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Effects of Brinkman number on thermal-driven convective spherical Dynamos.

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ABSTRACT: Brinkman number effects on the thermal-driven convective spherical dynamos are studied analytically. The high temperature of the Earth's inner core boundary is usually conducted by the viscous, electrically conducting fluid of the outer core to the core mantle boundary as the Earth cools. The problem considers conducting fluid motion in a rapidly rotating spherical shell. The consequence of this exponential dependence of viscosity on temperature is considered to be a thermal-driven convective phenomenon. A set of constitutive non-linear equations were then formulated in which the solutions for the flow variables were obtained by perturbation technique. The results illustrate enhancement of dynamo actions, demonstrating that magnetic field generation with time is possible. Moreover, the increased magnetic Prandtl number Pm with high Brinkman number shows dynamo actions for fixed Rayleigh and Taylor number values. The overall analyses succour our understanding of Earth's magnetic field generation mechanism often envisaged in the Earth's planetary interior.© JASEM

. NOMENCLATURE

$u_r, u_\theta, u_\vartheta$ = dimensional velocity components.

L = *Characteristic Length*

T = *Dimensional temperature*

t = *dimensional time*

P = *dimensional Pressure*

$\bar{p} = \frac{p}{\rho_0} + \varphi - \frac{1}{2}(\Omega x r)^2$ *(Modified pressure)*

B = *Magnetic field vector*

Ω = *Angular velocity*

ϕ = *Potential vector*

T_1^* = *dimensional SIC* temperature

T_0^* = dimensional CMB temperature

R_1 = *Dimensionless radius of the solid inner core*

R_2 = *Dimensionless radius of the fluid outer core*

ρ = *Fluid density*

$Pr = \frac{\nu}{\kappa}$ *(Prandtl number)*

$Pm = \frac{\nu}{\eta}$ *(Magnetic Prandtl number)*

σ = *Non-dimensional heat source*

$Ra = \frac{\alpha g_0 \beta_0 L^2}{\nu^2}$ *(Rayleigh number)*

$Ta = \left(\frac{2|\Omega|L^2}{\nu}\right)^2$ *(Taylor number)*

$\Gamma_d(Br) = \frac{L^2}{\nu}\frac{E}{RT_0^2}e^{\frac{-E}{RT_0}}$ *(Brinkman number)*

$\epsilon = \frac{RT_0}{E}$ *(Perturbation scale)*

$\Theta = \frac{E}{RT_0^2}(T - T_0)$ *(Non-dimensional temperature)*

$r = \frac{r'}{R_2}$ *(Normalized distance)*

Superscript

' Non-Dimensional quantities.

The search for an explanation to the origin of Earth's magnetic field gave birth to dynamo theory as suggested by Larmor in 1919. Different mechanisms by which the Earth's liquid core is stirred have been propounded to elucidate the Earth's main magnetic field regeneration (Ishihara and Kida,2002; Fearn and Rahman,2004). The authors employed the thermally driven magneto-hydrodynamic (MHD) dynamo systems with fundamental equations and governing parameters such as the magnetic Ekman number, modified Rayleigh number ($\widehat{Ra}$), Taylor number, magnetic Prandtl number and Roberts number.

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In order to explain the presence of the magnetic field several bewildering varieties of processes were propounded. Each of these generating processes must satisfy an important fundamental requirement, which is, that the energy lost by the electric currents must be replenished. Usually a dynamic dynamo theory basically starts with a velocity field which must satisfy the Navier Equation (Melchior,1986). Majority of these studies had resorted to numerical techniques in order to maintain a magnetic field when it is increasing with increase in time. Sometimes, how the Kinetic energy and magnetic energy grow can be used to assess the generation of magnetic field. Nimmo, *et al* (2004), examined the influence of Potassium on the core and geodynamo evolution. Their investigation reveals that rapid core cooling can form operation for a geodynamo but might create an inner core that is too large. However, an addition of Potassium into the core would perhaps retard inner core growth and would provide an additional source of entropy. The thermal driven mechanism involving heat sources alone is considered to be adequate to power the geodynamo. The compositional and thermal convection mechanisms can also be more efficient power sources for the geodynamo (Gubbins, *et al* ,2003). Sometimes, standard theory has the concept that fluid circulation within the Earth's fluid outer core may also be the source of the Earth's internal magnetic field. This stands as the magneto-hydrodynamic (MHD) theory in which a flowing charge – neutral but electrically conductive fluid will generate magnetic fields (Roberts and Glatzmaier ,2000). Moreover, it is believed that it is this circulation of the liquid iron within the core that lead to geomagnetic field which is sustained by the electric currents due to the continuous convection of the electrical conducting fluid. Planets like Uranus, Neptune, Saturn, Jupiter, the Earth, possibly Mercury and the Sun all have main magnetic fields. These fields could have created by convective motion inside the Planetary electrically conducting fluid cores or shells (Ivers ,2003). The presence of a magnetic field therefore signifies that such planets possess large-liquid cores; a core possibly rich in liquid metals, which are sources of free electrons and basically due to core rotation rate.

It is possible that in the absence of this fluid motion that any field generated, will decay in about 20,000 years; a process known as free decay modes (Moffatt,1978; Backus, *et al* ,1996). This convective motion can be driven by both the thermal and compositional buoyancy sources at the inner core boundary (Busse,2000; Roberts and Glatzmaier,2000).

Another most important process early in the formation of any planet that is deemed to influence its interior structure is gravity. It causes heavier constituents to sink to the core of the planet; and the process is known as chemical fractionation; which is grouped or classified as the gravitationally powered dynamo mechanism (Gubbins, *et al* ,2003; Loper,1978; Gubbins and Master,1979). In this article we shall examine the consequence of dimensionless parameters being complimentary to each other in terms of magnetic field being sustained in the system.

Problem Formulation And Governing Equations: The Earth is considered a concentric sphere (see figure 1) with fluid outer core with lower mantle at the outer boundary and solid inner core at the inner boundary (Jacobs,1953 and Jacobs,1986). The solid inner core is purely iron constituted. The vigorous convection current and swift cooling at the surface due to an adiabatic temperature gradient will lead to the solidification of the liquid iron at the center of the Earth (Melchior,1986). On further cooling the mantle solidifies from bottom upwards, and the fluid outer core maintains its original temperature insulated above by a rapidly thickening shell of silicates. Our system is composed of viscous incompressible conducting fluid that exhibits thermal conduction as heat flux flows through the core mantle boundary (CMB). The fluid is in between the rotating concentric spheres which form the basis of choosing spherical geometry configuration. It's characterized by magnetic diffusivity, thermal diffusivity, constant kinematic viscosity, magnetic permeability and density.

Consider ρ as the density of the fluid in between the concentric spheres, and u, T, P and B are the fluid velocity, temperature, pressure and magnetic field vector respectively, with Ω the angular velocity due to the Earth rotation and φ the potential, hence the mathematical statements that govern the flow of the fluid within this system of concentric spheres of radii R_1 *and* R_2 are given below:-

$$\frac{\partial\rho}{\partial t'} + div(\rho u') = 0 \qquad (1a)$$

$$\nabla . u' = 0 \qquad (1b)$$

$$\rho_0\left(\frac{\partial}{\partial t'} + u'.\nabla\right)u' + 2\rho_0\Omega \times u' = -\nabla p' - \rho_0\nabla\varphi + \tfrac{1}{2}\rho_0\nabla(\Omega\times r)^2 + \mu_0\nabla^2 u' + \Delta\rho g + J\times B' \qquad (2)$$

$$\left(\frac{\partial}{\partial t'} + u'.\nabla\right)T' = \kappa\nabla^2 T' + \sigma + QAe^{\frac{-E}{RT'}} \qquad (3)$$

$$\left(\frac{\partial}{\partial t'} - \eta\nabla^2\right)B' = \nabla \times (u' \times B') \qquad (4a)$$

$$\nabla . B' = 0 \qquad (4b)$$

Inertia, Coriolis, viscous, buoyancy and centrifugal forces exist within the Earth's planetary interior and they act on the fluid elements. On the hand, Lorentz force acts as an opposing force exerted by magnetic field on the moving conducting fluid. Several factors contribute to the total energy of the system as seen in Eq.(3). These include the heat source, rate of heat gain by the material element and viscosity effect with an exponential dependence on the inverse absolute temperature. Equation 4(a) is the magnetic transport equation which is a consequence of the motion of the conducting fluid particles and in the absence of velocity the diffusion term remains (Hughes and Brighton,1967; Moffatt,1978; Melchior,1986). Equations 1-4 are solved subject to the ensuing boundary conditions Eq. (4c, d)

$u' = 0,\ T' = T_1\ and\ \ B' = b_w$ at r = R_1

and (4c,d)

$u' = 0,\ T' = T_2\ and\ \ B' = 0$ at r = R_2

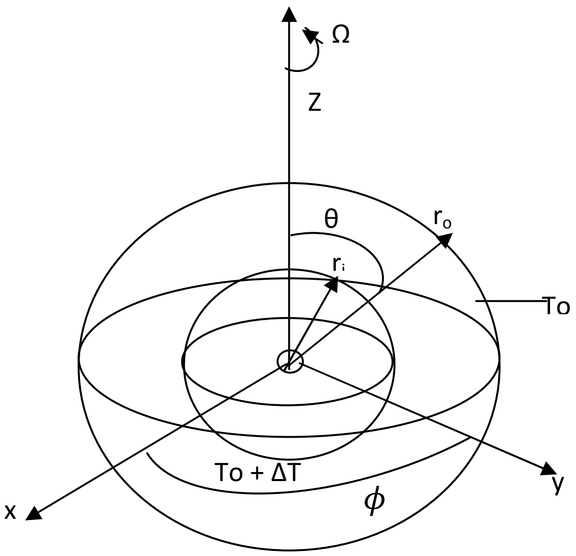

Fig: 1: Geometry of problem and symbols. The inner and outer boundaries are held at constant temperatures T_0 and $T_i = T_0 + \Delta T$ respectively.

Adopting the following non-dimensional quantities$(u_r, u_\theta) = \frac{\nu}{L}(u_r^*, u_\theta^*), t = \frac{L^2}{\nu}t^*, B = \frac{\eta(\mu_0\rho)^{\frac{1}{2}}}{L}B^*,\ \bar{p} = \frac{\rho\nu\eta}{L^2}\overline{p^*}, (r, \theta, \vartheta) = (r^*, \theta^*, \vartheta^*)L,$ $\varphi = gL\varphi^*, \varepsilon' = \frac{(T_1^* - T_0^*)\varepsilon'^*}{L}$, which are substituted into eqns. (1-4), the non-dimensional governing equations for the system are derived, where these quantities and variables are as defined in the nomenclature above. For the buoyancy term which produces a free convection, the associated dimensionless temperature is established as: $\Theta^* = \frac{Ea}{RT_0^2}(T^* - T_0^*)$, where $T^* - T_0^*$ is the characteristic temperature difference between the inner core and outer core boundaries.

$\nabla . u = 0$ (5)

$Pm\left(\frac{\partial}{\partial t} + u.\nabla - \nabla^2\right)u + PmTa^{\frac{1}{2}}|e_z xu = -\nabla\bar{p} + RaPm\theta_r + \frac{1}{Pm}(\nabla xB)xB$ (6)

$(\frac{\partial}{\partial t} + u.\nabla - \frac{1}{Pr}\nabla^2)\Theta = \Gamma_d e^{\frac{\Theta}{1+\epsilon\Theta}} + \varepsilon$ (7)

$\nabla . B = 0$ (8)

$(\frac{\partial}{\partial t} - \frac{1}{Pm}\nabla^2)B = \nabla x(uxB)$ (9)

The Equations (5-9) are solved subject to the following boundary conditions as stated below:

$u = 0,\ \Theta = \Theta_1,\ B = b_w$ at r = R_1
and (10a,b)
$u = 0,\ \Theta = \Theta_2,\ B = 0$ at r = R_2

METHOD OF SOLUTION.

Equations (5)-(9) show that the flow variables are highly coupled and non-linear. In this case some simplified assumptions are made. We observed that the magnetic Prandtl Pm is of the order of ϵ , which lies between 0 and 1, that is $0 < \epsilon < 1$. Taking a perturbation series expansion solution of the form:-

$f(r,\theta,t) = f_0(r,\theta,t) + \epsilon f_1(r,\theta,t) + \epsilon^2 f_2(r,\theta,t) + \cdots$ (11)

for all the dependent flow variables, Equations (5)-(9), therefore in non-dimensional form can be given by the following orders of approximations in ϵ. Thus

$u_0.\nabla\Theta_0 = Pr^{-1}\nabla^2\Theta_0 + \sigma + \Gamma_d(1 + \Theta_0)$ (12a)

$u_0.\nabla u_0 + Ta^{\frac{1}{2}}I_z \times u_0 = Pm^{-1}\nabla\bar{p} + \nabla^2 u_0 + Ra\Theta_0$ (12b)

$(\nabla^2 - P_m\bar{\lambda})B_0 = 0$ (12c)

subject to the following boundary conditions

$u_0 = 0,\ \Theta_0 = \Theta_{in},\ B_0 = \varphi$ $at\ r = 0.35$
and (13)
$u_0 = 0,\ \Theta_0 = \Theta_{out},\ B_0 = 0$ $at\ r = 1$

for the order-zero approximations, which represent the steady state situations of the fluid within the system and:

$\frac{\partial\Theta_1}{\partial t} + u_1.\nabla\Theta_0 + u_0.\nabla\Theta_1 = Pr^{-1}\nabla^2\Theta_1 + \sigma + \Gamma_d(\Theta_1 - \Theta_0^2)$ (14a)

$\frac{\partial u_1}{\partial t} + u_0.\nabla u_1 + u_1.\nabla u_0 + Ta^{\frac{1}{2}}I_z \times u_1 = \nabla^2 u_1 + Ra\Theta_1$ (14b)

$(\nabla^2 - Pm\bar{\lambda})b_1 = Pm(u_1.\nabla B_0 - B_0.\nabla u_1)$ (14c)

with the boundary conditions:

$u_1 = 0,\ \Theta_{in} = 0\ and\ b_1 = 0$ at r = 0.35
and (14d)
$u_1 \to 0,\ \Theta_{out} \to 0\ and\ b_1 \to \infty$ as r $\to$ 1

for the order- one approximations in $\epsilon, i.e\ \mathcal{O}(1)$, stand for the transient state situations. Equations (12) and (14), demonstrate dependences of the following physical non-dimensional parameters: Prandtl number (Pr), Rayleigh number (Ra), Taylor number (Ta), magnetic Prandtl number (Pm) and Brinkman number (Γ_d). To seek solutions for the variables described by $\mathcal{O}(0)$ of Equations (12) we start with the temperature equation. Assuming onset of convection and neglecting the nonlinear term due to Arrhenius energy contribution (Kono and Roberts,2001; Busse and Simitev,2004), the temperature and magnetic field equations as shown in Equations (12a) and (12c) have solutions of the form, based on the following standard method of mathematical expression in Abramowitz and Stegun,1972:

$\Theta_0(r) = a_1 j_0(r\sqrt{Pr\Gamma_d}) + a_2 y_0(r\sqrt{Pr\Gamma_d}) - \lambda r$ (15)

$$B_0(r) = g_1 i_n\left(r\sqrt{Pm\bar{\lambda}}\right) + g_2 \kappa_n\left(r\sqrt{Pm\bar{\lambda}}\right) \qquad (16)$$

Equations (15) and (16) are for the steady state temperature and magnetic field distributions respectively within the concentric spheres. $j_n(z) and\ y_n(z)$ are the known spherical Bessel functions of the first and second kind of order n respectively, while $i_n(z)$ and $\kappa_n(z)$ are the modified spherical Bessel function of the first and second kind of order n respectively (Abramowitz and Stegun, 1972; Arfken, 1985). Using the boundary conditions at the inner core (that is, when r $=R_1$) and at the outer core (when r $=R_2$) as mentioned earlier in Equation (13), the values of $a_1 and\ a_2$ and $g_1 and\ g_2$ are provided in Appendix shown below.

These solutions are the steady state temperature and magnetic field distributions inside the concentric spheres, when the onset of convection is zero, that is assuming u_0=0. Applying these results or solutions into the transient flow variables (Θ_1, u_1 and $b_1(r)$) due to the onset of convection which are expressed in the O(ϵ). Combing the transient equations (14a) and (14b) to have order-one flow field

$$(\nabla^2 - \alpha_2 - \alpha_1)(\nabla^2 + \gamma - \alpha_3)u_1 + Ra(Pr\nabla\Theta_0)u_1 = -\gamma Ra\Theta_0^2 \qquad (17)$$

Convection has begun as a result of temperature differential and when Ra is considered to be small, that is, Ra$< 1\ or\ Ra \to 0$, the velociity u_1 is expressed as

$$u_1 = u_{10} + Ra u_{11} + \cdots \qquad (18)$$

and emerging solutions to the different orders in this approximation approach, after a little algebraic manipulation become

$$u_{10}(r) = b_1 j_n\left(r\sqrt{Pr\Gamma_d}\right) + b_2 y_n\left(r\sqrt{Pr\Gamma_d}\right)$$
(18b)

and

$$u_{11}(r) = b_5 j_n\left(r\sqrt{Pr\Gamma_d}\right) + b_6 y_n\left(r\sqrt{Pr\Gamma_d}\right) + F_r(r)$$
(18c)

substituting these back into equation (18) gives

$$u_1(r) = [b_1 j_n\left(r\sqrt{Pr\Gamma_d}\right) + b_2 y_n\left(r\sqrt{Pr\Gamma_d}\right)] + Ra[b_5 j_n\left(r\sqrt{Pr\Gamma_d}\right) + b_6 y_n\left(r\sqrt{Pr\Gamma_d}\right) + F_r(r)]$$

that is

$$u_1(r) = c_1 j_n\left(r\sqrt{Pr\Gamma_d}\right) + c_2 y_n\left(r\sqrt{Pr\Gamma_d}\right) + RaF_r(r) \qquad (18d)$$

where all the associated constants were determined using the appropriate boundary conditions mentioned above, and $F_r(r)$, $\alpha_2, c_1, c_2, b_1, b_2, b_3, b_4, b_5, b_6$ are shown in the Appendix.

Pr, Γ_d, and γ as show in Eqn. (12) control the flow which will contribute to the nature of the solution to the velocity component. The temperature profiles will therefore depend on these quantities and they are causes of onset of convection that exists in the fluid outer core.

The solution to the velocity component of the flow model, Equation (18d) shows that the flow is controlled by the actions of Pr, Γ_d, and γ. These parameters are temperature dependent and are responsible for the initiation of convection observed in the fluid outer core layers. Applying these solutions of the temperature and velocity due to the initiation of convection we then derived the effect of the convective motion on the B-field within the Earth's core. And assuming a solution to the magnetic transport equation of the form:-

$$B(r,\theta,t) = B(r,\theta)e^{\bar{\lambda}t} \qquad (19)$$

and consequently following the method of solutions adopted for the temperature and the velocity, the solution to B-field equation becomes:

$b_1(r) = g_3 i_n\left(r\sqrt{Pm\bar{\lambda}}\right) + g_4 \kappa_n\left(r\sqrt{Pm\bar{\lambda}}\right) + Q(r)$ (20)

where g_3, g_4 *and* $Q(r)$ are given in the Appendix below

with, $k = \left(Pm\bar{\lambda}\right)^{1/2}$, $\alpha = \left(\sqrt{Ta}\right)^{1/2}$, $\gamma = \left(Pr\Gamma_a\right)^{1/2}$, $Z_1 = \int (i_1(r\sqrt{Pm\bar{\lambda}})F_r)dr$
and $Z_2 = \int (\kappa_1(r\sqrt{Pm\bar{\lambda}})F_r)dr$.

$B(r,\theta,t) = B_0(r,\theta) + b_1 e^{p_\alpha t}$
$=$
$(g_1 + g_3 e^{p_\alpha t}) i_n\left(r\sqrt{p_\alpha Pm}\right) +$
$(g_2 + g_4 e^{p_\alpha t}) k_n\left(r\sqrt{p_\alpha Pm}\right) + Q(r) e^{p_\alpha t}$ (21a)

$B(r,\theta,t) = g_5 i_n\left(r\sqrt{p_\alpha Pm}\right) + g_6 k_n\left(r\sqrt{p_\alpha Pm}\right) + Q(r) e^{p_\alpha t}$ (21b)

where, $g_5 = g_1 + g_3 e^{p_\alpha t}$ and $g_6 = g_2 + g_4 e^{p_\alpha t}$

The above mentioned solutions of the formulated problems are within the limits of our approximation. The steady state flow variables define the absence of convective motion, which typically shows that the magnetic field decays as one move away from the source. That is, the initiation of convection brings about the transient effects observed in the flow variables as well as introducing the dynamo effects through the Lorentz force term. We then displayed the obtained results graphically with the aid of Wolfram Mathematica software as shown in figures(2) to (7).

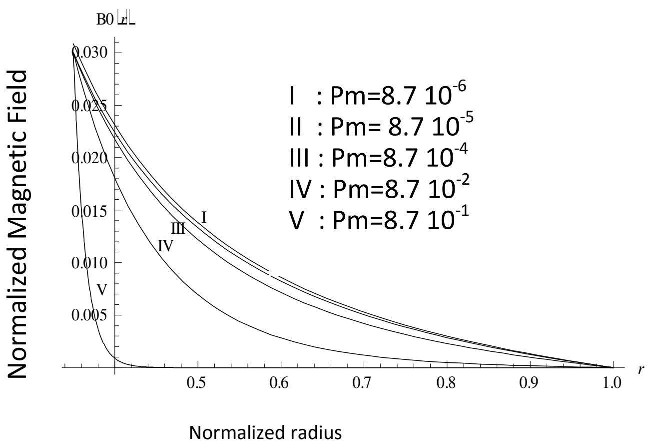

Fig: 2: Magnetic Fields decay in the core of the Earth at various values of magnetic Prandtl numbers(for δ=10).

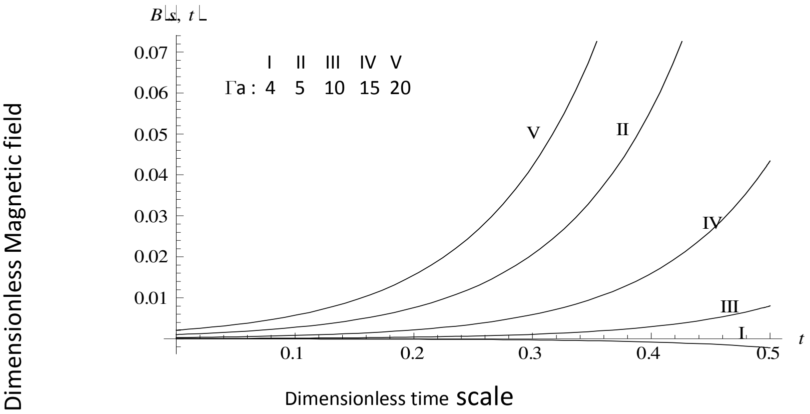

Fig: 3: Magnetic field regeneration due to the effect of convection heat tranfer motion (δ=10, σ=0.65, Pm=3.09, Ra=3.2x10^4, Ta=1.6x10^5, Pr=1.5).

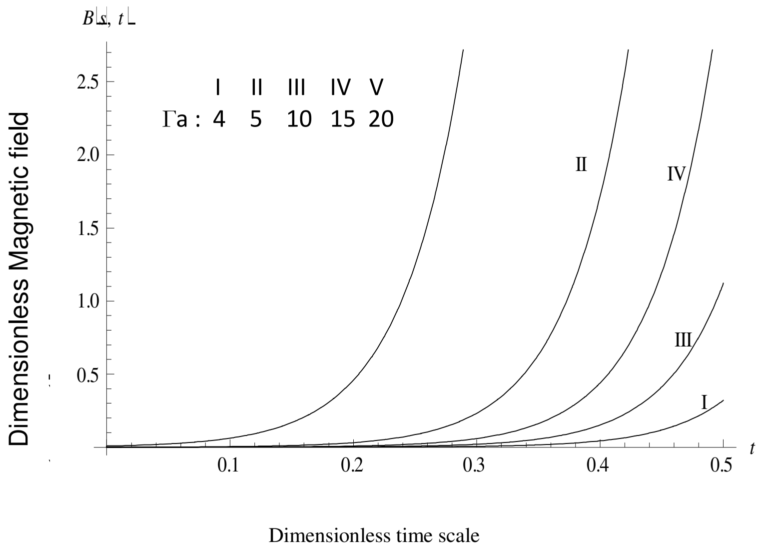

Fig:4: Magnetic field regeneration due to the effect of convection heat tranfer motion (δ=20, σ=0.65, Pm=8.7x10$^{-2.5}$, Ra=3.2x10^4, Ta=1.6x10^5, Pr=2.0).

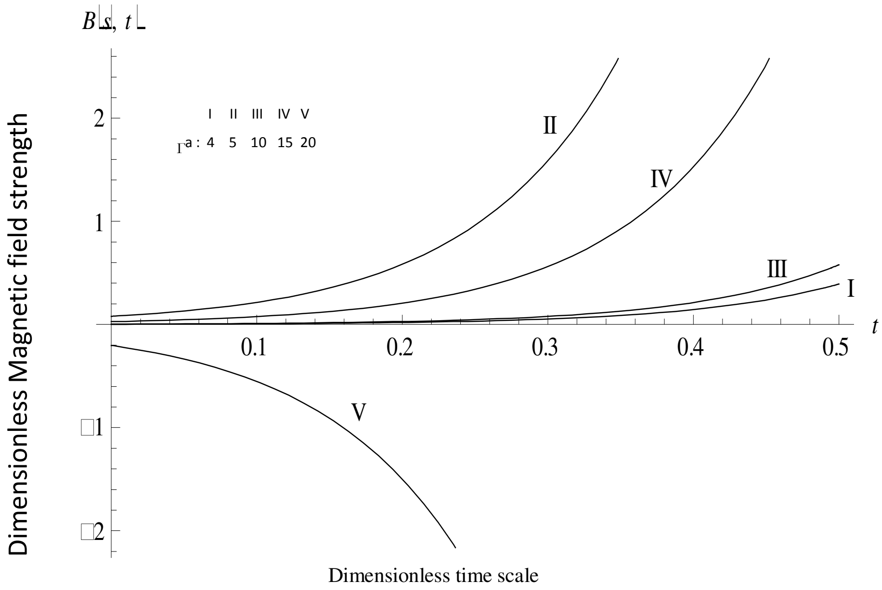

Fig: 5: Magnetic field regeneration due to the effect of convection heat tranfer motion (δ=10,σ=0.65,Pm=3.09,Ra=3.2×10^4,Ta=1.6×10^5, Pr=3.5).

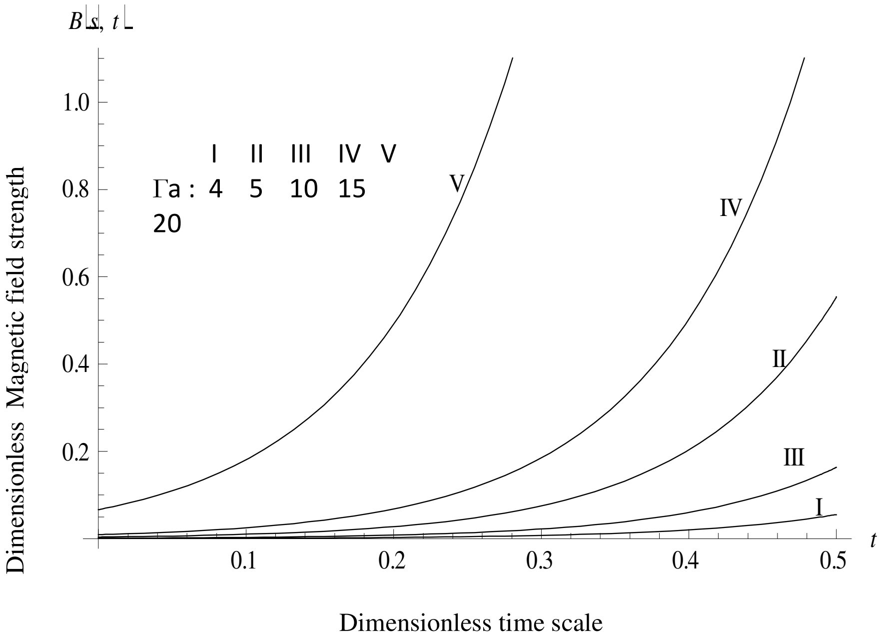

Fig: 6: Magnetic field regeneration due to the effect of convection heat tranfer motion (δ=10,σ=0.65,Pm=3.09,Ra=3.2×10^4,Ta=1.6×10^5,Pr=1.0).

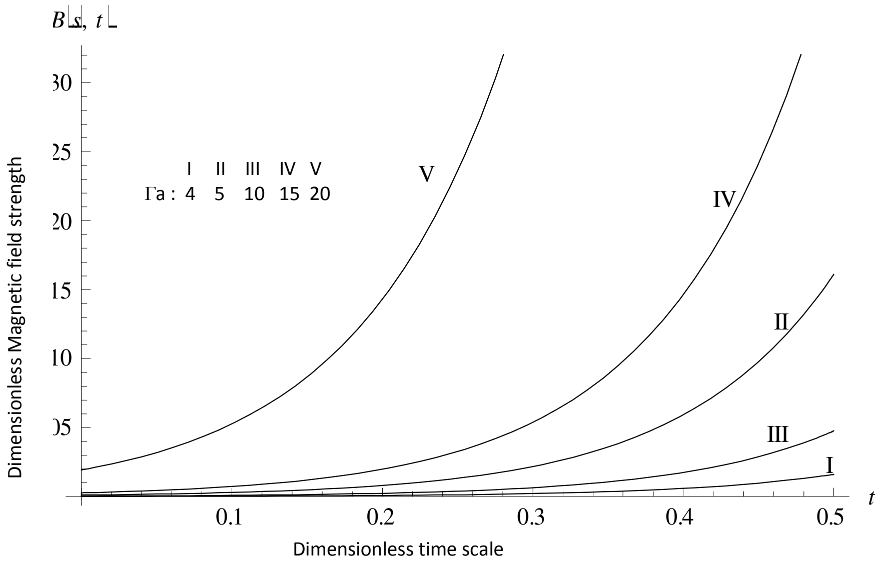

Fig: 7:Magnetic field regeneration due to the effect of convection heat tranfer motion(δ=10,σ=0.65,Pm=8.7x10^{-2},Ra=3.2x10^4,Ta=1.6x10^5, Pr=1.0).

RESULTS AND DISCUSSION

To understand the significance of the results obtained in Eqs.21(a & b) we present graphical representations which will demonstrate the influences of the dimensionless parameters. Eqs. (15) and (18d) are coupled in the above solution of the magnetic field. In the absence of the flow field, eq. (16) suffices which is the magnetic diffusion mode as shown in figure 2 above. No matter the magnitude of the active dimensionless parameter the magnetic field always shrinks or aproaches to zero as it moves far from the source. Here the magnetic Prandtl number (Pm) is the active quantity since the others including the energy consequence are not coupled to it. An increased magnetic Prandtl number becomes asymptotically curved more than when it is very very small, see curve V of figure 2. As long as there is no motion to drive the flow diffusion is certain. In this study we included the implication of shear stress due to frictional heating which is often neglected (Glaztmaier, *et a, 1999*; Ishihara and Kida,2002; Gubbins, *et al, 2003*; Ivers,2003; Fearn and Rahman,2004).

Observed and known temperature gradients within the Earth's deep interior (Gubbins and Masters, 1979, Kono and Roberts, 2001) establish the existence of convective heat motion in the core. To depict this, the heated fluid particles from the inner core due to latent heat of solidification and very high pressure of compaction, rise to the fluid outer core as denser fluid particles descend to the inner core. As these light fluid elements escape as helical fluid filaments convective motions manifest and on interacting with the seed magnetic field will result in dynamo mechanism (Ayeni, 1994). And since the fluid of the outer core is characterised as viscous it is temperature dependent (Turcotte and Schubert, 1982. More so, figure 2 shows magnetic field diffusion occuring at small positive growth rate value. The implication implies that without fluid flow which is the definite convective motion, the seed magnetic field once established will decay soon enough as shown by Moffatt,1978, Backus et al, 1996 and Ngwueke and Abbey, 2012. The significance then , according to Ngwueke and Abbey,2012 is that the convective heat transfer motion is responsible to the fluid velocity change which subsequently empowers the seed field growth. As seen in figure 3, Brinkman number in the range $4 \leq \Gamma a \leq 20$ support seed field regeneration however when the magnetic Prandtl number is increased or high and Prandtl number Pr>1.2 demonstrating strong parameter dependence proclivity for sustenance of the Earth's magnetic field (Simitev and Busse,2005). In contrast to the previous published paper, Ngwueke and Abbey(2012) dynamo actions manifested when Pm, Pr and δ were increased as seen in figure 4. Curve V looses its dynamo action when Pr number is raised to 3.5 and δ=10. Others were observed to sustain magnetic field (figure 5), that is when Γa≥20 the ability to support dynamo action fails. Moreover, at this point the thermal diffusivity is approaching zero, meaning that the thermal diffusivity, κ is by far less than kinematic viscosity, ν.

Still for various values of Brinkman number ranging 4< Γa<20 when Pm and Pr are increased in contrast to the geophysical realizable values only curve V does not support dynamo action (figure 5). But on reducing the Prandtl number to 1 we observe a total change where all the values of the Γa enhance dynamo mechanism as shown in figure 6. Each of these parameter groups usually demonstrates different temperature and velocity profiles. These profiles, however not presented in this paper are based on the solutions expressed by Equations (15 and 18d) and were characterized by Prandtl, Brinkman, Taylor and Rayleigh numbers. Their conditions affect the magnetic field of the conducting medium since their equations were coupled to the magnetic field equation (21d). As mentioned earlier when there is no flow and the Rayleigh number is deemed zero, that is, no associated heat the resulting situation will give rise to diffusion of the magnetic field.

The extent of the motion is determined by the interaction between the induced current of the liquid metal within the fluid outer core. This has significant implication on the convective motion of the conductive fluid which was thermal-driven in this our spherical system. Earth model values of Pr =1.0 and Pm = 3.09 and 8.7 x 10^{-2} produced dynamo actions as shown in figures 6 and 7 though the active convective parameter Ra is the same. Brinkman number, within the range $4 < \Gamma a < 20$ with the model values as shown will support magnetic field recreation. It is shown that a combination of Earth real value and model values can result in dynamo mechanism.

Conclusions: In this paper we have examined three outstanding issues usually encountered in dynamo theory, and proffered possible means of achieving dynamo mechanism

The non-linearity characteristics usually inherent in Earth's dynamo equations were solved by applying Perturbation technique making these equations tractable

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Dynamo sustenance depends on non-dimensional parameters being complementary to each other in other to obtain magnetic field recreation. Our study has demonstrated the complementary roles of magnetic Prandtl number, and Prandtl number to the active dimensionless parameter which is Brinkman number in this study.

The seed magnetic field rejuvenates for Br range between 4.0 and 20 when the Pr and Pm complement each other. This is shown to involve the convective term when it interacts with the magnetic field.

APPENDIX

$$Fr = \sqrt{Pr\Gamma_d Ta^{\frac{1}{2}}}\{\left(d_1 i_0\left(r\sqrt{Ta^{\frac{1}{2}}}\right) - d_2 k_0\left(r\sqrt{Ta^{\frac{1}{2}}}\right)\right) + \mathrm{Pr}\left(\sqrt{\xi}\mathrm{a}_1 c_1 \frac{\xi(1-\xi^2)}{1-\xi^3} j_0(r\sqrt{Pr\Gamma_d})j_n(r\sqrt{Pr\Gamma_d}) -\right.$$

a2c2ξξξ– 11– ξ3y0rPrΓd jnrPrΓd −λξ2c1jnrPrΓd +ξa1c2ξξ– 11– ξ3y0rPrΓd jnrPrΓd +ξa2c2ξξ– 11– ξ3y0rPrΓd ynrPrΓd −λξc2ynrPrΓd +...

$$a_1 = \frac{\Theta_{in} y_0(R_0\sqrt{Pr\Gamma_d}) - \Theta_{out} y_0(R_1\sqrt{Pr\Gamma_d}) + \lambda\left(R_1 y_0(R_0\sqrt{Pr\Gamma_d}) - R_0 y_0(R_1\sqrt{Pr\Gamma_d})\right)}{j_0(R_1\sqrt{Pr\Gamma_d}) y_0(R_0\sqrt{Pr\Gamma_d}) - j_0(R_0\sqrt{Pr\Gamma_d}) y_0(R_1\sqrt{Pr\Gamma_d})}$$

$$a_2 = \frac{\Theta_{out} j_0(R_1\sqrt{Pr\Gamma_d}) - \Theta_{in} j_0(R_0\sqrt{Pr\Gamma_d}) + \lambda(R_0 j_0(R_1\sqrt{Pr\Gamma_d}) - R_1 j_0(R_0\sqrt{Pr\Gamma_d}))}{j_0(R_1\sqrt{Pr\Gamma_d}) y_0(R_0\sqrt{Pr\Gamma_d}) - j_0(R_0\sqrt{Pr\Gamma_d}) y_0(R_1\sqrt{Pr\Gamma_d})}$$

$$b_1 = \frac{\Theta_{11}(R_1) \mathrm{y_n}(R_0\sqrt{\mathrm{Pr}\Gamma_\mathrm{d}}) - \Theta_{11}(R_0) \mathrm{y_n}(R_1\sqrt{\mathrm{Pr}\Gamma_\mathrm{d}})}{\mathrm{j_n}(R_1\sqrt{\mathrm{Pr}\Gamma_\mathrm{d}}) \mathrm{y_n}(R_0\sqrt{\mathrm{Pr}\Gamma_\mathrm{d}}) - \mathrm{j_n}(R_0\sqrt{\mathrm{Pr}\Gamma_\mathrm{d}}) \mathrm{y_n}(R_1\sqrt{\mathrm{Pr}\Gamma_\mathrm{d}})}$$

$$b_2 = \frac{\Theta_{11}(R_0) \mathrm{j_n}(R_1\sqrt{\mathrm{Pr}\Gamma_\mathrm{d}}) - \Theta_{11}(R_1) \mathrm{j_n}(R_0\sqrt{\mathrm{Pr}\Gamma_\mathrm{d}})}{\mathrm{j_n}(R_1\sqrt{\mathrm{Pr}\Gamma_\mathrm{d}}) \mathrm{y_n}(R_0\sqrt{\mathrm{Pr}\Gamma_\mathrm{d}}) - \mathrm{j_n}(R_0\sqrt{\mathrm{Pr}\Gamma_\mathrm{d}}) \mathrm{y_n}(R_1\sqrt{\mathrm{Pr}\Gamma_\mathrm{d}})}$$

$$c_1 = \frac{\Omega(R_1 y_n(R_0\sqrt{Pr\Gamma_d}) - R_0 y_n(R_1\sqrt{Pr\Gamma_d}))}{\mathrm{j_n}(R_1\sqrt{\mathrm{Pr}\Gamma_\mathrm{d}}) \mathrm{y_n}(R_0\sqrt{\mathrm{Pr}\Gamma_\mathrm{d}}) - \mathrm{j_n}(R_0\sqrt{\mathrm{Pr}\Gamma_\mathrm{d}}) \mathrm{y_n}(R_1\sqrt{\mathrm{Pr}\Gamma_\mathrm{d}})}$$

$$c_2 = \frac{\Omega(R_0 j_n(R_1\sqrt{Pr\Gamma_d}\,) - R_1 j_n(R_0\sqrt{Pr\Gamma_d}\,)}{j_n(R_1\sqrt{Pr\Gamma_d}\,)y_n(R_0\sqrt{Pr\Gamma_d}\,) - j_n(R_0\sqrt{Pr\Gamma_d}\,)y_n(R_1\sqrt{Pr\Gamma_d}\,)}$$

$$d_1 = \frac{\Omega(R_1 k_n\left(R_0\sqrt{Ta^{\frac{1}{2}}}\right) - R_0 k_n\left(R_1\sqrt{Ta^{\frac{1}{2}}}\right)) + Y_{R_0} k_n\left(R_1\sqrt{Ta^{\frac{1}{2}}}\right) - Y_{R_1} k_n\left(R_0\sqrt{Ta^{\frac{1}{2}}}\right)}{i_n\left(R_1\sqrt{Ta^{\frac{1}{2}}}\right)k_n\left(R_0\sqrt{Ta^{\frac{1}{2}}}\right) - i_n\left(R_0\sqrt{Ta^{\frac{1}{2}}}\right)k_n\left(R_1\sqrt{Ta^{\frac{1}{2}}}\right)}$$

$$d_2 = \frac{\Omega(R_0 i_n\left(R_1\sqrt{Ta^{\frac{1}{2}}}\right) - R_1 i_n\left(R_0\sqrt{Ta^{\frac{1}{2}}}\right)) + Y_{R_1} i_n\left(R_0\sqrt{Ta^{\frac{1}{2}}}\right) - Y_{R_0} i_n\left(R_1\sqrt{Ta^{\frac{1}{2}}}\right)}{i_n\left(R_1\sqrt{Ta^{\frac{1}{2}}}\right)k_n\left(R_0\sqrt{Ta^{\frac{1}{2}}}\right) - i_n\left(R_0\sqrt{Ta^{\frac{1}{2}}}\right)k_n\left(R_1\sqrt{Ta^{\frac{1}{2}}}\right)}$$

$Y_{R_1} =$

$\sqrt{Ta}\{\Pr\left(a_1 c_1\sqrt{\xi}\ \frac{\xi(1-\xi^2)}{1-\xi^3} j_0\left(R_1\sqrt{Pr\Gamma_d}\,\right) j_0\left(R_1\sqrt{Pr\Gamma_d}\,\right) -\right.$

a2c2ξ ξ1−ξ1−ξ3y0R1PrΓd j0R1PrΓd

*−λc1ξj0R1PrΓd +*a1*c2ξ ξξ−11−ξ3y0R1PrΓd*

j0R1PrΓd−…}

$Y_{R_0} =$

$\sqrt{Ta}\{\Pr\left(a_1 c_1\sqrt{\xi}\ \frac{\xi(1-\xi^2)}{1-\xi^3} j_0\left(R_0\sqrt{Pr\Gamma_d}\,\right) j_0\left(R_0\sqrt{Pr\Gamma_d}\,\right)\right.$

a2c2ξ ξ1−ξ1−ξ3y0R0PrΓd j0R0PrΓd

*−λc1ξj0R0PrΓd +*a1*c2ξ*

ξξ−11−ξ3y0R0PrΓd j0R0PrΓd−…}

$\alpha_2 = Ta^{\frac{1}{2}}$;

$$b_3 = \frac{\Omega(R_1\kappa_n(\alpha R_0) - R_0\kappa_n(\alpha R_1)) + Y_{R0}\kappa_n(\alpha R_1) - Y_{R1}\kappa_n(\alpha R_0)}{i_n(\alpha R_1)\kappa_n(\alpha R_0) - i_n(\alpha R_0)\kappa_n(\alpha R_1)}$$

$$b_4 = \frac{\Omega(R_0 i_n(\alpha R_1) - R_1 i_n(\alpha R_0)) + Y_{R1} i_n(\alpha R_0) - Y_{R0} i_n(\alpha R_1)}{i_n(\alpha R_1)\kappa_n(\alpha R_0) - i_n(\alpha R_0)\kappa_n(\alpha R_1)}$$

$$b_5 = \frac{\Omega\left(R_1 y_n(R_0\sqrt{Pr\Gamma_d}) - R_0 y_n(R_1\sqrt{Pr\Gamma_d})\right) + F_{R0} y_n(R_1\sqrt{Pr\Gamma_d}) - F_{R1} y_n(R_0\sqrt{Pr\Gamma_d})}{j_n(R_1\sqrt{Pr\Gamma_d})y_n(R_0\sqrt{Pr\Gamma_d}) - j_n(R_0\sqrt{Pr\Gamma_d})y_n(R_1\sqrt{Pr\Gamma_d})}$$

$$b_6 = \frac{\Omega\left(R_0 j_n(R_1\sqrt{Pr\Gamma_d}) - R_0 j_n(R_0\sqrt{Pr\Gamma_d})\right) + F_{R1} j_n(R_0\sqrt{Pr\Gamma_d}) - F_{R0} j_n(R_1\sqrt{Pr\Gamma_d})}{j_n(R_1\sqrt{Pr\Gamma_d})y_n(R_0\sqrt{Pr\Gamma_d}) - j_n(R_0\sqrt{Pr\Gamma_d})y_n(R_1\sqrt{Pr\Gamma_d})}$$

$$G_1 = \frac{\varphi\kappa_n(R_0\sqrt{Pm\bar{\lambda}})}{i_n\left(R_1\sqrt{Pm\bar{\lambda}}\right)\kappa_n\left(R_0\sqrt{Pm\bar{\lambda}}\right) - i_n\left(R_0\sqrt{Pm\bar{\lambda}}\right)\kappa_n\left(R_1\sqrt{Pm\bar{\lambda}}\right)}$$

$$G_2 = \frac{-\varphi i_n\left(R_0\sqrt{Pm\bar{\lambda}}\right)}{i_n\left(R_1\sqrt{Pm\bar{\lambda}}\right)\kappa_n\left(R_0\sqrt{Pm\bar{\lambda}}\right) - i_n\left(R_0\sqrt{Pm\bar{\lambda}}\right)\kappa_n\left(R_1\sqrt{Pm\bar{\lambda}}\right)}$$

$$G_3 = \frac{Q(R_0)K_n\left(R_1\sqrt{Pm\bar{\lambda}}\right) - Q(R_1)K_n\left(R_0\sqrt{Pm\bar{\lambda}}\right)}{i_n\left(R_1\sqrt{Pm\bar{\lambda}}\right)K_n\left(R_0\sqrt{Pm\bar{\lambda}}\right) - i_n\left(R_0\sqrt{Pm\bar{\lambda}}\right)K_n\left(R_1\sqrt{Pm\bar{\lambda}}\right)}$$

$$G_4 = \frac{Q(R_1)i_n\left(R_0\sqrt{Pm\bar{\lambda}}\right) - Q(R_0)i_n\left(R_1\sqrt{Pm\bar{\lambda}}\right)}{i_n\left(R_1\sqrt{Pm\bar{\lambda}}\right)K_n\left(R_0\sqrt{Pm\bar{\lambda}}\right) - i_n\left(R_0\sqrt{Pm\bar{\lambda}}\right)K_n\left(R_1\sqrt{Pm\bar{\lambda}}\right)}$$

$F_{R_0} = \sqrt{Pr\Gamma_d Ta^{\frac{1}{2}}}\{\left(d_1 i_0\left(R_0\sqrt{Ta^{\frac{1}{2}}}\right) -\right.$

*d2k0R0Ta12 +*Pr*ξ*a1*c1 ξ1−ξ21−ξ3j0R0PrΓd*
jnR0PrΓd−a2c2ξξξ−11−ξ3y0R0PrΓd jnR0PrΓd
−λξ2c1jnR0PrΓd +ξa1c2ξξ−11−ξ3y0R0PrΓd
jnR0PrΓd +ξa2c2ξξ−11−ξ3y0R0PrΓd
ynR0PrΓd−λξc2ynR0PrΓd+…

$F_{R_1} = \sqrt{Pr\Gamma_d Ta^{\frac{1}{2}}}\{\left(d_1 i_0\left(R_1\sqrt{Ta^{\frac{1}{2}}}\right) -\right.$

$\left. d_2 k_0\left(R_1\sqrt{Ta^{\frac{1}{2}}}\right)\right) +$

$$\Pr\Bigg(\sqrt{\xi}a_1c_1\,\frac{\xi(1-\xi^2)}{1-\xi^3}j_0\big(R_1\sqrt{Pr\Gamma_d}\,\big)j_n\big(R_1\sqrt{Pr\Gamma_d}\,\big)-$$
$$a_2c_2\sqrt{\xi}\,\frac{\xi(\xi-1)}{1-\xi^3}y_0\big(R_1\sqrt{Pr\Gamma_d}\,\big)j_n\big(R_1\sqrt{Pr\Gamma_d}\,\big)-$$
$$\lambda\xi^2c_1j_n\big(R_1\sqrt{Pr\Gamma_d}\,\big)+$$
$$\sqrt{\xi}a_1c_2\,\frac{\xi(\xi-1)}{1-\xi^3}y_0\big(R_1\sqrt{Pr\Gamma_d}\,\big)j_n\big(R_1\sqrt{Pr\Gamma_d}\,\big)+$$
$$\sqrt{\xi}a_2c_2\,\frac{\xi(\xi-1)}{1-\xi^3}y_0\big(R_1\sqrt{Pr\Gamma_d}\,\big)y_n\big(R_1\sqrt{Pr\Gamma_d}\,\big)-$$
$$\lambda\xi c_2y_n\big(R_1\sqrt{Pr\Gamma_d}\,\big)\Bigg)+\cdots$$

$Q(r) = Pm\Big\{c_2g_1\left(\frac{1}{1-\gamma\kappa}\right)\left(j_n(\gamma r)i_n(\kappa r)-\right.$
κj1γri1κr− c1g111−γκjnγrκnκr+κj1γrκ1κr+c2 g111−γκynγrinκr+κy1γri1κr− c2g211−γκκy1γr κnκr− ynγrκnκr+…

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