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Static and dynamic behaviors of laminated composite plates resting on elastic foundation

Ghazoul T *, Benatta M A , Bachir Bouiadjra M

1 Laboratoire des structures et matériaux avancés dans le génie civil et travaux publics, Université de SIDI BEL ABBES, BP 89 cité ben Mhidi, Sidi bel abbés, Algérie.
* Corresponding Author : taherghazoul@gmail.com

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Abstract. This present study consists to analyze the mechanical buckling and the free vibration stabilities of antisymmetric cross-ply and angle-ply laminated composite plates using a refined high order shear deformation theory of four variables against five in other high order theories. Among the advantages of this new theory it takes into consideration the shearing effect in the calculation of deformation without the need for shear correction factors and giving rise to a variation of the shear stresses along the thickness and satisfying the zero shear stresses condition in faces of the plate. The laminate resting on the Pasternak elastic foundation, including a shear layer and Winkler spring, are considered. The equations of the motion are derived from Hamilton's principal. The closed form solution of simply supported rectangular plates has been obtained by using the Navier method. In addition, the effects of various parameters of the laminated composite plate on static buckling and dynamic are presented.

Key words: Laminate composite plates, buckling, free vibration, elastic foundation, refined theory.

1. Introduction

The ability to control the selection of the proper composite materials, the number of plies, and the fiber orientation give the laminated composite plates much attention in industry applications and fields of technology. This promising feature allows designers to acquire desired structural responses under a given set of operating conditions. Many shear deformation theories accounting for transverse shear effects have been developed to overcome the deficiencies of the classical plate theory (CPT). The first-order shear deformation theories (FSDT) based on Reissner (1945) and Mindlin (1951) account for the transverse shear effects by the way of linear variation through the thickness. In references (Srinivas and Rao, 1970; Bert, 1974) a shear correction factors are required to rectify the equilibrium conditions and satisfying the zero shear stresses condition at the top and bottom faces of the plate. In order to overcome the limitations of the first-order shear deformation theories (FSDT), higher-order shear deformation theories (HSDT) were developed, among them the Reddy's theory is the most widely used due to its high efficiency and simplicity (Reddy and Phan, 1985; Mallikarjuna and Kant, 1993). Shimpi (2002) for isotropic plates developed a refined plate theory using only two unknown, then it was extended by Shimpi and Patel (2006a-b) for orthotropic plates.

In the present study, an attempt is made to check the efficiency of four variable refined shear deformation theories for the buckling and free vibration analysis of cross-ply laminated composite plates resting on elastic foundation. The theory satisfies zero shear stress conditions at top and bottom surfaces of the plates and does not need shear correction factor. The elastic foundation is modeled as two parameter Pasternak foundation. Governing equations and boundary conditions are obtained using the Hamilton principle. A closed form solution is obtained by using a double trigonometric series technique developed by Navier.

2. Theoretical formulation

2.1. Main Assumptions

Consider a rectangular plate of total thickness h composed of n orthotropic layers with the coordinate system as shown in Figure 1. The principal assumption of the refined plates theory is that the transverse displacement w includes two components of bending w_b and shear w_s. These components are functions of coordinates (x, y) and time t only.

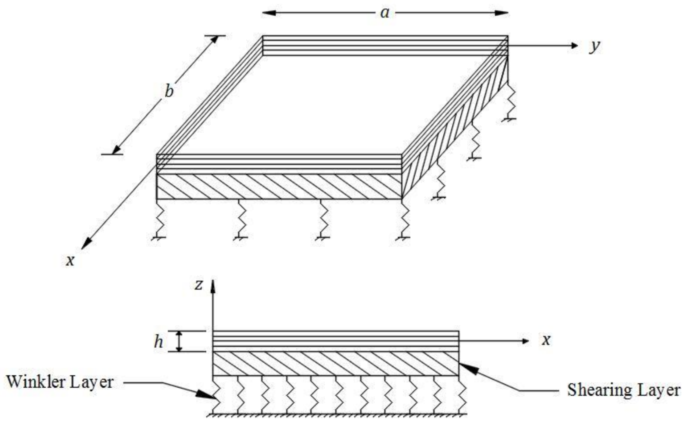


Fig 1. Geometry and coordinate system of laminated plate on elastic foundation.

2.2. Kinematics

The displacement field can be obtained as

$$\begin{aligned} u(x,y,z,t) &= u_0(x,y,t) - z\frac{\partial w_b(x,y,t)}{\partial x} - f(z)\frac{\partial w_s(x,y,t)}{\partial x} \\ v(x,y,z,t) &= v_0(x,y,t) - z\frac{\partial w_b(x,y,t)}{\partial y} - f(z)\frac{\partial w_s(x,y,t)}{\partial y} \\ w(x,y,z,t) &= w_b(x,y,t) + w_s(x,y,t) \end{aligned} \tag{1}$$

where u_0 and v_0 denote the in-plane displacements in the directions of x and y, respectively; w_b and w_s are the bending and shear components of the transverse displacement, respectively; $f(z)$ represents shape function determining the distribution of the transverse shear strains and stresses along the thickness and is used as

$$f(z) = \frac{1}{p}\left(\frac{2}{h}\right)^{p-1} z^p \,,\ \left(p = 3,5,\ldots\right) \tag{2}$$

2.3. Constitutive relations

Since the laminate is made of several orthotropic layers with their material axes oriented arbitrarily with respect to the laminate coordinates, the constitutive equations of each layer must be transformed to the laminate coordinates (x, y, z). The stress-strain relations in the laminate coordinates of the k^th layer are given as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{xz} \end{Bmatrix}^k = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} & 0 & 0 \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} & 0 & 0 \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} & 0 & 0 \\ 0 & 0 & 0 & \bar{Q}_{44} & \bar{Q}_{45} \\ 0 & 0 & 0 & \bar{Q}_{45} & \bar{Q}_{55} \end{bmatrix}^k \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \quad (3)$$

Where $\{\sigma\}^k$; $\{\varepsilon\}$ and $\left[\bar{Q}_{ij}\right]^k$ are the stress vector, strain vector and transformed stiffness matrix, respectively. With the non-zero strains components associated with the displacement field in Equation (1) can be defined as follows

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z \begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} + f(z) \begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix}, \quad \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xy} \end{Bmatrix} = g(z) \begin{Bmatrix} \gamma_{yz}^s \\ \gamma_{xz}^s \end{Bmatrix}, \quad g(z) = 1 - f'(z) \quad (4)$$

Where

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial y} \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} \end{Bmatrix}, \begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w_b}{\partial x^2} \\ -\dfrac{\partial^2 w_b}{\partial y^2} \\ -2\dfrac{\partial^2 w_b}{\partial x \partial y} \end{Bmatrix}, \begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w_s}{\partial x^2} \\ -\dfrac{\partial^2 w_s}{\partial y^2} \\ -2\dfrac{\partial^2 w_s}{\partial x \partial y} \end{Bmatrix}, \begin{Bmatrix} \gamma_{yz}^s \\ \gamma_{xz}^s \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial w_s}{\partial y} \\ \dfrac{\partial w_s}{\partial x} \end{Bmatrix} \quad (5)$$

2.4. Governing Equation

The application of Hamilton principle leads to

$$\begin{aligned} &\int_0^a\int_0^b\int_{-h/2}^{h/2}\left[\sigma_x\delta\varepsilon_x+\sigma_y\delta\varepsilon_y+\sigma_{xy}\delta\gamma_{xy}+\sigma_{yz}\delta\gamma_{yz}+\sigma_{xz}\delta\gamma_{xz}\right]dzdydx \\ &+\int_0^a\int_0^b\left[K_w\left(w_b+w_s\right)\left(\delta w_b+\delta w_s\right)-K_s\left(\frac{\partial^2\left(w_b+w_s\right)}{\partial x^2}+\frac{\partial^2\left(w_b+w_s\right)}{\partial y^2}\right)\left(\delta w_b+\delta w_s\right)\right]dydx \\ &-\int_0^a\int_0^b q\left(\delta w_b+\delta w_s\right)dydx+\rho\int_0^a\int_0^b\int_{-h/2}^{h/2}\left[\frac{\partial^2 u}{\partial t^2}\delta u+\frac{\partial^2 v}{\partial t^2}\delta v+\frac{\partial^2\left(w_b+w_s\right)}{\partial t^2}\left(\delta w_b+\delta w_s\right)\right]dzdydx \\ &-\int_0^a\int_0^b\left[N_{xx}^0\frac{\partial^2\left(w_b+w_s\right)}{\partial x^2}+N_{yy}^0\frac{\partial^2\left(w_b+w_s\right)}{\partial y^2}+2N_{xy}^0\frac{\partial^2\left(w_b+w_s\right)}{\partial x\partial y}\right]\left(\delta w_b+\delta w_s\right)dydx \end{aligned} \quad (6)$$

Where N_x^0; N_y^0; N_{xy}^0 are in-plane applied loads. ρ is the composite density. K_w and K_s are the Winkler and shear layer spring constants, In this study, the laminated rectangular plate is taken to be simply supported at the edges. The solution of Equations (6) can be obtained analytically by using the following boundary conditions:

Cross-ply laminated plate:	Angle -ply laminated plate:		
$v_0 = w_b = w_s = N_x = M_x^b = M_x^s = 0$	$u_0 = w_b = w_s = N_x = M_x^b = M_x^s = 0$	at *x=0,a*	
$u_0 = w_b = w_s = N_y = M_y^b = M_y^s = 0$	$v_0 = w_b = w_s = N_y = M_y^b = M_y^s = 0$	at *y=0,b*	(7)

The following approximate solution that satisfies the boundary conditions is applied as

Cross-ply laminated plate:

$$u_0(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}U_{mn}\ e^{i\omega t}\cos(\alpha x)\sin(\beta y)$$
$$v_0(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}V_{mn}\ e^{i\omega t}\sin(\alpha x)\cos(\beta y)$$
$$w_b(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}W_{bmn}\,e^{i\omega t}\sin(\alpha x)\sin(\beta y)$$
$$w_s(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}W_{smn}\ e^{i\omega t}\sin(\alpha x)\sin(\beta y)$$

Angle -ply laminated plate:

$$u_0(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}U_{mn}\ e^{i\omega t}\sin(\alpha x)\cos(\beta y)$$
$$v_0(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}V_{mn}\ e^{i\omega t}\cos(\alpha x)\sin(\beta y)$$
$$w_b(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}W_{bmn}\,e^{i\omega t}\sin(\alpha x)\sin(\beta y)$$
$$w_s(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}W_{smn}\ e^{i\omega t}\sin(\alpha x)\sin(\beta y) \tag{8}$$

In which U_{mn}, V_{mn}, W_{bmn} and W_{smn} are arbitrary parameters to be determined. ω is the Eigen frequency associated with (m, n) the Eigen mode, and $\alpha = m\pi/a$ and $\beta = n\pi/b$. The analytical solutions can be obtained from the following equations for buckling and free vibration problem

$$\left(\begin{bmatrix} s_{11} & s_{12} & s_{13} & s_{14} \\ s_{12} & s_{22} & s_{23} & s_{24} \\ s_{13} & s_{23} & s_{33}+K & s_{34}+K \\ s_{14} & s_{24} & s_{34}+K & s_{44}+K \end{bmatrix} + \omega^2 \begin{bmatrix} m_{11} & 0 & 0 & 0 \\ 0 & m_{22} & 0 & 0 \\ 0 & 0 & m_{33} & m_{34} \\ 0 & 0 & m_{34} & m_{44} \end{bmatrix}\right)\begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{bmn} \\ W_{smn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{9}$$

Where

$$\begin{aligned}
s_{11} &= A_{11}\alpha^2 + A_{66}\beta^2, s_{12} = \alpha\beta\left(A_{12}+A_{66}\right) \\
s_{22} &= A_{66}\alpha^2 + A_{22}\beta^2 \\
s_{33} &= D_{11}\alpha^4 + 2\left(D_{12}+2D_{66}\right)\alpha^2\beta^2 + D_{22}\beta^2 \\
s_{34} &= D_{11}^s\alpha^4 + 2\left(D_{12}^s+2D_{66}^s\right)\alpha^2\beta^2 + D_{22}^s\beta^4 \\
s_{44} &= H_{11}^s\alpha^4 + 2\left(H_{12}^s+2H_{66}^s\right)\alpha^2\beta^2 + H_{22}^s\beta^4 + A_{55}^s\alpha^2 + A_{44}^s\beta^2
\end{aligned}$$

$$\left.\begin{aligned} s_{13} &= -B_{11}\alpha^3 \\ s_{14} &= -B_{11}^s\alpha^3 \\ s_{23} &= B_{11}\beta^3 \\ s_{24} &= B_{11}^s\beta^3 \end{aligned}\right\}\quad \text{For antisymmetric cross-ply}$$

$$\left.\begin{aligned} s_{13} &= -\left(3B_{16}\alpha^2\beta + B_{26}\beta^3\right) \\ s_{14} &= -\left(3B_{16}^s\alpha^2\beta + B_{26}^s\beta^3\right) \\ s_{23} &= -\left(B_{16}\alpha^3 + 3B_{26}\alpha\beta^3\right) \\ s_{24} &= -\left(B_{16}^s\alpha^3 + 3B_{26}^s\alpha\beta^3\right) \end{aligned}\right\}\quad \text{For antisymmetric angle-ply} \tag{10}$$

$$\begin{aligned}
m_{11} &= m_{22} = I_1, m_{33} = I_1 + I_3\left(\alpha^2+\beta^2\right) \\
m_{34} &= I_1 + I_5\left(\alpha^2+\beta^2\right), m_{44} = I_1 + I_6\left(\alpha^2+\beta^2\right) \\
K &= N_x^0\alpha^2 + N_y^0\beta^2 + 2N_{xy}^0\alpha\beta
\end{aligned}$$

With

$$(A_{ij}, B_{ij}, D_{ij}, B_{ij}^s, D_{ij}^s, H_{ij}^s) = \sum_{k=1}^{N}\int_{h_{k-1}}^{h_k}\bar{Q}_{ij}^k(1, z, z^2, f(z), z\,f(z), f^2(z))dz \quad (i,j)=(1,2,6)$$

$$A_{ij}^s = \sum_{k=1}^{N}\int_{h_{k-1}}^{h_k}\bar{Q}_{ij}^k g^2(z)dz \quad (i,j)=(4,5) \tag{11}$$

$$\left(I_1, I_2, I_3, I_4, I_5, I_6\right) = \sum_{k=1}^{N}\int_{h_{k-1}}^{h_k}\rho^k\left(1, z, z^2, f(z), zf(z), f^2(z)\right)dz$$

3. Results and discussion

The lamina properties used in this study are given in Table 1. The following non-dimensionalizations are used in presenting the numerical results in tabular and graphical form:

$$k_w = \frac{K_w a^4}{E_2 h^3},\ k_s = \frac{K_s a^2}{E_2 h^3}$$
$$\bar{\varpi} = \omega \frac{a^2}{h}\sqrt{\frac{\rho}{E_2}},\ \hat{N} = N_{cr}\left(\frac{a^2}{E_2 h^3}\right) \tag{10}$$

Table 1. Lamina properties.

E_1	$G_{12}=G_{13}$	G_{23}	$\upsilon_{12}=\upsilon_{13}$
From 3 to 40 E_2	0.6 E_2	0.5 E_2	0.25

3.1. Free vibration analysis

In order to verify the accuracy of the present analysis, some numerical examples were solved. As a first example, the fundamental natural frequencies of a simply supported anti-symmetric cross-ply (0/90)n plates were calculated by varying the number of plies n and the orthotropic ratio E1/E2. Figure 2 shows the comparison between the present results and the solutions given by Reddy (1989) for different values of orthotropic ratio. The results clearly indicate that the fundamental natural frequencies predicted by the present model and Reddy (1989) are almost identical.

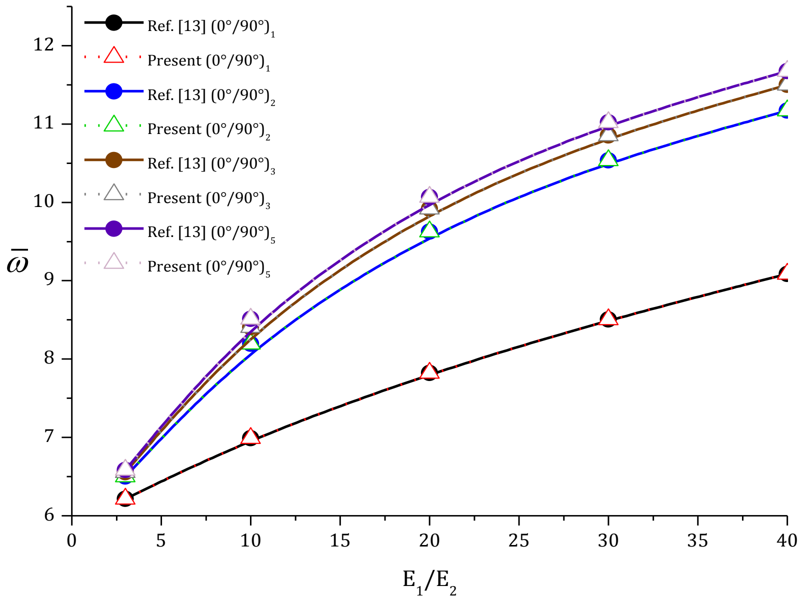


Fig 2. Comparison of the Fundamental Frequency $\bar{\varpi}$ of simply supported anti-symmetric cross-ply square laminates (a/h=10, Kw=Ks=0).

In order to validate the present model in the case of elastic foundation, the results for the fundamental natural frequency parameter of isotropic plate with different values of thickness-to-length ratios and different values of elastic coefficients are compared in Table 2 with those obtained by Baferani et al. (2011) and Thai and Choi (2011). Excellent agreement of the three methods can be seen.

Table 2. Comparison of the Fundamental Frequency $\bar{\omega} = \omega h \sqrt{\rho / E_2}$ of Isotropic Square Plates.

Kw	Ks	sources	h/a			
			0.05	0.1	0.15	0.2
0	0	Ref. [10]	0.0291	0.1134	0.2454	0.4154
		Ref. [11]	0.0291	0.1135	0.2454	0.4154
		Present	0.0291	0.1134	0.2452	0.4150
0	100	Ref. [10]	0.0406	0.1599	0.3515	0.6080
		Ref. [11]	0.0406	0.1599	0.3515	0.6080
		Present	0.0406	0.1597	0.3512	0.6075
100	0	Ref. [10]	0.0298	0.1162	0.2519	0.4273
		Ref. [11]	0.0298	0.1163	0.2519	0.4273
		Present	0.0298	0.1162	0.2515	0.4269
100	100	Ref. [10]	0.0411	0.1619	0.3560	0.6162
		Ref. [11]	0.0411	0.1619	0.3560	0.6162
		Present	0.0411	0.1617	0.3557	0.6156

Figure 3 shows the effect of the thickness ratio a/h on the dimensionless natural frequencies of both cross-ply and angle-ply simply supported laminates with and without the elastic foundation. As shown in this figure, the dimensionless natural frequencies increase with increasing thickness ratio and number of plies. Besides, the frequencies of laminates increase when foundation parameters increase, this increase is more significant in cases of Cross-ply laminates.

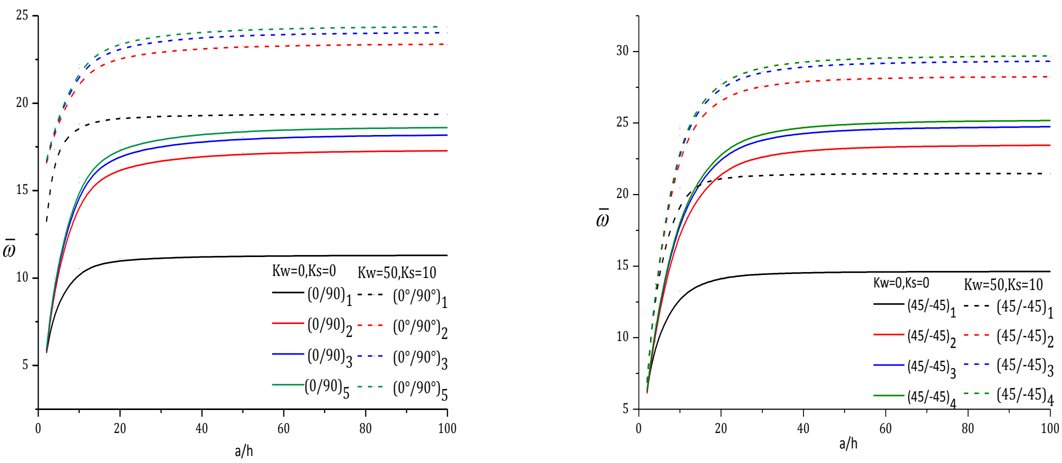


Fig 3. The effect of side-to-thickness ratio on fundamental frequency $\bar{\omega}$ of simply supported anti-symmetric $(0/90)_n$ cross-ply and $(45/-45)_n$ angle-ply square laminates on the elastic foundation (E1/E2=40).

Figure 4 illustrates the variations of dimensionless fundamental frequencies of anti-symmetric cross-ply (0/90/0/90) laminate on elastic foundation against orthotropic ratio E1/E2. It is seen that an increase in the degree of orthotropy produces an increase in the fundamental frequency. Additionally, it is observed that the frequencies of laminates increase when foundation parameters increase.

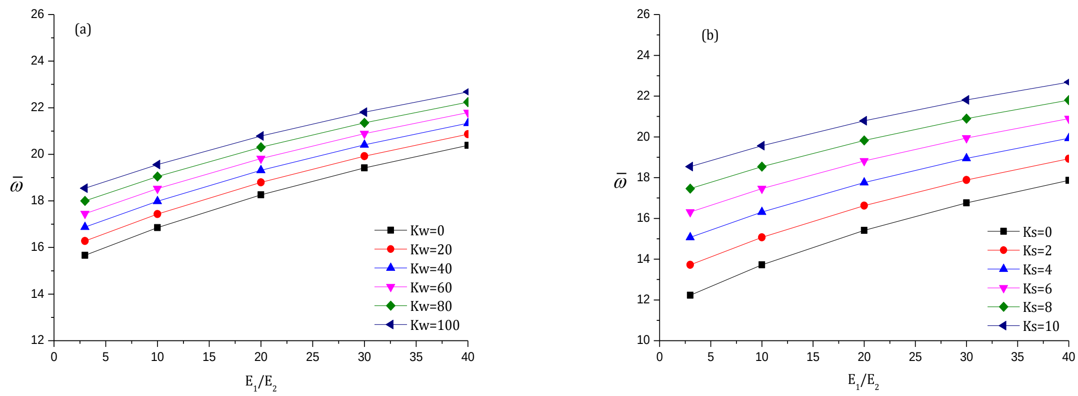


Fig 4. Variation of fundamental frequency $\bar{\omega}$ versus orthotropy ratio for simply supported anti-symmetric laminated (0/90/0/90) square plate on the elastic foundation (a/h=10, (a): Ks=10, (b): Kw=100).

The effects of angle θ on the dimensionless fundamental frequencies of angle-ply $(\theta/-\theta)$ laminate with different values of foundation parameters are shown in Figure 5. It is found that the curves are symmetric to the line of $\theta = 45°$ and it presents the highest values of fundamental frequencies for all cases.

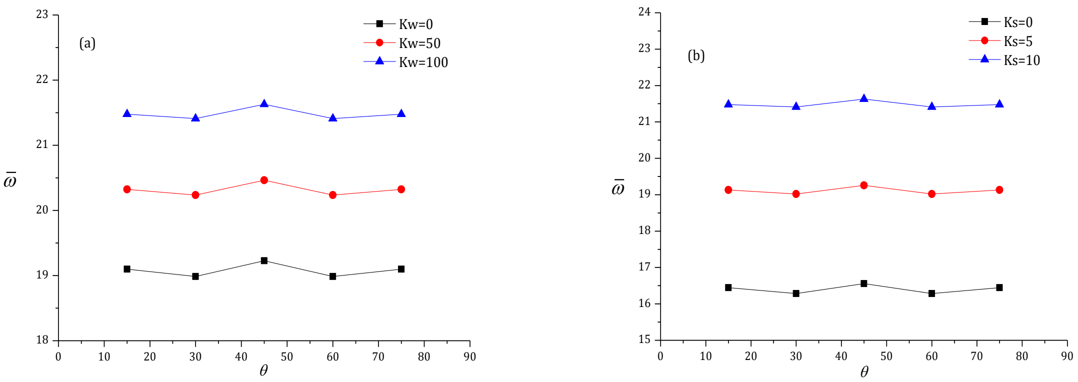


Fig 5. Effect of the lamination angle θ on fundamental frequency $\bar{\omega}$ of simply supported anti-symmetric laminated $(\theta/-\theta)_1$ angle-ply square plate on the elastic foundation (a/h=10, (a): Ks=10, (b): Kw=100).

3.2. Buckling analysis

For buckling analysis, it is also begun with numerical validation to verify the accuracy of the present mathematical models in predicting buckling analysis of Laminated Composite Plates with Elastic Foundation. A simply supported anti-symmetric cross-ply (0/90)n square laminates subjected to uniaxial compressive load is considered. Figure 6 shows a comparison between the present solutions and the solutions obtained by Reddy and Khdeirt (1989). From this figure, it can be seen, the present solutions are almost identical with solutions obtained by Reddy and Khdeirt (Reddy, 1989) for all different values of orthotropic ratio.

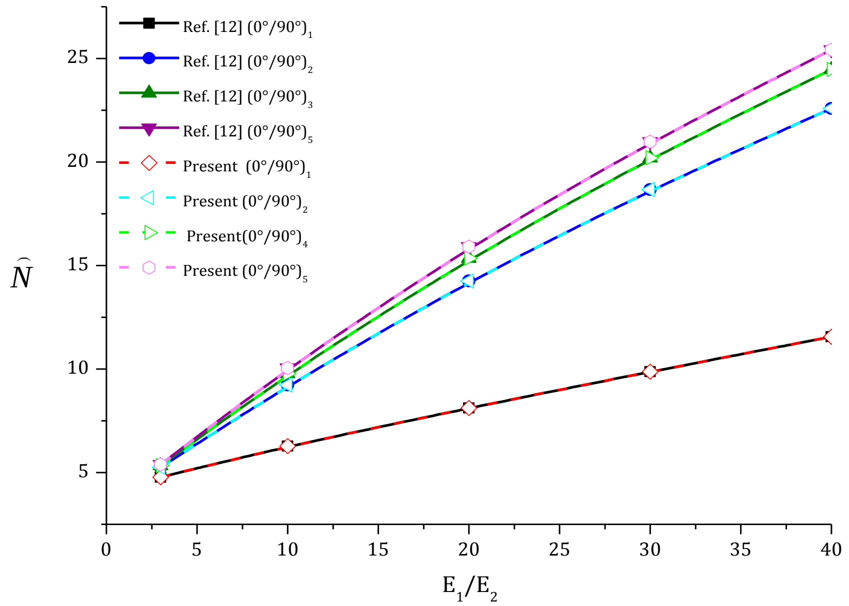
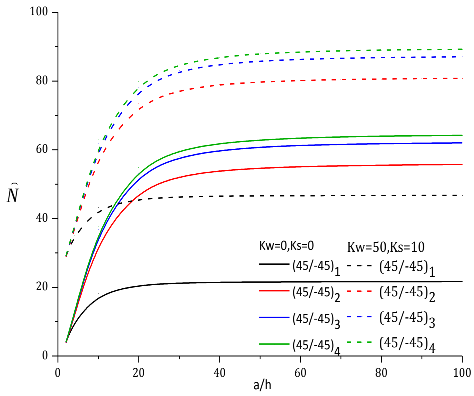


Fig 6. Comparison of the uniaxial buckling load $\widehat{N}$ of simply supported anti-symmetric Cross-ply square laminates (a/h=10, Kw=Ks=0).

Figure 7 contains plots of dimensionless uniaxial critical buckling load versus thickness ratio a/h of a simply supported anti-symmetric cross-ply and angle-ply with and without the elastic foundation. The results show that the dimensionless buckling loads increase with increasing thickness ratio and number of plies. Also, it can be observed that the dimensionless buckling loads of laminates increase when foundation parameters increase, this increase is more significant in cases of Cross-ply laminates.



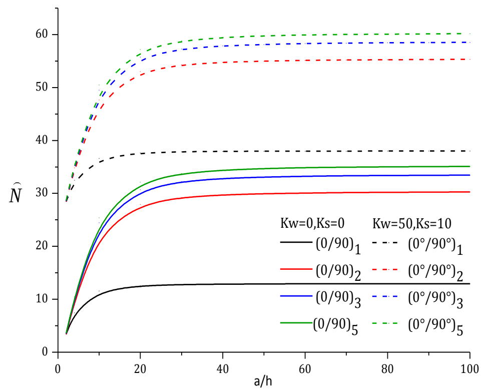


Fig 7. The effect of side-to-thickness ratio on uniaxial buckling load $\widehat{N}$ of simply supported anti-symmetric $(0/90)_n$ cross-ply and $(45/-45)_n$ angle-ply square laminates on the elastic foundation (E1/E2=40).

The effect of orthotropy ratio E1/E2 on dimensionless uniaxial critical buckling loads of anti-symmetric cross-ply (0/90/0/90/0/90) laminate on elastic foundation is shown in Figure 8. It is seen that an increase in the degree of orthotropy leads to an increase in the dimensionless buckling loads. Additionally, it is observed that the dimensionless buckling loads increase when foundation parameters increase.

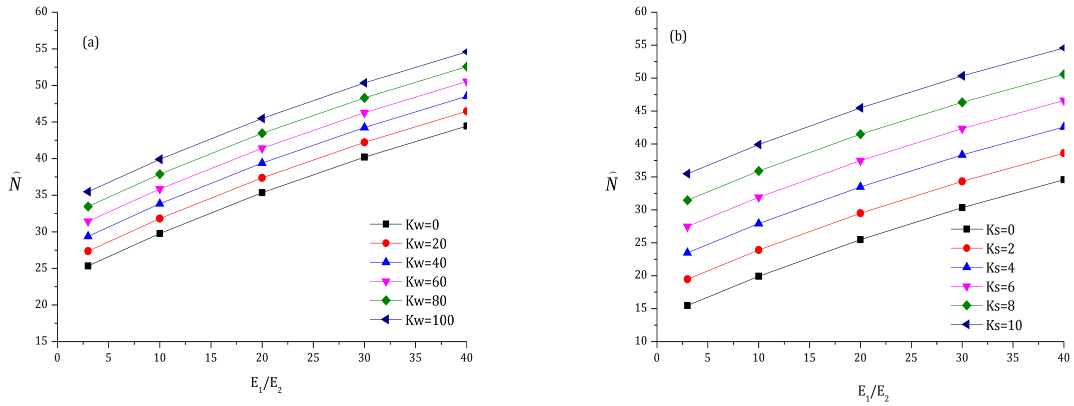


Fig 8. Variation of uniaxial buckling load $\hat{N}$ versus orthotropy ratio for simply supported anti-symmetric laminated (0/90/0/90/0/90) square plate on the elastic foundation (a/h=10, (a): Ks=10, (b): Kw=100).

Figure 9 contains plots of dimensionless uniaxial critical buckling loads versus lamination angle θ of angle-ply $(\theta/-\theta)_1$ laminate with different values of foundation parameters. It is found that the curves are symmetric to the line of $\theta = 45°$ where the latter presents the largest values of dimensionless buckling loads for all cases.

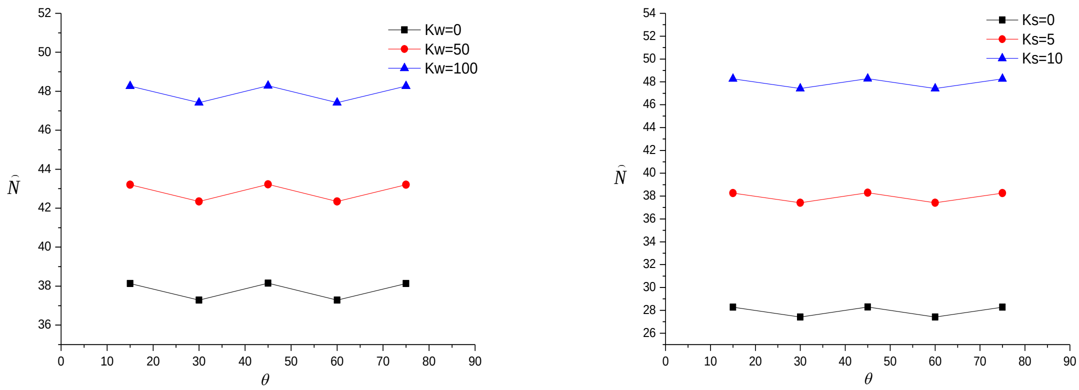


Fig 9. Effect of the lamination angle θ on uniaxial buckling load $\hat{N}$ of simply supported anti-symmetric laminated $(\theta/-\theta)_1$ angle-ply square plate on the elastic foundation (a/h=10, (a): Ks=10, (b): Kw=100).

4. Conclusions

In this paper, a refined higher order shear deformation theory has been used to determine natural frequencies and buckling loads of simply supported antisymmetric cross-ply and angle-ply laminated rectangular plates on two parameters elastic foundation. From the results and discussion it can be concluded that the theory proposed is accurate and efficient in predicting the vibration and the buckling responses of laminated plates.

The influence of the side to thickness ratio, number of layers, lamination angle, orthotropic ratio and elastic foundation parameters on both nondimensional fundamental frequencies and critical buckling loads is studied. The results showed that these parameters have significant influence on the buckling and free vibration characteristics of antisymmetric laminated composite plates.

It is also concluded that the presence of elastic foundation increases buckling load and frequencies of antisymmetric laminates plates. While, The Winkler stiffness does not have as much of effect on natural frequency and buckling loads as shear layer stiffness.

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