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Application of Fuzzy theory to project scheduling with critical path method

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ABSTRACT: In this paper, we analyze the project scheduling problem using fuzzy theory. The crisp activity durations are modeled as triangular fuzzy sets. Fuzzy forward pass was carried out to determine fuzzy activity earliest start, fuzzy event earliest time and fuzzy activity earliest finish times. In order to overcome the occurrence of negative fuzzy numbers which occurs in fuzzy backward pass using fuzzy subtraction, we apply a modified fuzzy backward pass technique which uses a recursive relation to obtain the fuzzy event latest; fuzzy activity latest start and fuzzy activity latest finish times. Through numerical examples, we determine the criticality of the project activities and hence the critical path(s). The results obtained using the present method is compared with those obtained using other methods used in literature.© JASEM

In the real world, project activity time estimation is often inherently vague. The decision makers are forced to use crisp activity times to estimate expected project completion time when bidding for a project and in other cases, the uncertainty was handled by using stochastic probability -based PERT method (Malcoln et al, 1959) In the use of crisp activity durations, more often than not, decision makers are not able to complete the project on schedule leading to increased project cost. The use of optimistic, most likely and pessimistic time estimates to model activity duration was impracticable in many instances as it was difficult to get the distribution of probabilities of activity duration times since the project was being carried out for the first time. The decision maker usually estimates activity durations by using very vague statements such as "an activity duration is almost 6 months", or "between 4 and 6 months". This type of estimation of activity cannot be handled by traditional methods such as CPM and PERT but proves to be amenable to fuzzy theory solution methodology.

A number of researchers have utilized fuzzy set theory to analyze project networks. Chen et al. [1997] incorporated time-window constraint and time schedule constraint into the traditional activity network and developed a linear time algorithm for finding the critical path in an activity network with these time constraints. Dubois et al. (2003a) studied latest starting times and floats in activity networks with ill known durations. Shankar et al (2010) developed an analytical method for finding critical path in a fuzzy critical path by developing a new defuzzification formula for fuzzy set whose members are not equal. Hsian and Lin (2009) developed a fuzzy PERT approach to evaluate plant construction project scheduling risk under uncertain resource capacity. Shankar et al (2010) presented an analytical method for finding the critical path in a fuzzy project network. They presented a new defuzzification formula for trapezoidal fuzzy numbers and applied it to the float time of each activity in order to find the critical path. Mikaeilvand et al (2010) applied a novel ranking method based on centre of mass on network projects and compared results with other ranking methods.

In this paper, we determine activity criticality in a fuzzy project network using a modified backward pass based on a recursive methodology. Through numerical examples, the applicability of fuzzy theory to project network analysis is demonstrated. The solution obtained is compared with those using other methods.

FUZZY FUNDAMENTALS

Let R be the space of real numbers. A fuzzy set of numbers is a set of ordered pairs $\{(x,\mu_{\tilde{A}}(x)) | x\in R\}$, where $\mu_{\tilde{A}}(x) : R \to [0,1]$ and is upper semi continuous. $\mu_{\tilde{A}}(x)$ is called the membership function of the fuzzy set. A convex fuzzy set is a fuzzy set in which Eq. (1) and (2) holds

$$\forall x, y\in R, \forall \lambda \in [0,1] \quad (1)$$

$$\mu_{\tilde{A}}(\lambda x+(1-\lambda)y) \ge \min[\mu_{\tilde{A}}(x),\mu_{\tilde{A}}(y)] \quad (2)$$

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A fuzzy set $\tilde{A}$ is called positive if its membership function is such that $\mu_{\tilde{A}}(x) = 0, \forall x \le 0$.

A triangular fuzzy set $\tilde{A}$ is a convex fuzzy set which is defined as $\tilde{A} = (x, \mu_{\tilde{A}}(x))$ where

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a}{b-a} & a \le x \le b \\ \dfrac{c-a}{c-b} & b \le x \le c \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

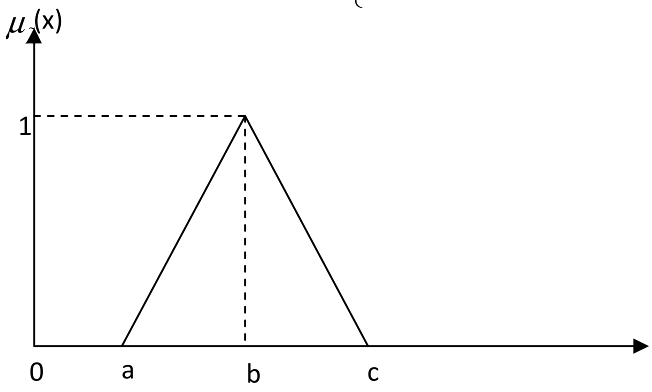

Fig: 1: A triangular fuzzy number

The triangular fuzzy set $\tilde{A}$ is given by the set of numbers (a,b,c) where $0 \le a \le b \le c$

FUZZY SET ARITHMETIC OPERATIONS

Given two triangular fuzzy sets $\tilde{A} = [a_1, b_1, c_1]$ and $\tilde{B} = [a_2, b_2, c_2]$, fuzzy arithmetic and subtraction are implemented as follows

$$\tilde{A} \oplus \tilde{B} = [a_1, b_1, c_1] \oplus [a_2, b_2, c_2] = [(a_1 + a_2), (b_1 + b_2), (c_1 + c_2)] \quad (4)$$

$$\tilde{A} \; \Theta \; \tilde{B} = [a_1, b_1, c_1] \Theta [a_2, b_2, c_2] = [(a_1 - c_2), (b_1 - b_2), (c_1 - a_2)] \quad (5)$$

For two trapezoidal fuzzy sets $\tilde{A} = [a_1, b_1, c_1, d_1]$ and $\tilde{B} = [a_2, b_2, c_2, d_2]$

$$\tilde{A} \oplus \tilde{B} = [(a_1 + a_2), (b_1 + b_2), (c_1 + c_2), (d_1 + d_2)] \quad (6)$$

$$\tilde{A} \; \Theta \; \tilde{B} = [(a_1 - d_2), (b_1 - c_2), (c_1 - b_2), (d_1 - a_2)] \quad (7)$$

FUZZY FORWARD PASS

Consider a project network with activity times t_{ij} where the vertices (V) represent the set of node numbers and the direct edges represent the activities. An activity is represented by one and only one arrow with its tail event at node i and its head event at node j $(i, j \in V)$ where $i < j$. In fuzzy forward pass, we compute the earliest fuzzy time $(\tilde{E}_j)$ of event j, the earliest fuzzy start time $(\tilde{ES})$ of activity (i, j), the earliest fuzzy finish time of activity (i, j) and the fuzzy completion $(\tilde{T}_F)$ time of the project. The earliest fuzzy time of event j can be obtained by implementing the CPM forward pass methodology in the fuzzy environment using the expression

$$\tilde{E}_j = (e_j^1, e_j^2, e_j^3) = \max_{i \in p(j)} \left[\tilde{E}_i \oplus \tilde{t}_{ij} \right] \qquad i \in p(j) \ne \phi \quad (8)$$

In the expression above, $i \in p(j)$ denotes the set of activities i which precedes the node j. When there are no proceeding events $(p(j) = \phi)$ to the event under inspection which corresponds uniquely to the starting node in the project, the fuzzy time of starting the project $(\tilde{T}_S)$ is given as

$$\tilde{T}_S = (t_S^1, t_S^2, t_S^3) = (0,0,0) \quad (9)$$

The fuzzy earliest start and fuzzy earliest finish times of the activities are computed using the expression

$$\tilde{ES}_{ij} = \left(es^1_{ij}, es^2_{ij}, es^3_{ij}\right) = \tilde{E}_i \quad (10)$$

$$\tilde{EF}_{ij} = \left(ef^1_{ij}, ef^1_{ij}, ef^1_{ij}\right) = \tilde{ES}_{ij} \oplus \tilde{t}_{ij} \quad (11)$$

The fuzzy project completion time $\left(\tilde{T}_C\right)$ is obtained by using the expression

$$\tilde{T}_C = \left(t^1_C, t^2_C, t^3_C\right) = \underset{i\in V}{MAX}\ \tilde{E}_i \quad (12)$$

FUZZY BACKWARD PASS

In crisp environment, backward pass is used to calculate the latest event times as well as the latest finish times and fuzzy latest start time of activities. In fuzzy environment, we calculate the fuzzy latest time of event i $\left(\tilde{L}_i\right)$, the fuzzy latest finish time $\left(\tilde{LF}_{ij}\right)$ of activity (i, j) and the fuzzy latest start time $\left(\tilde{LS}_{ij}\right)$ of activity (i, j) In fuzzy backward pass, the occurrence of negative fuzzy numbers is possible since unlike in crisp environment, $\tilde{A} \oplus \tilde{B} \Theta \tilde{B} \neq \tilde{A}$. To overcome this problem, a recursive algorithm is used in the backward pass to obtain positive triangular fuzzy representations of $\tilde{L}_i$, $\tilde{LS}_{ij}$ and $\tilde{LF}_{ij}$. The recursive algorithm is given as

$$\tilde{L}_i = \left(\tilde{L}^1_i, \tilde{L}^2_i, \tilde{L}^3_i\right)$$

(13) $$\tilde{L}_i = \max_{j\in s(i)}\left[0, \min\left(\tilde{L}^3_j - t^3_{ij}\right)\right] \quad (14)$$

$$\tilde{L}^2_i = \max\left[0,\ \min\left(\tilde{L}^3_i,\ \min_{j\in s(i)}\left(\tilde{L}^2_j - t^2_{ij}\right)\right)\right]$$

(15) $$\tilde{L}^1_i = \max\left[0,\ \min\left(\tilde{L}^2_i,\ \min_{j\in s(i)}\left(\tilde{L}^1_j - t^1_{ij}\right)\right)\right]$$

(16)

The fuzzy latest finish time $\left(\tilde{LF}_{ij}\right)$ is by obtained using the expression

$$\tilde{LF}_{ij} = \left(\tilde{LF}^1_{ij}, \tilde{LF}^2_{ij}, \tilde{LF}^3_{ij}\right) = \tilde{L}_j \quad (17)$$

The fuzzy latest start time of an activity (i, j) is computed by using the analogy of the procedure in the backward pass in crisp CPM but in fuzzy environment using the expression

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$$\tilde{LS}_{ij} = \tilde{LF}_{ij} \Theta\ t_{ij} \quad (18)$$

Since backward pass is known to produce negative fuzzy numbers which are infeasible in fuzzy time domain, a recursive algorithm similar to that used in obtaining $\tilde{L}_j$ is used. The recursion formula is shown in the equation below.

$$\tilde{LS}_{ij} = \left(\tilde{LS}^1_{ij}, \tilde{LS}^2_{ij}, \tilde{LS}^3_{ij}\right) \quad (19)$$

$$\tilde{LS}^3_{ij} = \max_{j\in s(i)}\left[0, \min\left(\tilde{LF}^4_{ij} - t^3_{ij}\right)\right] \quad (20)$$

$$\tilde{LS}^2_{ij} = \max\left[0,\ \min\left(\tilde{LF}^3_{ij},\ \min_{j\in s(i)}\left(\tilde{LF}^2_{ij} - t^2_{ij}\right)\right)\right] \quad (21)$$

$$\tilde{LS}^1_{ij} = \max\left[0,\ \min\left(\tilde{LF}^2_{ij},\ \min_{j\in s(i)}\left(\tilde{LF}^1_{ij} - t^1_{ij}\right)\right)\right] \quad (22)$$

The fuzzy total float is used as a measure of criticality of an activity. The fuzzy total float is computed using the relation

$$\tilde{TF}_{ij} = \tilde{LF}_{ij} \text{ -}\tilde{ES}_{ij} \text{ -}t_{ij} = \left(\tilde{TF}^1_{ij}, \tilde{TF}^2_{ij}, \tilde{TF}^3_{ij}\right) \quad (23)$$

However since fuzzy subtraction could result in infeasible total float, we employ the recursive algorithm stated below to obtain positive fuzzy total float numbers.

$$\tilde{TF}^4_{ij} = \max\left[0, \left(\tilde{LF}^4_{ij} - \tilde{ES}^4_{ij} - t_{ij}\right)\right] \quad (24)$$

$$\tilde{TF}^3_{ij} = \max\left[0, \min\left(\tilde{TF}^4_{ij}, \left(\tilde{LF}^3_{ij} - \tilde{ES}^3_{ij} - t_{ij}\right)\right)\right] \quad (25)$$

$$\tilde{TF}^2_{ij} = \max\left[0, \min\left(\tilde{TF}^3_{ij}, \left(\tilde{LF}^2_{ij} - \tilde{ES}^2_{ij} - t_{ij}\right)\right)\right] \quad (26)$$

$$\tilde{TF}^1_{ij} = \max\left[0, \min\left(\tilde{TF}^2_{ij}, \left(\tilde{LF}^1_{ij} - \tilde{ES}^1_{ij} - t_{ij}\right)\right)\right] \quad (27)$$

NUMERICAL EXAMPLE

In order to demonstrate the proposed method, we apply the procedure described to a hypothetical project network. The fuzzy activity durations along with the precedence relationships are presented in table 1. Figure 1 shows the precedence relationships in the fuzzy project network

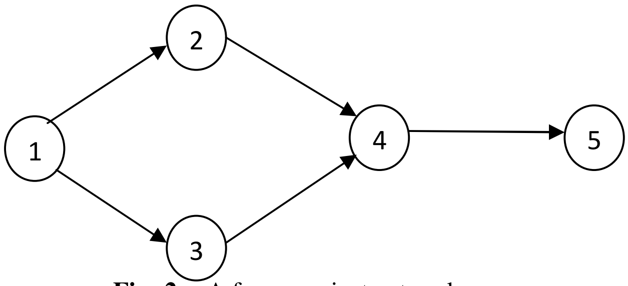

Fig: 2: A fuzzy project network

Table 1: Fuzzy activity time for each activity in the project

Activity	1-2	1-3	2-4	3-4	4-5
Duration	Approximately Between 2 and 3 days	Approximately Between 2 and 3 days	Approximately Between 3 and 4 days	Approximately Between 7 and 8 days	Approximately Between 2 and 3 days

Using the node numbering in figure 2, we compute the earliest event times as shown below

$\tilde{E}_1 = \tilde{T}_s = (0,0,0)$, since this is the starting node of the project

Using Eq. (11), $\tilde{E}_2$, $\tilde{E}_3$, $\tilde{E}_4$ and $\tilde{E}_5$ can be computed noting that nodes 2 and 3 have only node 1 as predecessor node each, node 4 has nodes 2 and 3 as predecessors and node 5 has node 4 as predecessor.

$$\tilde{E}_2 = \tilde{E}_1 + t_{12} = (1,2,4)$$

$$\tilde{E}_3 = \tilde{E}_1 + t_{13} = (1,2,4)$$

$$\tilde{E}_4 = \max\left(\left(\tilde{E}_2 + t_{24}\right),\left(\tilde{E}_3 + t_{34}\right)\right) = (7,9,13)$$

$$\tilde{E}_5 = \tilde{E}_4 + t_{45} = (8,11,17)$$

The fuzzy early start time $\left(\tilde{ES}_{ij}\right)$ of the activities in the network is computed using Eq. (10)

$$\tilde{ES}_{12} = \tilde{E}_1 = (0,0,0)$$

$$\tilde{ES}_{13} = \tilde{E}_1 = (0,0,0)$$

$$\tilde{ES}_{23} = \tilde{E}_2 = (1,2,4)$$

$$\tilde{ES}_{34} = \tilde{E}_3 = (1,2,4)$$

$$\tilde{ES}_{45} = \tilde{E}_4 = (7,9,13)$$

The fuzzy project completion time $\left(\tilde{T}_F\right)$ can be computed using Eq. (12)

$$\tilde{T}_F = MAX\left(\tilde{E}_1,\tilde{E}_2,\tilde{E}_3,\tilde{E}_4,\tilde{E}_5\right) = (8,11,17)$$

The fuzzy earliest finish times $\left(\tilde{EF}_{ij}\right)$ of the activities is computed using Eq. (11)

$$\tilde{EF}_{12} = \tilde{ES}_{12} + t_{12} = (1,2,4)$$

$$\tilde{EF}_{i3} = \tilde{ES}_{13} + t_{13} = (1,2,4)$$

$$\tilde{EF}_{24} = \tilde{ES}_{24} + t_{24} = (3,5,9)$$

$$\tilde{EF}_{34} = \tilde{ES}_{34} + t_{34} = (7,9,13)$$

$$\tilde{EF}_{45} = \tilde{ES}_{45} + t_{45} = (8,11,17)$$

The fuzzy latest start times $\left(\tilde{LS}_{ij}\right)$ and latest finish times $\left(\tilde{LF}_{ij}\right)$ are computed using the recursive relations in Eq (19) to Eq. (22) and Eq. (13) to Eq. (17) respectively. Based on the recursive relation for computing $\left(\tilde{LS}_{ij}\right)$, we have for example

$$\tilde{LS}_{45}^{3} = \max\left(0,(17-4)\right) = 13$$

$$\tilde{LS}_{45}^{2} = \max\left(0,\min(13,9)\right) = 9$$

$$\tilde{LS}_{45}^{1} = \max\left(0,\min(9,7)\right) = 7$$

The fuzzy latest start time of activity 4-5 s therefore given as $(7,9,13)$.The computation of $\tilde{LS}_{12}$ is presented below

$$\tilde{LS}_{12}^{3} = \max(0,4) = 4$$

$$\tilde{LS}_{12}^{2} = \max\left(0,\min(4,4)\right) = 4$$

$$\tilde{LS}_{12}^{1} = \max\left(0,\min(4,4)\right) = 4$$

The fuzzy latest start time $\tilde{LS}_{12}$ is computed to be $(4,4,4)$. Based on the procedure described the fuzzy activity times $\tilde{E}S_{ij}$, $\tilde{E}F_{ij}$, $\tilde{L}S_{ij}$ $\tilde{L}F_{ij}$ as well as the activity total floats $\tilde{T}F_{ij}$ are computed and presented in Table 2

Table 2: Fuzzy project times and total float for network using triangular fuzzy set representation of activity durations

Activity	Fuzzy duration (days)	$\tilde{E}S_{ij}$	$\tilde{E}F_{ij}$	$\tilde{L}S_{ij}$	$\tilde{L}F_{ij}$	TF_{ij}
1-2	1,2,4	0,0,0	4,4,4	1,2,4	5,6,8	4,4,4
1-3	1,2,4	0,0,0	0,0,0	1,2,4	1,2,4	0,0,0
2-4	2,3,5	1,2,4	5,6,8	3,5,9	7,9,13	4,4,4
3-4	6,7,9	1,2,4	1,2,4	7,9,13	7,9,13	0,0,0
4-5	1,2,4	7,9,13	7,9,13	8,11,17	8,11,17	0,0,0

NUMERICAL EXAMPLE 2

The network representing a project is shown in Figure 3. The fuzzy activities durations are presented in Table 3

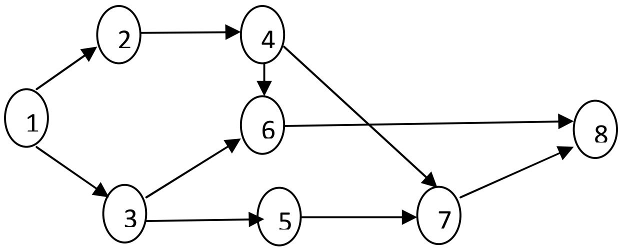

Fig: 3: A fuzzy project network

Based on the procedure described in the previous sections, the activity times, namely $\tilde{E}S_{ij}$, $\tilde{E}F_{ij}$, $\tilde{L}S_{ij}$ $\tilde{L}F_{ij}$, and $\tilde{T}F_{ij}$ are obtained as shown in Table 3

Table 3: Fuzzy project times and total float for network using triangular fuzzy set representation of activity durations

Activity	Fuzzy duration (days)	$\tilde{E}S_{ij}$	$\tilde{L}S_{ij}$	$\tilde{E}F_{ij}$	$\tilde{L}F_{ij}$	TF_{ij}
1-2	2,3,4	0.0.0	0,0,0	2,3,4	2,3,4	0,0,0
1-3	1,2,4	0,0,0	3,2,2	1,2,4	4,5,6	2,2,2
2-4	1,3,5	2,3,4	2,3,4	3,6,9	3,6,9	0,0,0
3-5	1,2,3	1,2,4	4,5,6	2,4,7	5,7,9	2,2,2
3-6	2,5,7	1,2,4	4,5,6	3,7,11	7,10,15	3,3,4
4-6	3,4,6	3,6,9	4,6,9	6,10,15	7,10,15	0,0,0
4-7	3,4,5	3,6,9	3,7,9	6,10,14	6,11,14	0,0,0
5-7	1,4,5	2,4,7	5,7,9	3,8,12	6,11,14	2,2,2
6-8	2,5,6	6,10,15	7,10,15	8,15,21	9,15,21	0,0,0
7-8	3,4,7	6,10,14	6.11,14	9,14,21	9,15,21	0,0,0

Table 4: Comparison of fuzzy total float and defuzzified total float using present method and Sireesha and Sharikar (2010)

Activity	Fuzzy duration (days)	Total float Present Method	Defuzzified total float Present method	Total float Sireesha and Shankar (2010)	Defuzzified total float Sireesha and Shankar (2010)
1-2	2,3,4	0,0,0	0	-13.0.13	0
1-3	1,2,4	2,2,2	2	-11.2.15	2
2-4	1,3,5	0,0,0	0	-13,0,13	0
3-5	1,2,3	2,2,2	2	-11,2,15	2
3-6	2,5,7	3,3,4	3.33	-9,2,16	3.00
4-6	3,4,6	0,0,0	0	-13,0,13	0
4-7	3,4,5	0,0,0	0	-13,1,12	0
5-7	1,4,5	2,2,2	2	-11,2,15	2
6-8	2,5,6	0,0,0	0	-13,0,13	0
7-8	3,4,7	0,0,0	0	-13,1,12	0

RESULTS AND DISCUSSION

The fuzzy theory has been applied using a modified backward pass to the analysis of project scheduling problems. Table 2 shows the activity times as well as float times obtained using the present method. There are two paths in the network namely, 1-2-4-5 and 1-3-4-5. The table shows that activities 1-3, 3-4 and 4-5 are critical since their fuzzy total floats are equal to $(0,0,0)$ When these fuzzy total floats are defuzzified they give a crisp float equal to 0. The critical path is therefore 1-3-4-5. This finding is consistent with the solution obtained using the #chain approach developed by Udosen (1997) when the crisp activity durations are used. A comparison of the total floats shows that the non critical activities 1-2 and 2-4 have a defuzzified total floats equal to 3 and 4 respectively. The corresponding total floats for these activities using the #Chain approach equals 3 and 3. The difference is attributable to the membership function used in obtaining the fuzzy activity duration times of the project.

Table 4 shows the fuzzy activity floats and defuzzified activity total floats obtained using the present modified backward pass method and those obtained by Sireesha and Sharikar (2010) who solved the same problem using the ranking method. The table shows that all activities but 3-6 and 5-7 are critical using the present method. There are four paths in the network namely 1-2-4-5-7-8, 1-2-4-6-8, 1-3-6-8 and 1-3-5-7-8. The paths 1-2-4-7-8 and 1-2-4-6-8 are critical since they contain activities which have a defuzzified total float equal to 0. The defuzzified total floats obtained by Sireesha and Sharikar (2010) shows that only activities 3-6 and 5-7 have non zero total float. However Sireesha and Sharikar (2010) reported only 1-2-4-6-8 as being critical. This may be due to the criterion used in ranking paths in the network even though the defuzzified total floats shows two fuzzy critical paths. The defuzzified total floats using both methods are identical (equal to 2) for activity 5-7, while that defuzzified total float obtained for activity 3-6 is 3.33 for the present method and 3 for the ranking method used by Sireesha and Sharikar (2010).

Conclusion: A modified fuzzy backward pass has been used to obtain the total float, critical activities and critical path of fuzzy project network. The method has been shown to be effective in determining critical activities in a project when activity durations are uncertain and can be represented as fuzzy sets.

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