$) is greater than that for USD/XAF ($\beta_1 = 0.608172$). This indicates that volatility shocks will take a longer time to die out with the CNY/XAF exchange rate than the USD/XAF exchange rate. While the CNY/XAF exchange rate have been found to exhibit leverage effects, no leverage effects have been found in the USD/XAF exchange rate. This means bad news causes the CNY/XAF exchange rate to be more volatile than good news and vice versa for the USD/XAF exchange rate. The volatility of the CNY/XAF exchange rate shows a larger reaction to past negative shocks than to positive shocks of the same size. This is the reverse with the volatility of the USD/XAF exchange rate. The consequence of this result is that, an unanticipated decrease in the CNY/XAF exchange rate would lead to a higher level of uncertainty when compared to the level of uncertainty that would result from an unanticipated increase. On the contrary, an unanticipated increase in the USD/XAF exchange would instead produce a higher level of uncertainty than an unanticipated decrease. These results are very useful to investors as the underlying model can help them to make proper investment decisions. In the design of appropriate exchange rate policies, Cameroon's monetary authorities and BEAC should take into consideration the fact that the exchange rate market is very volatile and reacts differently to both good and bad news. This is particularly important in financial transactions between Cameroon and China on the one hand, and Cameroon and the United States of America on the other hand since the results of this research indicate that the USD/XAF and CNY/XAF exchange rates respond differently to good and bad news. BEAC should also garner enough financial reserves to be able to absorb financial shocks to the foreign exchange structure and be able to offer timely intervention to reduce or mitigate the effects of persistent volatility on the Cameroonian economy

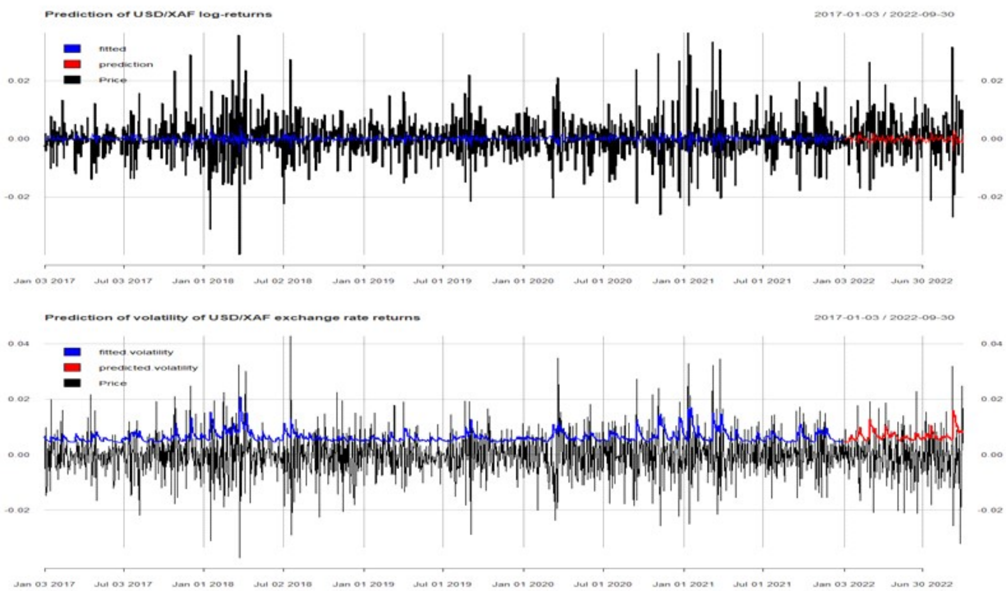


Fig 7: Out-of-sample prediction of USD/XAF log returns and volatility

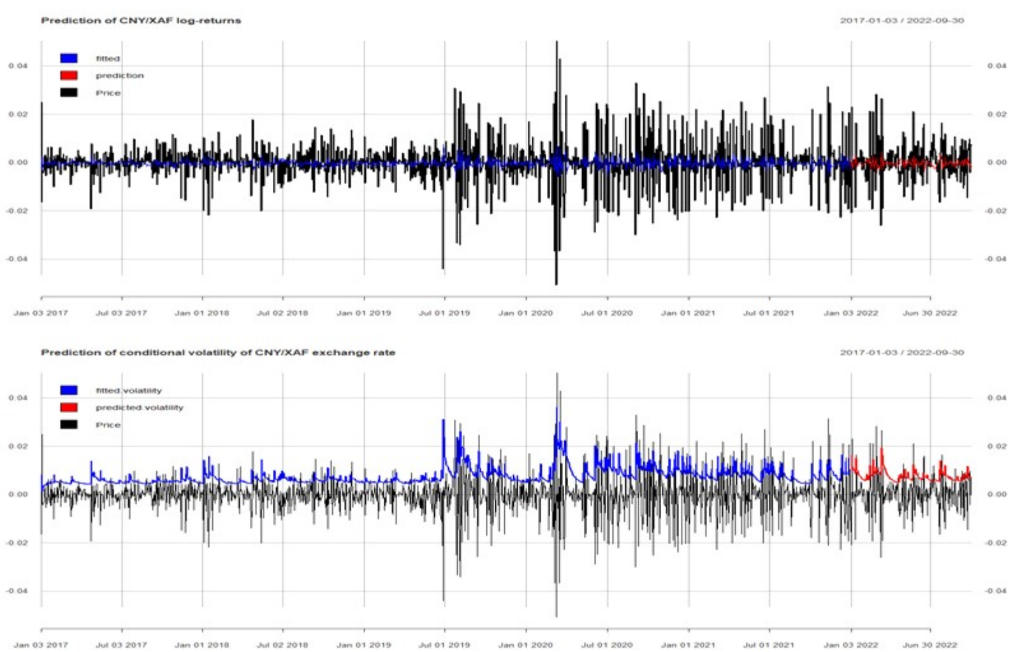


Fig 8: Out-of-sample prediction of CNY/XAF log returns and volatility

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APPENDIX A

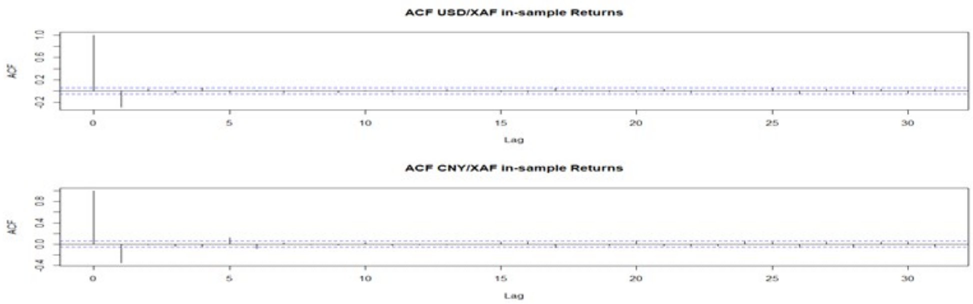

Fig A1. ACF of USD/XAF and CNY/XAF Exchange Rate Returns

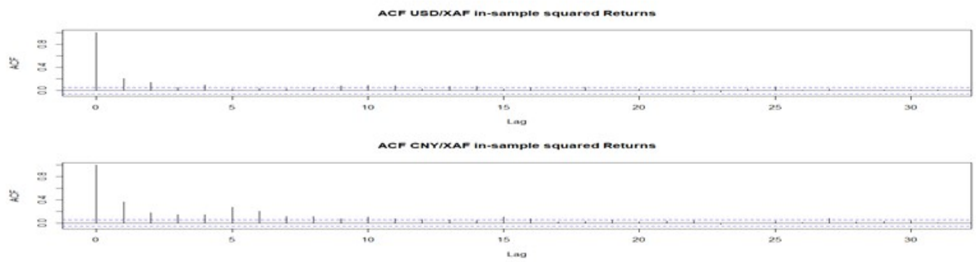

Fig A2. ACF of USD/XAF and CNY/XAF Exchange Rate Returns Squared

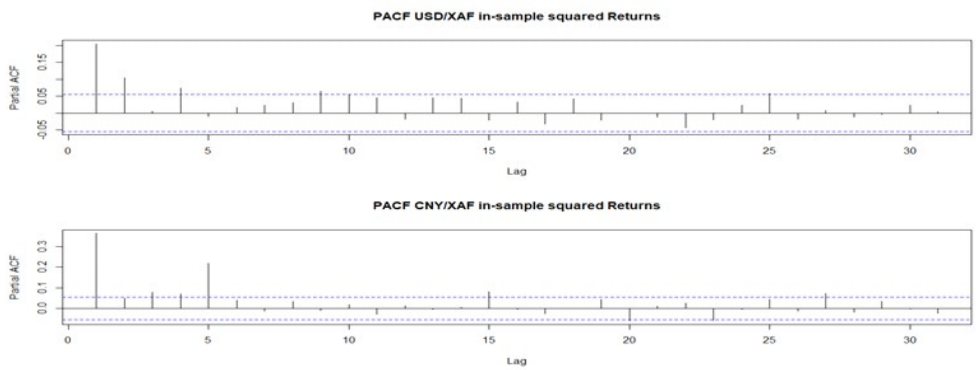

Fig A3. PACF of Squared Returns

Table A1. Estimation Results of GARCH models for USD /XAF Daily Exchange Rate Returns

Model	ND	SND	STD	SSTD	GED	SGED	Mini. AIC
ARMA(0,1)+GARCH(1,1)	-7.373709	-7.393089	-7.561264	-7.562324	-7.585710	**-7.588435**	**-7.588435**
ARMA(0,1)+GARCH(1,2)	-7.371869	-7.391240	-7.559745	-7.560889	-7.584084	**-7.586896**	
ARMA(0,1)+GARCH(2,1)	-7.372003	-7.391416	-7.559521	-7.560592	-7.583908	**-7.586656**	
ARMA(0,1)+GARCH(2,2)	-7.373613	-7.392298	-7.558211	-7.559355	-7.582616	**-7.585320**	
ARMA(0,1)+EGARCH(1,1)	-7.3597	-7.3781	-7.5565	-7.5583	-7.5917	**-7.5944**	**-7.5944**
ARMA(0,1)+EGARCH(1,2)	-7.3580	-7.3764	-7.5555	-7.5575	-7.5904	**-7.5935**	
ARMA(0,1)+EGARCH(2,1)	-7.3575	-7.3766	-7.5559	-7.5581	-7.5896	**-7.5929**	
ARMA(0,1)+EGARCH(2,2)	-7.3584	-7.3774	-7.5550	-7.5574	-7.5885	**-7.5919**	
ARMA(0,1)+GJR-GARCH(1,1)	-7.3719	-7.3915	-7.5601	-7.5621	-7.5944	**-7.5968**	**-7.5968**
ARMA(0,1)+GJR-GARCH(1,2)	-7.3701	-7.3897	-7.5586	-7.5607	-7.5927	**-7.5953**	
ARMA(0,1)+GJR-GARCH(2,1)	-7.3698	-7.3906	-7.5577	-7.5599	-7.5915	**-7.5941**	
ARMA(0,1)+GJR-GARCH(2,2)	-7.3708	-7.3912	-7.5562	-7.5585	-7.5900	**-7.5926**	

KEY: ND=Normal Distribution, SND=Skewed Normal Distribution, STD=Student's t Distribution, SSTD=Skewed Student's t Distribution, GED=Generalized Error Distribution, SGED=Skewed Generalized Error Distribution

Table A2. Estimation Results of GARCH models for CNY/XAF Daily Exchange Rate Returns

Model	ND	SND	STD	SSTD	GED	SGED	Min. AIC
ARMA(1,1)+GARCH(1,1)	-7.098824	-7.125723	-7.300183	-7.304759	-7.315650	**-7.316468**	**-7.321300**
ARMA(1,1)+GARCH(1,2)	-7.102625	-7.131592	-7.306356	-7.311508	-7.319575	**-7.321300**	
ARMA(1,1)+GARCH(2,1)	-7.097521	-7.125150	-7.298933	-7.303731	-7.314311	**-7.315230**	
ARMA(1,1)+GARCH(2,2)	-7.103409	-7.132007	-7.304821	-7.309973	-7.318040	**-7.319858**	
ARMA(1,1)+EGARCH(1,1)	-7.0879	-7.1310	-7.2983	-7.3086	-7.3307	**-7.3328**	**-7.3383**
ARMA(1,1)+EGARCH(1,2)	-7.0907	-7.1341	-7.3054	-7.3156	-7.3341	**-7.3369**	
ARMA(1,1)+EGARCH(2,1)	-7.0902	-7.1335	-7.3083	-7.3195	-7.3355	**-7.3383**	
ARMA(1,1)+EGARCH(2,2)	-7.0915	-7.1332	-7.3069	-7.3181	-7.3340	**-7.3366**	
ARMA(1,1)+GJR-GARCH(1,1)	-7.1093	-7.1433	-7.3017	-7.3141	-7.3362	**-7.3387**	-7.3441
ARMA(1,1)+GJR-GARCH(1,2)	Convergence Problem	-7.1477	-7.3054	-7.3167	-7.3388	**-7.3415**	
ARMA(1,1)+GJR-GARCH(2,1)	-7.1148	-7.1529	-7.2987	-7.3057	**-7.3425**	-7.3420	
ARMA(1,1)+GJR-GARCH(2,2)	-7.1133	-7.1514	-7.3096	-7.3102	-7.3408	-7.3441	