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Probabilistic slope stability analysis: a case study of Ain Bouzian landslide within North-East Algeria

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Abstract. The aim of this study is to carry out a probabilistic analysis of slope stability within homogenous soil profile of Ain Bouzian landslides located north-east Algeria. This region suffers of many instability problems, which is aggravated during the construction of East-West motorway. The random variables for the soil considered in this study are the Young modulus E, cohesion C and friction angle φ. Stochastic soil properties values are used to evaluate the reliability and to assess sensitivity of the system, for two hydrological conditions, with and without water table within the soil mass. The deterministic model is based on Strength Reduction Method (SRM) analysis using the finite difference code FLAC3D.

Key words: Factor of safety; failure probability, probabilistic method, Monte Carlo simulation, strength reduction method.

1. Introduction

The problems of the landslides have emerged, since the human being begins to disrupt the delicate balance of nature. The building of massive projects (motorways, Dams etc….) has generated often problems of slope instabilities. With the development of economic construction, more and more attention has been paid to the reasonable assessment of slope stability. At present, the deterministic methods of slope analysis can mainly be classified into two categories: One is the limit equilibrium method (LEM); the other is numerical approaches, such finite element method (FEM) or finite difference method (FDM) (Yang et al., 2012).Though, conventional deterministic slope analysis does not account for assessing uncertainty in an explicit manner and relies on conservative parameters. The account for this uncertainty in design need statistical definitions of material properties, probabilistic analysis, and global response quantified in terms of reliability and probability of failure (Hicks, 2005).

Slope stability engineering is perhaps the geotechnical subject that most dominated by uncertainty. Geological anomalies, inherent spatial variability of soil properties, scarcity of representative data, changing environmental conditions, unexpected failure mechanisms, simplifications and approximations adopted in geotechnical models, and human mistakes in design and construction are all factors contributing to uncertainty (El-Ramly et al., 2002).

Probabilistic modeling of slope stability problems has been carried out since seventies, albeit mainly through the use of limit equilibrium methods combined with various statistical approaches (El-Ramly et al., 2002).It is worth to notice that the Monte Carlo Simulation (MCS) method is the most used technique to conduct probability studies. This methodology is well-known to be very expensive because of the great number of calls; of the deterministic model, required for the probabilistic analyses (Houmadi et al., 2012).In this paper, an efficient probabilistic method called *Collocation-based Stochastic Response Surface Method* (CSRSM) is used. This method is based on the approximation of the system response by a Polynomial Chaos

Expansion (PCE). It aims to replace a complex deterministic model by a meta-model which is an approximate explicit analytical formula. This makes it easy to apply MCS methodology on the meta-model without the need to call the original deterministic numerical model.

Beside the CSRSM allows to rigorously calculating the contribution of each random variable in the variability of the system response using Sobol indices (Sudret, 2008; Mollon et al., 2011). This is very important because one can detect the uncertain variables that have the major contribution in the variability of the system response, and thus can lead to a reduction in the number of uncertain parameters that should be handled by the geotechnical engineer.

This paper is organized as follows: First, a brief review of *Collocation-based Stochastic Response Surface Method* (CSRSM) and the Sensitivity analysis are presented. Then a brief presentation of study area where landslide has occurred along with geological formation of the site. Finally, deterministic and probabilistic analyses are presented. The study ends up with a conclusion on the main findings.

2. Revue of strength reduction method (SRM)

In the last few decencies, there are two main types of slope stability analysis method, one is based on the rigid body motion of limit equilibrium method, and the second is based on the numerical approaches. The latter once are currently adopted in several well-known geotechnical software.

The Strength Reduction Method (SRM) was first put forward by Zienkiewicz et al. (1975), which developed in 1990s (Griffiths and Lane, 1999). Griffiths and Lane (1999) indicated that SRM is a powerful alternative to the traditional limit equilibrium methods, and should be seriously considered by geotechnical practitioners. This approach is one of the most important slope stability analysis methods. Among its main advantages it's suitability in dealing with non-homogeneous soil profiles and complicated slopes. By combining reduction theory and elastic-plastic method to analyze the slope stability, the nonlinear constitutive relationship of rock and soil mass are considered, the critical failure surface is detected automatically from the localized shear strain; it requires no assumption on the inter-slice shear force distributions; and it is applicable to many complex conditions (Zeng and Xiao, 2017). SRM is typically applied with Mohr-Coulomb failure criterion for computing factor of safety, by progressively reducing the original shear strength parameters in order to bring the slope to a state of failure. This method implemented in *FLAC3D* (Fast Lagrangian Analysis of Continua), it is a three-dimensional explicit finite-difference program, to carry out a series of simulation of three-dimensional formations of soil, rock or others. The process of strength reduction technique can be described as follows. For Mohr-Coulomb failure criterion, the shear strength $\boldsymbol{\tau_f}$ is given:

$$\tau_f = c + \sigma_n \cdot \tan\varphi \tag{1}$$

Where $\boldsymbol{\sigma_n}$ is the effective normal stress, **c** is the cohesion, and $\boldsymbol{\varphi}$ is the angle of internal friction.

The reduced shear strength $\boldsymbol{\tau_m}$ along the failure surface is expressed as follows:

$$\tau_m = \frac{c_f}{F} \tag{2}$$

By substituting, we get:

$$\tau_m = c/F + \sigma_n \cdot \frac{\tan\varphi}{F} \tag{3}$$

After strength reduction the strength parameters can be expressed as respectively:

$$C^{trial} = \frac{1}{F^{trial}} C \tag{4}$$

$$\varphi^{trial} = arctan\left(\frac{1}{F^{trial}} \tan\varphi\right) \tag{5}$$

The value of F^{trial} is adjusted until the slope fails, whereas the ultimate F^{trial} is the factor of safety.

$$FOS = \frac{c}{c_{FOS}} = \frac{\tan\varphi}{\tan\varphi_{FOS}} \tag{6}$$

3. Methodology

3.1. The study area

Ain Bouzian landslide is one of many instability problems encountered in Skikda region, it is located halfway between the two cities, Skikda to the North (47 km) and Constantine to the South, within eastern part of northern Algeria, which have been sighted alongside of the Ain Bouzian-El Harrouch section of the East-West motorway as shown in (Fig .1).

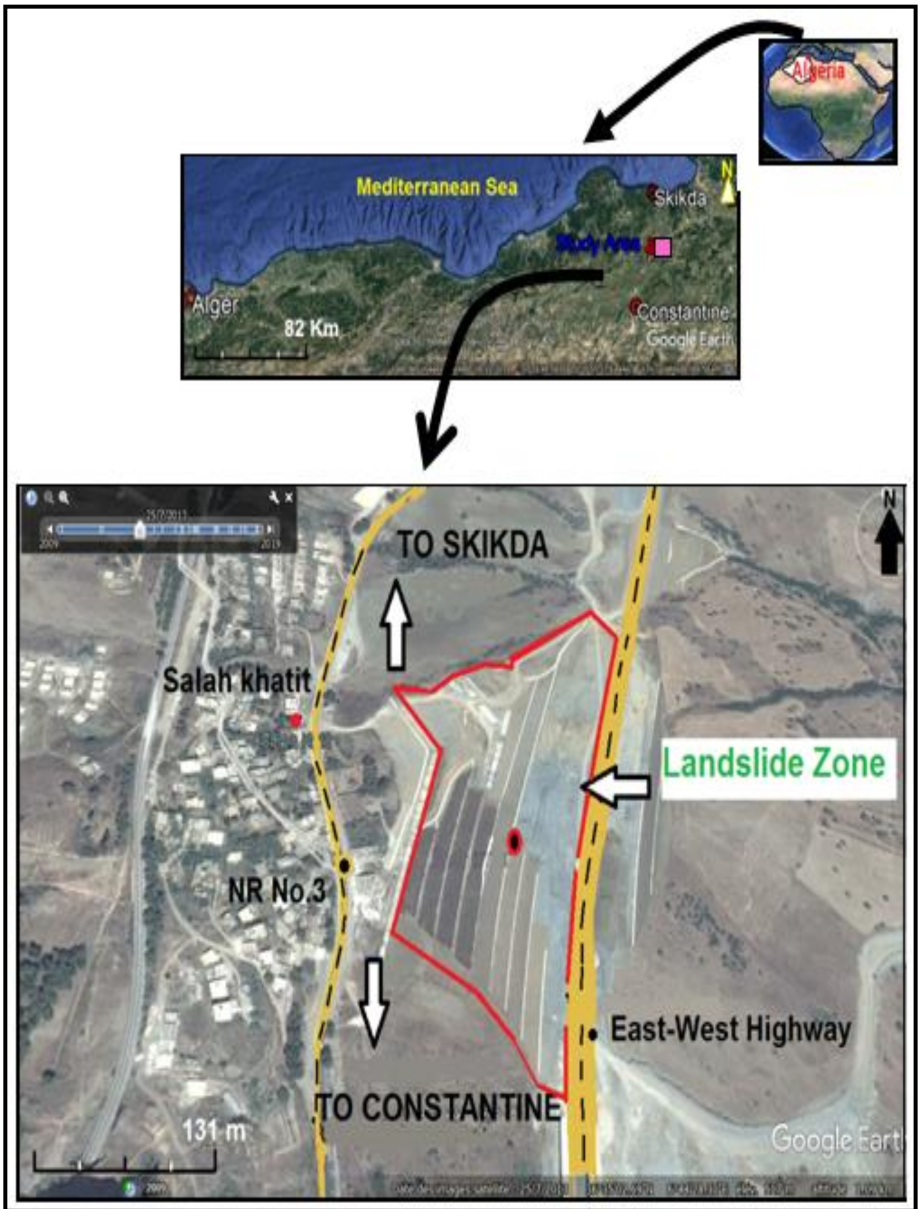

Fig .1. (a) ;(b) and (c) Map of Algeria - Study area-Ain Bouzian showing location of landslide (google earth 2021)

3.2. Field investigation

From a geological point of view (Figs. 2, 3), the area is a part of The Maghrebain Chain (Bouillin, 1986) with a complex geological formations, it is the buffer zone between the Kabylides (the Internal Zones) to the north and the Tellian over thrust (External Zones) to the South (Durand Delga, 1969), this area is a paleogeographic boundary that has been active during Mesozoic Era. Furthermore the region of Ain Bouzian as the mountains of the northern part of Algeria has been affected by the alpine orogeny with large and highly complex tectonic movements which results from the additional effects at least three different tectonic phases: before the Eocene, during middle to upper Eocene and during the upper Oligocene (fig.1). According to Raoult (1974) and Benabbas (2007), at the North slope of DJ Kantour from the national road 03 to the plain of Oued Neça is consists of the flysch-formations , Mauritian Flysch of the Ziane type of Turonian-Senonian age (greenish yellow marls with centimeter levels of slightly carbonate sandstone pelites, and varicolored clays) .

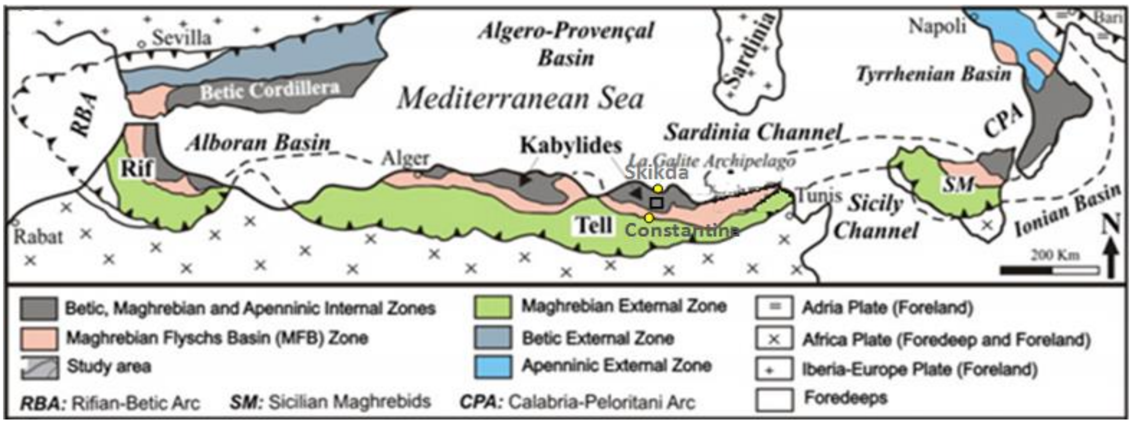

Fig. 2. General sketch of the Maghrebian Chain and location of the study area (Belayouni et al. 2012)

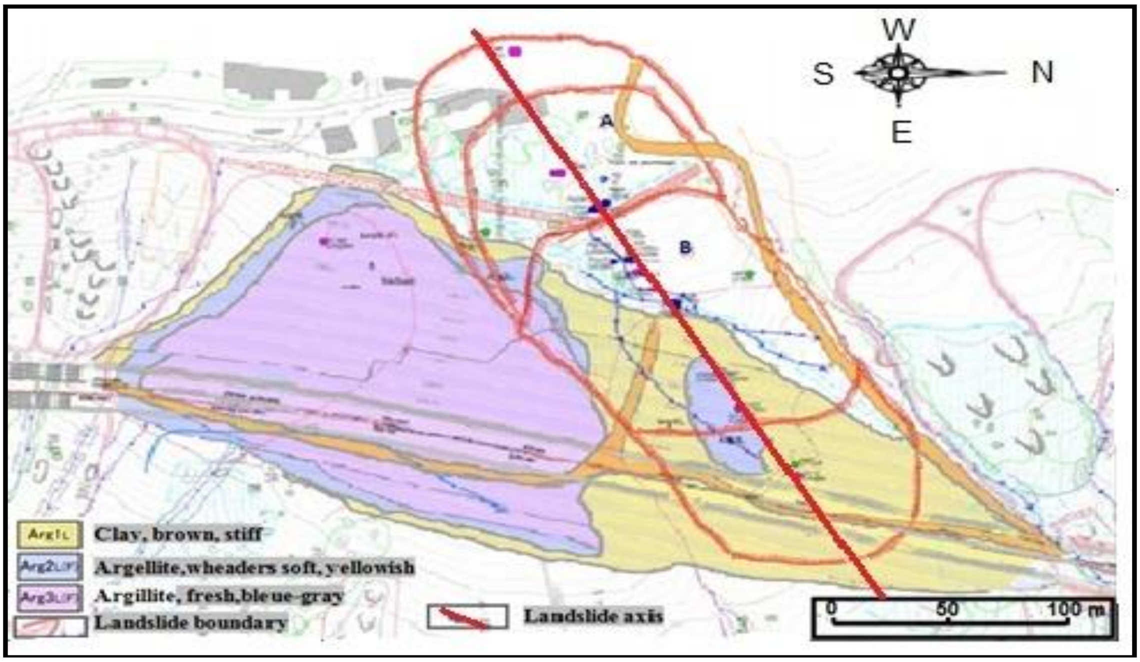

Fig. 3. Geology and geotechnical investigation on a major slope failure on a Est-Ouest highway (Ain Bouzian)

3.3. Experimental work

The hydrogeological (piezometric measurement) and geotechnical investigations carried out in site lead to identify the causes and mechanism of instability: representative soil samples were collected from different parts of the slope. These soil samples were prepared and tested in laboratory to determine the average values of material properties which are given in Table 1. According to the core drilling and the observation of slope, the local geology is predominated by the presence of yellowish brown weathered argillite, clay soil and blue-gray fresh argillite highly fragmented. According to inclinometer measurement, the shape of the sliding surface is located approximately between the layers of fresh and rather altered argillites, within a depth of 17m below ground surface. According to piezometric measurement, the level of the water table is quite high, around 1.4m below ground level, which indicate the significant effect of water table presence on the triggering of landslide.

4. Deterministic analysis

In this section the results obtained for FOS, by deterministic slope stability analysis, using both methods SRM and LEM, are presented for both cases, with and without water table. The physical and mechanical parameters of the soil (Table 1) were determined through in situ and laboratory tests. Using this data the numerical simulations for slope stability analysis were performed. The failure criterion adopted is the Mohr-coulomb criterion in the study. In order to initialize ground stress conditions the elastic parameters should be specified, a high elastic modulus was chosen to ensure that the model reaches equilibrium in a short time period.

Table 1. Mechanics parameters of the slope

item	γ_h [kN/m^3]	C [kN/m^2]	Φ [°]	E [MPa]	ν
Soil	20	3e4	20	7.38	0.3

Figure 4 represents the slope profile with a total height equal to 60 m with a varying inclination angle (43.53°,23.26° and 09°),the geometric size and boundary conditions are also illustrated.

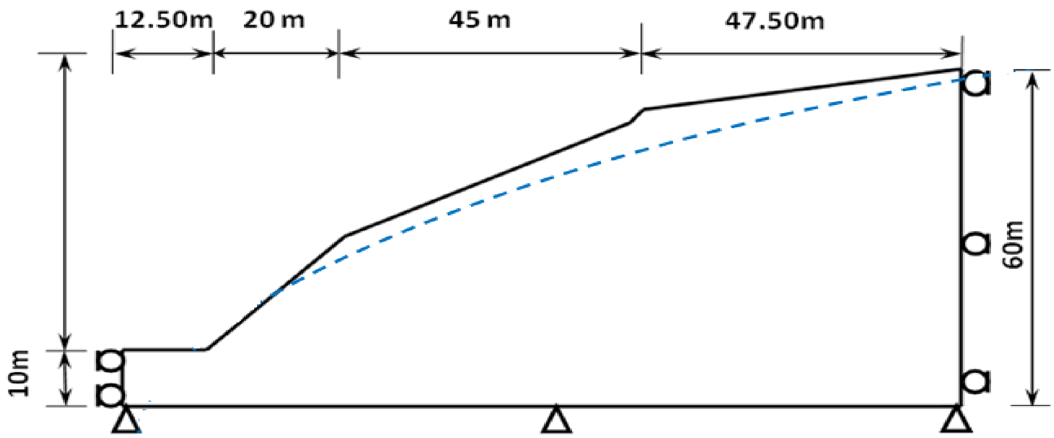

Fig. 4. Model to evaluate slope stability analysis.

The result of safety factors for the slope, computed using the Strength Reduction Method (SRM) and limit equilibrium methods (LEM), are grouped in Table 2, for both case, the safety factors values obtained by the two approaches are almost similar, the major difference is less than 8%. As expected the factor of safety in the case of absence of water table is greater than those obtained for saturated cases, the large difference is around 16 % (SRM), however all values are less than 1.5 which means that the slope, in its state, is unstable.

Table 2. FOS Calculated by LEM and SRM.

cases studies	SRM	Ordinary	Bishop	Janbu	Morgenstern Price
With water table	0.92	0.938	0.954	0.938	0.954
Without water table	1.10	1.017	1.072	1.008	1.039

5. Collocation-based stochastic response surface method (CSRSM)

CSRSM is a general and powerful method for probabilistic analysis (Isukapalli et al., 1998; Sudret , 2008; Mollon et al., 2011; Houmadi et al., 2012). CSRSM aims to determine the full probability distribution function (PDF) of the system response by accounting for the uncertainties of the input parameters by their probability distributions.

The evaluation of probability distribution of the system response must be performed as follows:

- the number and positions of the collocation points in the standard space of random variables, depends on the order of the chosen polynomial chaos;
- the collocation points must be transformed into physical space of random variables (non-normal and/or correlated);
- each collocation point available in the physical space must be computed, using the corresponding deterministic mechanical model of the system response (by performing the Monte Carlo-simulation method on the PCE);
- the PDF of the approximated system response is subsequently computed, and the statistical moments of this distribution are deduced.

Two methods are available for computing the PC expansion coefficients, from selected deterministic evaluations of the model function, whose unknown coefficients are computed either by a regression approach or by a projection approach (Isukapalli et al., 1998; Mollon et al., 2011). Note that the size P of the PCE is given by:

$$P = \frac{(m+P)!}{m!.P!} \tag{7}$$

Thus, the number N of available collocation points that are result from all possible combinations of the roots depends on the number of the random variables M and order p of the chosen PCE, $N = (P+1)^M$. The number of points required for an acceptable solution is directly related to the invariability of the information matrix$A = \Psi^{\mathrm{T}}\Psi$, which represents the generic terms. Herein, $\Psi$is the matrix of size N ∗ P whose coefficients are given by:

$$\Psi_{ij} = \Psi_j(\xi^i) \ , i = 1, \dots\dots, Q; j = 0 \dots\dots, P-1,$$

It is necessary to specify the choice of the experimental design X**,** the output random variable may be expressed by a PCE as follows:

$$Y_{app} = \sum_{J=0}^{p-1} a_j . \Psi_j(\xi^i) \tag{8}$$

The unknown coefficients of the PCE can be computed using the following equation:

$$a = Argmin \frac{1}{Q} \sum_{i=1}^{Q} \left\{ h(X^i) - \sum_{J=0}^{p-1} a_j . \Psi_j(\xi^i) \right\}^2 = (\Psi^T * \Psi)^{-1} * \Psi^T * Y \tag{9}$$

In which a is a vector containing the unknown coefficients, Y is a vector containing the system responses as calculated by the deterministic model at the different collocation points and Ψ is a matrix of size N*P whose elements are the multivariate Hermite polynomials.

In the regression method, it is necessary to specify the choice of the experimental design to improve the accuracy of the results. The optimal number of regression points, for any number of input random variable M and degree of PC expansion p which is smaller than the empirical rule is given by the following equation.

$$\mathrm{N} = (\mathrm{M} - 1) \tag{10}$$

In the following parts, CSRSM is used for reliability and the global sensitivity evaluation of slope stability using meta-model. The computational deterministic model for slope stability is based on FLAC3D code. The Polynomial Chaos Expansion (PCE) can be performed, whose unknown coefficients are determined by a regression approach.

The Monte Carlo simulation gives the system response through the evaluation of corresponding PDF and failure probability PF which provide an accurate and intrinsic representation of safety factor FOS. This can be carried out faster than the other numerical approach, due to the high efficiency of the PCE. The coefficients of the PCE are computed by solving the linear system of equations using MATLAB.

5.1 Reliability analysis using CSRSM

Probabilistic analysis is commonly adopted to quantify the uncertainties of input parameters by computing failure probabilities of system responses. Precisely, input parameters are treated as random variables following prescribed distributions, thus the corresponding model response (slope safety factor in this case) is also a random variable, and hence reliability analysis is carried out. The limit state/performance function, g(X) is defined in terms of the basic random variables Xi, and the functional relationship among them. The failure condition is defined as $G(x)=FS-1$;the function of the uncertain input parameters and the factor of safety FS is given by Eq. (6). The "probability of failure", is regarded as the probability of unacceptable values for the analytical and statistical models yielded by the performance function. In this study, reliability evaluation has been carried out, using Monte-Carlo simulations, which is expressed as:

$$PF = \frac{1}{N_{mc}} \sum_{i=1}^{N_{mc}} I(G(x)) \quad (11)$$

where N_{mc} is the number of MCS samples.

Stochastic soil properties values presented in Table 3 are used to evaluate the reliability and to assess sensitivity of the system, for the two hydrological conditions.

Table 3. Stochastic soil properties on the slope stability analysis

Parameters	Mean	Coefficient of variation (%)	Distribution type
E [MPa]	7.38	9.78	Log-normal
c [kN/m^2]	3E4	18.50	Log-normal
φ [°]	19.76	13.45	Log-normal

To get a good estimate of probability of failure through this method, 100,000 simulations were performed. These values are then used for the study of the system reliability for slope stability analysis. The effect of the presence of groundwater on slope reliability is also investigated in this section. The position of the water table in the slope is indicated in the previous section (determinist Analysis).As displayed in Table 1 for both cases, we have the same statistical input values for soil properties. The obtained result for PDFs, of the corresponding safety factors (FOS), are shown in Table 4, for both hydrological conditions. It is clear that the PDFs shapes tend to be Gaussian distribution. We can conclude that, the presence of the water table generates higher variability of safety factor (FOS), which illustrated by an increase in the mean value around 15 %. Meanwhile the water table in the soil mass induces a substantial increase in the P_F (Table 4) more than three times that obtained in the case of absence of water table. The corresponding difference in $P_F$is significantly large around 67%. Which means that the presence of water table within the soil mass increases safety factor uncertainty and failure risk for the slope. Most results of the analysis are presented in Table 4.

Table 4. Slope reliability analysis for both case study and its statistical moments

case study	Mean	Standard deviation	Skewness	Kurtosis	COV (%)	P_F (%)
With groundwater Table	1.11	0.021	0.471	0.423	0.131	22.20
Without groundwater Table	0.94	0.016	0.536	0.5891	0.136	68.70

5.2. Sensitivity analysis using CSRSM

Global Sensitivity analysis based on Sobol indices, aims to quantify the effects of each input random variable on the response system variability as well their interaction (Huber et al., 2011), through computing the Sobol' indices, which have proven to be the most efficient sensitivity measures for general computational models.
Sudret (2015) has proposed an original post-processing of polynomial chaos expansions for sensitivity analysis, which can be established analytically, using, the Sobol' decomposition of a truncated PC expansion *Y.* Once the PC basis has been set up , and the coefficients have been calculated, the series expansion will be post processed, in order to compute the statistical moments of the model response (mean value, standard deviation, etc.), and the probability density function.

Notice that the sum of all the Sobol indices of a model is always equal to unity. It is now easy to derive sensitivity from the above representation. These indices, called PC-based Sobol' indices and denoted by $SUi_{1.........}i_s$,are defined as:

$$SU_{i_{1,\dots\dots,}is} = \frac{\sum_{\alpha \epsilon j_{i_{1,\dots\dots\dots}is}} f_\alpha^2 E[\Psi_\alpha^2]}{D_{PC}} \tag{12}$$

There are several methods for the quantification of the accuracy of the PCE, that depends mainly on its order. The calculation of the coefficient of determination R^2 is commonly used to control the quality of approximation, by normalizing the mean squared difference between the model and the surrogate (Sudret, 2008). The coefficient of determination R^2, is calculated as follows:

$$R^2 = 1 - \Delta_{PCE} \tag{13}$$

where Δ_{PCE} is given by:

$$\Delta_{PCE} = \frac{\left(1/_J\right) \sum_{i-1}^{J} \left[Y(\xi^{(i)} - Y_{app}(\xi^{(i)})\right]^2}{Var(Y)} \tag{14}$$

And

$$Var(Y) = \frac{1}{J-1} \sum_{i-1}^{J} \left[Y\left(\xi^{(i)}\right) - \bar{Y}\right]^2 \tag{15}$$

$$\bar{Y} = \frac{1}{J} \sum_{i-1}^{J} Y\left(\xi^{(i)}\right) \tag{16}$$

In these equations, J is number of collocation points used to evaluate the unknown coefficients of the PCE. M, ξ is the standard normal random vector which is defined by J realization $\left\{\xi^{(i)} = \left(\xi_1^{(i)}, \dots, \xi_m^{(i)}\right), \dots \xi^{(J)} = \left(\xi_1^{(i)}, \dots, \xi_m^{(J)}\right)\right\}$.

In this study, a global sensitivity assessment based on Sobol indices, relative to the three random variables (E, C, φ) for the system response represented by (FOS), were performed using the Monte Carlo simulation (MCS) via a PCE within CSRSM. According to the numerical results, a PCE order M=7 with corresponds coefficient of determination $R^2 \approx 0,999$ are obtained with the number of calls equals 209.

6. Parametric study

In order to investigate the effect of COV of certain random variables on the reliability and the sensitivity of the system response related to safety, a parametric study is carried out. The random variables for the soil considered in this study are the Young modulus E, cohesion C and friction angle φ. So the COV of cohesion C and friction angle φincreased or decreased by 10%, the COV of Young modulus E increased or decreased by 20%. The sensitivity of the system response is estimated, using Sobol indices. Three values of each input parameter are considered (Table 3), and their effects on the possibility of slope failure are assessed for both cases with and without water table.

6.1. Their Effect on the PDF of Factor of Safety

Figures 5, 6 and 7 illustrate the PDF associated for each parameter variations in both cases with and without water table. From the first examination of PDF curves, we can observe, for both hydrological cases, the parameter variations have similar effects on the PDF shapes. The effect of variation of Young modulus is trivial and all curves overlap, the system response almost remain the same. While for the other two parameters (C, φ) when increasing COV(φ) or COV(C), the PDF curves shift away from being Gaussian distribution, the corresponding PDF changes, associated with friction angle variations, are more significant.

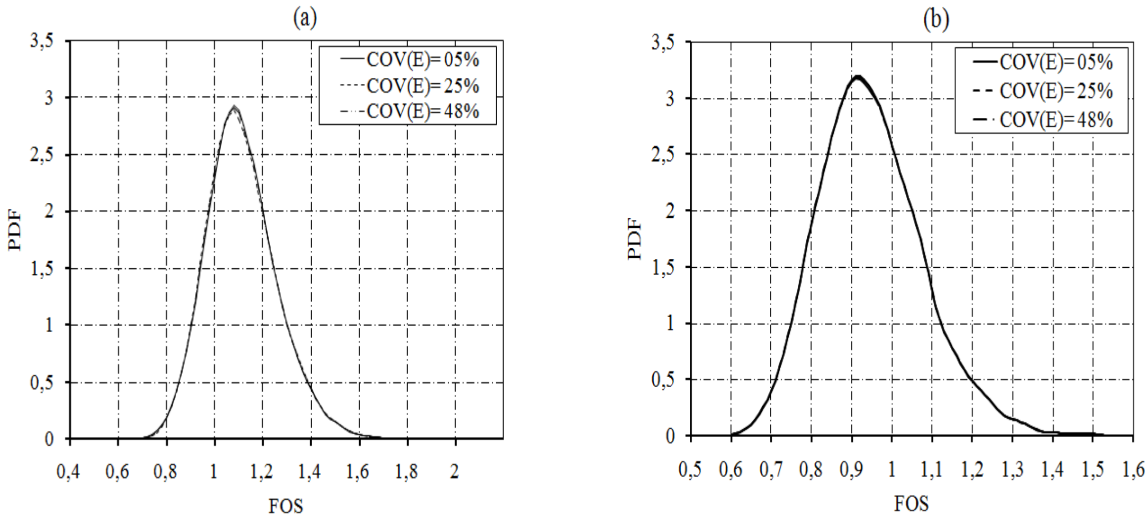

Fig. 5. Influence of the coefficients of variation of Young modulus COV (E) on the PDFs of (FOS) : (a) without groundwater; (b) with groundwater

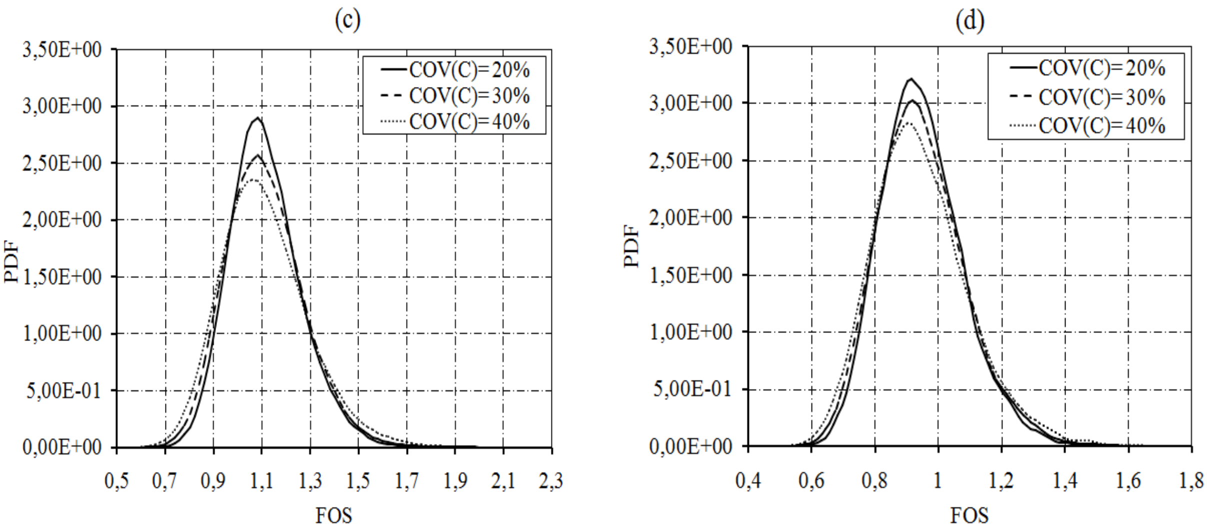

Fig . 6. Influence of the coefficients of variation of Cohesion COV(C) on the PDFs of FOS: (c) without groundwater; (d) with groundwater.

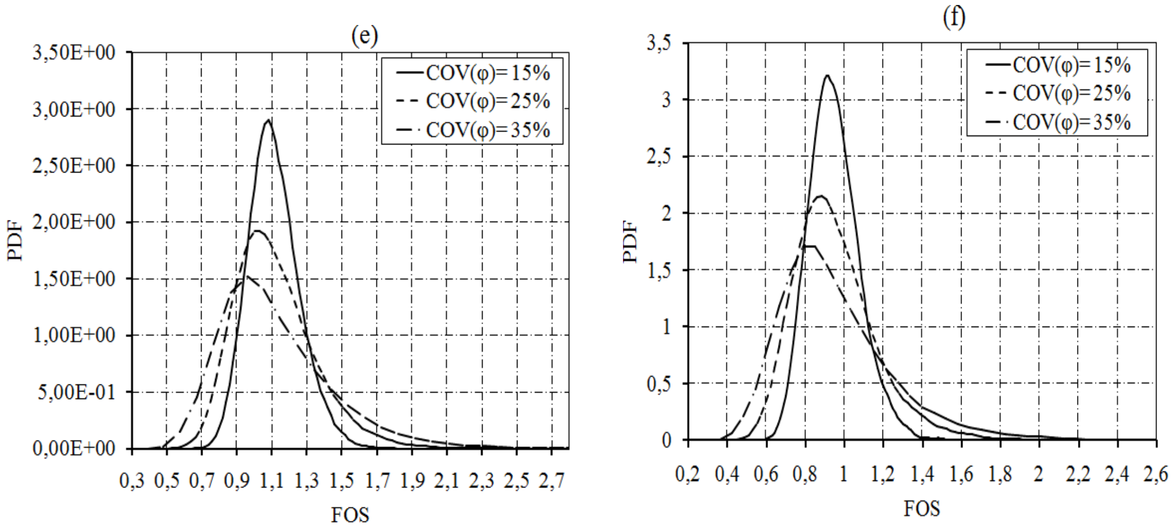

Fig. 7. Influence of the coefficients of variation of friction angle COV (φ) on the PDFs of FOS: (e) with groundwater; (f): without groundwater.

6.2 Effect of variability of geotechnical input parameters on sensitivity analysis

Based on methods of global sensitivity analysis and using Sobol indices, a study is realized in order to quantify the influence of each input random variables onto the system behavior. The computation of the Sobol indices is achieved by Monte Carlo simulation, using a combination of input random variable COV (φ and c).The effect of the coefficients of variation on Sobol indices are presented in figures 8 and 9 ,for COV (φ) and COV (C)respectively and both hydrological conditions, (with and without water table).

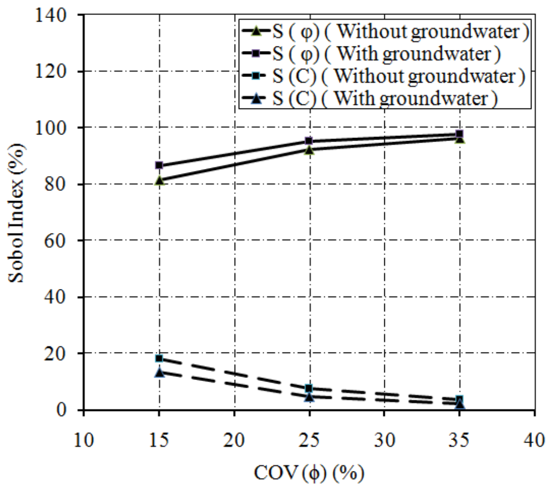

Fig . 8. Influence of the coefficients of variation of friction angle COV (φ) on the Sobol index of S (C) and S (φ) for both cases.

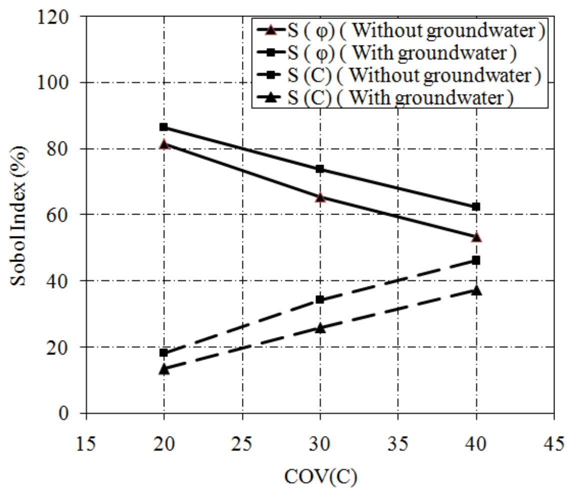

Fig .9. Influence of the coefficients of variation of Cohesion COV (C) on the Sobol Index of S (C) and S (φ) for both cases.

As we can see in figures 8 and 9, with the increases of COV (φ) and COV (C) for both models, it is clear that the random variable φ has higher effect on the system response; its weight in the variability is more than 80% for both hydrological cases. While the random variable of cohesion C has less effect with values around 13% and 18%, for both system responses, with and without water table respectively. The random variable of elastic modulus E has negligible influence onto the system responses for both cases. Therefore we can conclude that the random variable φ plays major role into the variability of system response in slope stability analysis, thus great care should be paid during geotechnical evaluation of this parameter.

7. Conclusion

Collocation-Based Stochastic Response Surface Method (CSRSM) was applied for the probabilistic study of slope stability analysis problems within homogenous soil profile of Ain Bouzian landslide. It intends to investigate the system response, reliability of slope failure and global sensitivity due to soil parameters variability and correlations for two hydrological conditions. From the previous results and discussions we can draw the main conclusions:

- The deterministic analysis shows that the slope is instable for both cases with and without water table; with the shape of critical failure surfaces are almost circulars. The results obtained from the two approaches for safety factor are practically similar and indicate the slope is instable.
- From parametric study, with the increase of COV(C), the PDF curves of safety factor remain almost Gaussian with an increase in the main values. Whereas for an increase of COV(φ), the PDF curves tend to shift away from Gaussian shape with a reduction in the main value of factor of safety. Moreover we can put forward that :
  - The Young modulus has no effect on the system response which is logic. In fact this parameter is an elastic propriety of the soil mass and can be input as a deterministic parameter.
  - The major contribution to the system responses is induced by the variation of friction angle.
  - The other strength parameter of the soil, the cohesion, has minor effect on the system response.
- We can conclude that the presence of water table reduces the probability of failure P_F when increasing COV(φ) and COV(C), while we get the opposite impact in the case of absence of water table.
- From global sensitivity analysis based on Sobol indices, we can realize that the contribution of each parameter (C and φ) increases with the increase of its coefficients of variation and reduces the effect of the other parameter. However the angle of friction always has the major contribution on the system response.

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