Computer–Aided Design of the Critical Speed of Shafts

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ABSTRACT: A computer aided design software for the analysis of the critical speed of shaft, is developed and presented in this work. The software was designed using the principles of object oriented programming, and implemented with the Microsoft Visual Basic Language. The package was tested on a number of benchmark design problems. The solutions obtained were highly accurate. Also, the software exhibited very high efficiency. To improve on the method of presenting these solutions, graphical features were incorporated. This enhances the ability to visualize results. @ JASEM

All rotating shafts, even in the absence of external load, deflect during rotation. The magnitude of the deflection depends upon stiffness of the shaft and its supports, the total mass of shaft and attached part, the unbalance of the mass with respect to the axis of rotation, and the amount of damping in the system. As long as the deflections are minimal, the shaft can still operate satisfactorily. However, as the speeds increase, the shafts become unstable. Thus, there is the need to determine these critical speeds at which instability sets in. With the deflection, now considered as a function of speed, only the lowest (first) and occasionally and the second are of interest to the designer, the others will usually be so high as to be well out of range of the operating speed

At the first critical speed, the shaft will bend to the simplest shape possible. At the second critical speed it will bend to the second simplest shape possible etc.for example a shaft supported at its end and having a large (compared to shaft) masses attached, will bend according to the configurations shown in Figures 1 and Figure 2 at the first and second critical speed respectively.
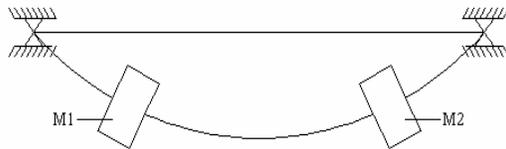

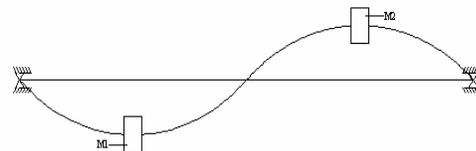**Fig.1:** Loading configuration at the first critical speed

Fig. 2: A variant of the Loading configuration at the first critical speed

We wish to state that to the best of our knowledge there is no documented computer-aided design work for resolving the critical speed of shafts. The authors would have conducted a comparative assessment of our results with any that was available. But the results presented here are highly accurate as they were solutions to bench mark examples and problems adopted from (Hall et al, 2000). This work is a continuation of efforts by the authors to develop software for the design of machine components. Earlier works had considered the design of: some types of gears (helical, and spur), design of Rolling Bearings, machine vibrations design see (Akpobi, 1998; Akpobi; Ardey, 2002; Akpobi; Airiohuodion 2005). And quite recently, software for the design of Flywheels, was developed and reported see (Akpobi; Lawani, 2006). The design for the critical speeds of shafts, is well expounded in standard machine design texts, see (Hall et al, 2000; Shigley; Mischke, 2001). In this work the problem we focused on, was to develop and implement a computer software that would greatly enhance an accurate and efficient design for the critical speeds of shafts. Also we

addressed the problem of reducing the computational difficulties usually encountered in manually designing for the critical speeds of shafts. And then provided a visual display of the solutions, so as to easily and speedily interpret these solutions. The natural frequency of the shaft is very nearly the same as the critical speed and is usually taken as the same thing .There is a difference usually quite small due the gyroscopic action of the mass.

DESIGN OF SOFTWARE MODEL

In designing the software, all the parameters necessary for proper analysis of the critical speed of shafts, were carefully assembled, and used in developing the programme that forms the software in this present work. In the software, the allowable loads were defined and the corresponding induced obtained. The concept of object oriented programming technique was employed in the design of the software. This principle was then implemented using Microsoft Visual Basic language (Microsoft

Incorporated, 1998; Byron, 2002). These design parameters are presented as follows:

SHAFT WITH SINGLE ATTACHED MASS
If the shaft mass is small compares to the attached mass, the first critical speed (W_c) can be calculated approximately as:

$$W_c = \sqrt{k/m} \qquad 1$$

Where k shaft spring constant, and m is mass
Also

$$W_c = \sqrt{g/\delta} \qquad 2$$

where g is acceleration due to gravity (m/s²)and δ is static deflection (m)

SHAFT OF CONSTANT CROSS SECTION
If the shaft is simply supported at the end, with no mass involved other than that of the Shaft itself, the first critical speed will be

$$W_c = \sqrt{5/4(g/\delta_{max})} \qquad 3$$

where δ_{nax} is the maximum static deflection (m)

SHAFT OF NEGLIGIBLE MASS CARRYING SEVERAL CONCENTRATED LOAD
For shaft of negligible mass carrying several concentrated masses, the first critical speed is obtained using the Rayleigh Ritz equation (Equation 4).

$$w_c = \sqrt{\frac{g_1^j \sum(w_n \delta_n)}{j_1 \sum(w_n \delta_n)}} \qquad 4$$

where δ_n is static deflection (m) at the nth mass
w_n is the weight of the nth mass and j is the total number of masses.
The Rayleigh Rita equation above can be used for estimating the critical speed of shafts with distributed loads. The method involves breaking the distributed masses into a series of masses m_1, m_2, m_3……m_n, after which each of the masses is taken to be a concentrated load having its weight acting at its center.

THE DUNKERLEY EQUATION
Another approximation for the first critical speed of a multiple mass system is

$$1/w_c^2 = 1/w_1^2 + 1/w_2^2 + 1/w_3^2 + \text{........} \qquad 5$$

where w_1 is the first critical speed with only mass 1 present, w_2 is the first critical speed with only mass 2 present, e.t.c.
Both the Rayleigh- Ritz and the Dunkerley equations are approximations to the first natural frequency of

vibration, which is assumed to be nearly equal to the critical speed of vibrations. In general, however, the Dunkerley equation underestimates while the Rayleigh- Ritz equation overestimates the natural frequency.

HIGHER CRITICAL SPEEDS FOR MULTIPLE MASS SYSTEM
This requires much more extensive computations than is necessary for the determination of the lowest (first) critical speed. Several methods have been developed to for this purpose. For a two mass system is given as, we have:

$$1/w^4 - (a_1 m_1 + a_{22} m_2)1/w^2 + (a_{11} a_{22} - a_{12} a_{21}) w_1 m_2 = 0 \qquad 6$$

where $a_{(\cdot)}$ are the influence coefficients,

This is a quadratic equation having two positive roots
$1/w_1$ and $1/w_2$

For any multi-mass system, the frequency equation is obtained by setting the following determinants equal to zero.

$$\begin{bmatrix} (a_{11} - \frac{1}{w^2}) & (a_{12} m_2) & (a_{13} m_3) & .... \\ (a_{12} m_1) & (a_{22} m_2 - \frac{1}{w^2}) & (a_{23} m_3) & .... \\ (a_{31} m_1) & (a_{32} m_2) & (a_{33} m_3 - \frac{1}{w^2}) & .... \\ .... & .... & .... & .... \end{bmatrix} \qquad 7$$

PROGRAMME DESCRIPTION
The program was designed and implemented using Microsoft Visual Basic object oriented programming language. The structure of the program is such that there are three stages. These stages include the input stage, analysis and output stage.

INPUT STAGE
At this stage, the user enters the required information about the Critical speed of the shaft, in the input interface form. The programme is well written in such a way that the software requires minimal input to carryout its analysis.

ANALYSIS
After the required data is in entered, the analysis carried out. Once the data is entered, the software computes all the parameters required in analyzing the critical speed of shaft in less than 2 seconds depending on the speed of the processor.

OUTPUT
The software is designed such that the critical speed parameters are outputted numerically and graphically with the accompanying description of the result.

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PROGRAMME'S ALGORITHM OR PSEUDO-CODE

The software was designed using the following algorithm or the programme's pseudo-code:

If Option.value = 1 Then
  Display introduction
End If

If Option.value = 2 Then
  If the system has only one mass, Then
    Input known parameters from: K, m. compute desired parameters using Equations 1
    and 2
    Compute values for initial speed and generate plots of initial speed against mass.
  End If
End If

If Option.value = 3 Then
  If the uniform shaft carries no mass Then
    Input known parameters δ max
    Compute desired parameters using equation 3
    Compute values for the critical speed of the shaft and generate a plot of critical
    speed against δ max .
  End If
End If

If Option.value = 4 Then
  If deflection of the mass are known then
    Input known parameters:
$\delta_1, \delta_2, \delta_3...., \delta_n, \ w_1, \ w_2, \ w_3,...., w_n$
    Compute desired parameters using equation 4
    Compute value for the critical speed of the shaft and generate a plot of critical
    speed against w_1 or w_2.
  Else If the influence coefficient is known Then
    Input known
parameters: $a_{11}, \ a_{22}, \ a_{21} = \ a_{12}, \ w_1, \ w_2,...., w_n$
    Compute desired parameters using equation 6
  End If
  End If
End If

If Option.value = 5 Then

  If the system has only one mass Then
    If the system has spring support Then
      If considering a rigid support Then
      Input known parameters: $K, \delta_b, w, \ l_1, \ l_2$
      Compute desired parameter using Equations 4
and 5
    Else If considering a spring support then
        Click the check.value = 1
        Compute desired parameters and obtain value for critical speed and the effect of spring support on the system.
  End If
End If
End If
End If
End If

EXAMPLES TO ILLUSTRATE THE USE OF THE SOFTWARE

To illustrate the effectiveness and numerical accuracy of the software, the following Examples were considered.

EXAMPLE 1
Calculate the critical speed of rotating shaft carrying a mass M_1 of 50kg having a shaft spring constant of 250Nm, as shown in Fig. 4.
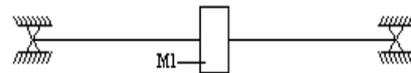
Fig. 4: Loading configuration for Example 1

SOLUTION
The out put from the software for this analysis on inputting values for
Mass of 50kg, spring constant of 250Nm is as follows:
The critical speed is 2.236 rad/s
The software also gave as output a graph of:
Varying critical speed (rad/s) against mass (kg)
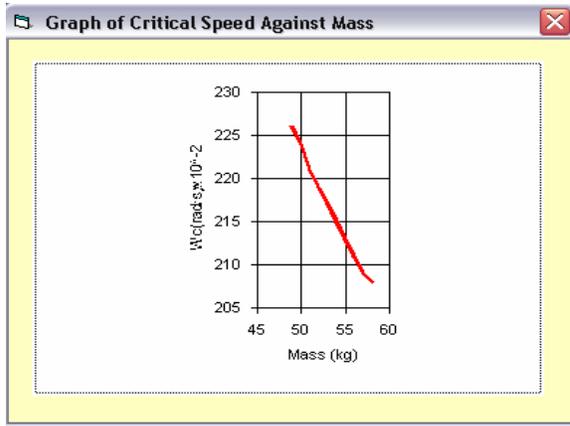
The screen shot of the solution in shown Fig. 5

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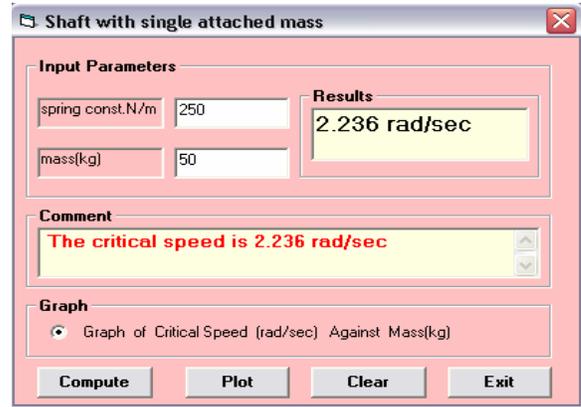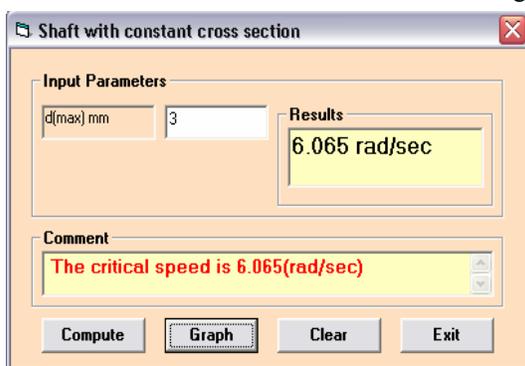**Fig. 5:** Screenshot Of The Result For Example 1

Fig. 6: Screenshot of the graph of critical speed against mass for Example 1

EXAMPLE 3
Compute the critical speed of a rotating shaft with no mass attached to it having a maximum static deflection of 0.03mm caused by a uniformly distributed load

SOLUTION
The out put from the software for this analysis on inputting value for
 Maximum static deflection of 0.03mm is as follows: The critical speed is 6.065(rad/s)
The software also gave as output a graph of:
Varying critical speed (rad/s) against Maximum deflection (mm)

The screen shot of the solutions are shown in Figs. 7 and 8

Fig. 7: screenshot of the result for Example 2.
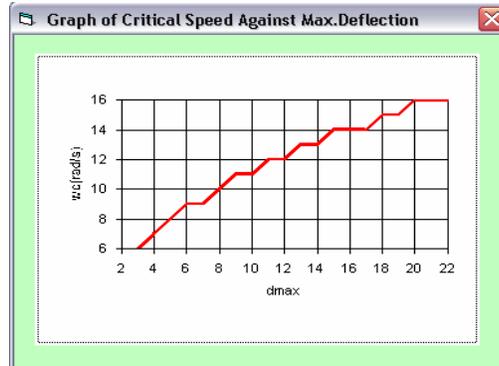
Fig. 8: Screenshot of the graph of critical speed against maximum deflection for Example 2

EXAMPLE 3
The shaft shown in Fig. 8 has attached to it a gear weighing 220N and flywheel weighing 440N, while the static deflection δ_1 and δ_2 have been found to be 0.03mm and 0.008 mm respectively. Determine the critical speed ignoring the mass of the shaft.
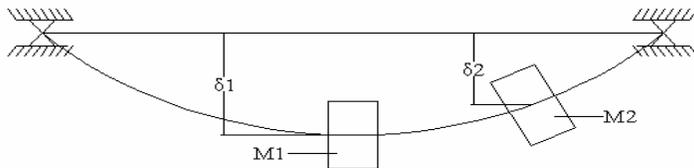

Fig. 9: Loading configuration for Example 3

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SOLUTION

The out put from the software for this analysis on inputting values for

Gear of weight, 220N, Flywheel of weight, 440N, static deflection for gear is 0.03 and for flywheel is 0.008 is as follows:

The software also gave as output a graph of:

Varying critical speed (rad/s) against weight 1 (N)

Varying critical speed (rad/s) against weight 2 (N)

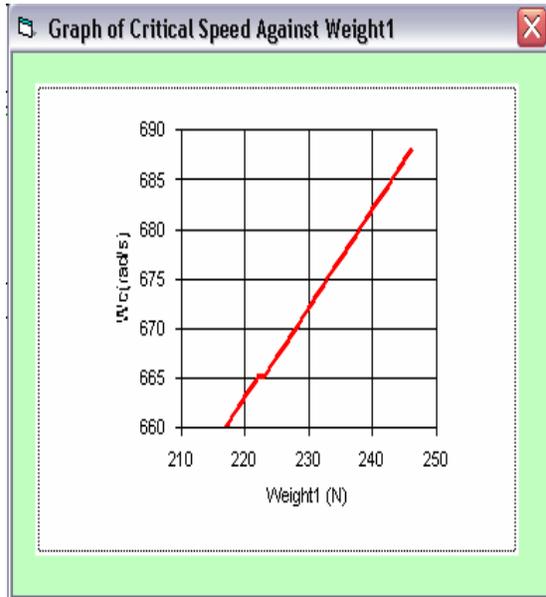The screen shot of the solutions are shown in Figs. 10-12

Fig. 10: Screenshot of the graph of critical speed against weight 1 for Example 3   **Fig.11:** screenshot of the result for Example 3
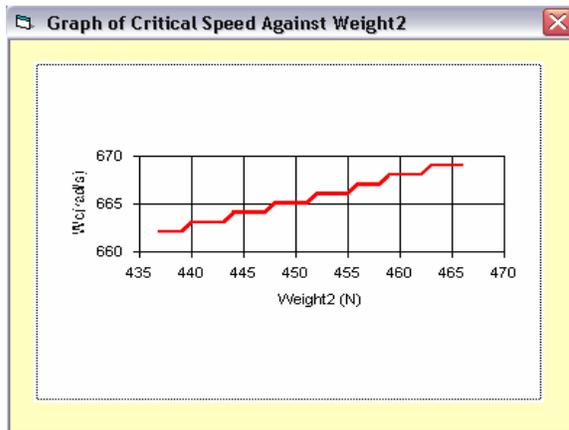

Fig. 12: Screenshot of the graph of critical speed against weight 2 for Example 3

EXAMPLE 4

The masses m_1 and m_2, attached to the shaft of Fig.12 weighing 625N and 270N respectively through a deflection analysis, the influence coefficients of the shaft has been found to be

A_{11}= 1.142E-8 m/N

A_{22}= 6.853E-8 m/N

A_{12}=21=2.284E-8 m/N
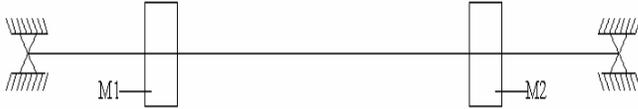
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Fig. 13: Loading configuration for Example 4

SOLUTION
The out put from the software for this analysis on inputting values for
Weight 1 of 625N, weight 2 of 270N, Influence coefficients of A_{11}= 1.142E-8 m/N
A_{22}= 6.853E-8 m/N and A_{12}=21=2.284E-8 m/N is as follows:
The critical speed is618.544rad/s

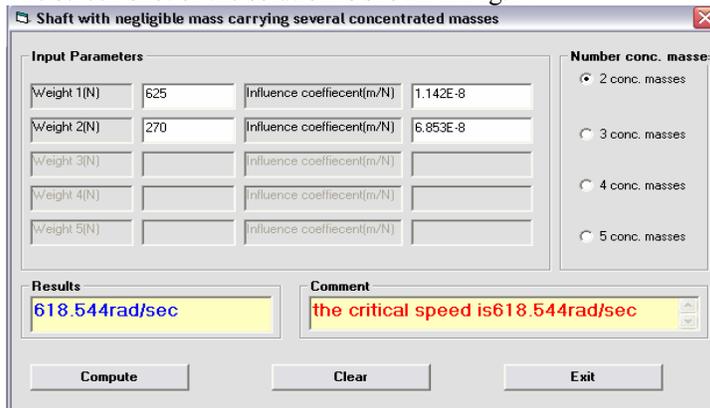The screen shot of the solution is shown in Fig.14

Fig.14: screenshot of the result for Example 4.

EXAMPLE 5
The bearing support for the shaft is shown in Fig.12. It has flexibility equivalent to a spring constant of k 44MN/m in any direction perpendicular to the shaft axis. Due to bending, the shaft itself has a deflection db of 0.046mm under the 1350N load. What effect does the flexibility of the support have on the critical speed?
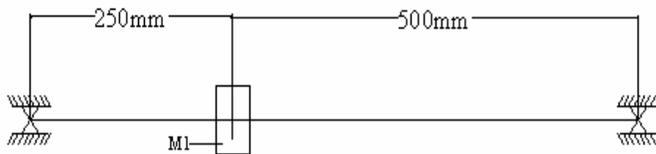

Fig. 15

SOLUTION
The out put from the software for this analysis on inputting values for
Spring constant of 44MN/m, Deflection of 0.046mm, load of 1350N,Distance between support A and the mass is (L_1) = 250mm,while that between support B and mass
 (L_2) =500mm is as follows:

The reaction at the support A is 900 N and the reaction at support B is 450 N.
The critical speed due to rigid support is 461.802 rad/s.

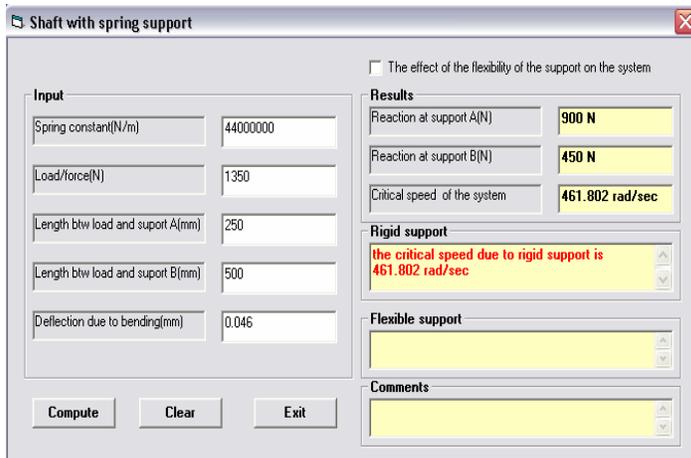The screen shot of the solution is shown in Figs. 16 and 17

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Fig. 16: screenshot of the result for Example 5

The software also produced as output:
The critical speed due to flexible support is 394.464 rad/s
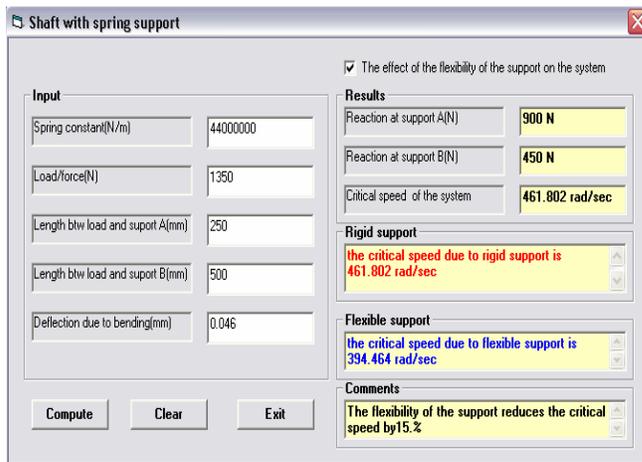The flexibility of the support reduces the critical speed by15.0%

Fig. 17: screenshot of the result for Example 5.

Discussion: This software was tested for this paper using 5 Examples, in which the problems on critical for systems with single mass, multi-mass having rigid support or spring support and systems with known static deflections, known influence coefficient were analyzed. The results displayed by the software depend on the problem definition and provides the designer with the exact numerical values required to design for the critical speeds of shaft.

The software also outputted graphical features for the following:
- Critical speed (rad/s) against mass (kg)
- Critical speed (rad/s) against maximum deflection (mm)
- Critical speed (rad/s) against weight 1 (N)
- Critical speed (rad/s) against weight 2 (N)

The results for the various Examples shown in Figs. 5 -17 are highly accurate and even better than results obtain from the text (Hall, et al) from which they were adopted. The accuracy obtained was due to the use of double precision (12 decimal places) variables in storing computed values before formatting final answers to the users designed level of decimal places.

Conclusion: We have presented in this work, a computer-aided design software that allows for easy design and analysis of the critical speed of shaft. The number of practical problems formulated and solved accurately using this software, shows that it is flexible, accurate, and robust. The graphical features incorporated in the software, also enhances the ability of the user to visualize and interpret solutions easily.

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