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Introduction of New General Laplace-Type Integral Transform: The Iyeme-Okpo Transform

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Abstract: In this paper, a new general Laplace-type integral transform called Iyeme-Okpo transform that generalizes all the existing Laplace-type integral transforms has been introduced. Thus, existing integral transforms such as Laplace, Sumudu, Natural, Jafari, Elzaki, Mahgoub, Kamal, Mohand, Sawi, Aboodh, HY, Anuj, Y, Soham, G, Kushare, Emad-Falih, ZZ, SEE, Iman, R, and Formable transforms are special cases of this general transform. Also, we can introduce new Laplace-type integral transforms using this general transform.

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Historically, the concept of an integral transform originated from the celebrated Fourier integral formula. The importance of integral transforms is that they provide powerful operational methods for solving initial value problems and initial-boundary value problems for linear differential and integral equations (Debnath and Bhatta, 2007). Laplace-type integral transforms are those that have the exponential kernel. There are, generally, two classes of Laplace-type integral transforms, namely: one-parameter Laplace-type integral transforms and two-parameter Laplace-type integral transforms. More than thirty-one-parameter Laplace-type integral transforms have been proposed by different scholars. These include the Elzaki transform (Elzaki, 2011), the Aboodh transform (Aboodh, 2013), the Mahgoub transform (Mahgoub, 2013), the Kamal transform (Abdelilah and Hassa, 2016), the Mohand transform (Mahgoub, 2017), the Jafari transform (Jafari, 2020), the Sawi transform (Mahgoub, 2019), the Iman transform (Iman, 2023), just to mention few. For two-parameter Laplace-type integral transforms, there are about ten existing ones, namely: the Sumudu transform, (Watugala, 1993), the Natural transform (Khan and Khan, 2008), the Y-transform (Okpo and Adie, 2023), the Shehu transform (Maitama and Zhao, 2019), the R-transform (Iyeme *et al*., 2024), the ZZ transform (Zafar, 2016), the NE transform (Musta, 2023), etc. This brings the total number of Laplace-type integral transform to over forty. The objective of this paper is to introduce a general Laplace-type integral transform to be called the Iyeme-Okpo transform such that all the existing Laplace-type integral transforms become special cases of it.

Definition
Let $f(t)$ be an integrable function defined for $t \geq 0$, $p(s) \neq 0$ and $q(s)$ are positive real functions. The

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Iyeme-Okpo transform of $f(t)$ is the function $M(u,s)$ defined by

$$D\{f(t)\} = M(u,s) = p(s)\int_0^\infty f(ut)e^{-q(s)t}\,dt \quad (1)$$

provided the integral on the right-hand side of (1) exists.

Some existing Laplace-type integral transforms

In this section, we give the definitions of some existing Laplace-type integral transforms as follows:

$Laplace\ Transform$: $L\{f(t)\} = \int_0^\infty f(t)e^{-st}dt$

$Sumudu\ Transform$: $S\{f(t)\} = \int_0^\infty f(ut)e^{-t}dt$

$Natural\ Transform$: $N\{f(t)\} = \int_0^\infty f(ut)e^{-st}dt$

$Elzaki\ Transform$: $E\{f(t)\} = s\int_0^\infty f(t)e^{-\frac{t}{s}}dt$

$Mahgoub\ Transform$: $M\{f(t)\} = s\int_0^\infty f(t)e^{-st}dt$

$Kamal\ Transform$: $K\{f(t)\} = \int_0^\infty f(t)e^{-\frac{t}{s}}dt$

$Mohand\ Transform$: $M\{f(t)\} = s^2\int_0^\infty f(t)e^{-st}dt$

$Sawi\ Transform$: $S\{f(t)\} = \frac{1}{s^2}\int_0^\infty f(t)e^{-\frac{t}{s}}dt$

$Abaoub - Shkheam\ Transform$: $Q\{f(t)\} = \int_0^\infty f(ut)e^{-\frac{t}{s}}dt$

$Aboodh\ Transform$: $L\{f(t)\} = \frac{1}{s}\int_0^\infty f(t)e^{-st}dt$

$HY\ Transform$: $HY\{f(t)\} = s\int_0^\infty f(t)e^{-s^2t}dt$

$Anuj\ Transform$: $\wedge\{f(t)\} = s^2\int_0^\infty f(t)e^{-\frac{t}{s}}dt$

$Kharrat - Toma\ Transform$: $B\{f(t)\} = s^3\int_0^\infty f(t)e^{-\frac{t}{s^2}}dt$

$Jafari\ Transform$: $T\{f(t)\} = p(s)\int_0^\infty f(t)e^{-q(s)t}dt$

$G - Transform$: $G\{f(t)\} = s^{\propto}\int_0^\infty f(t)e^{-\frac{t}{s}}dt$

$Formable\ Transform$: $R\{f(t)\} = s\int_0^\infty f(ut)e^{-st}dt$

$Soham\ Transform$: $S\{f(t)\} = \frac{1}{s}\int_0^\infty f(t)e^{-s^{\propto}t}dt$

$Emad - Sara\ Transform$: $ES\{f(t)\} = \frac{1}{s^2}\int_0^\infty f(t)e^{-st}dt$

$NE\ Transform$: $\{f(t)\} = \frac{1}{s}\int_0^\infty f(ut)e^{-st}dt$

$Y - Transform$: $Y\{f(t)\} = \frac{1}{s^2}\int_0^\infty f(ut)e^{-\frac{t}{s}}dt$

$Kushare\ and\ Fundo\ Transform$: $K\{f(t)\} = \frac{1}{s}\int_0^\infty f(t)e^{-\frac{t}{s^2}}dt$

$Sadik\ Transform$: $S\{f(t)\} = \frac{1}{s^\beta}\int_0^\infty f(t)e^{-s^\alpha t}dt$

$Kushare\ Transform$: $K\{f(t)\} = s\int_0^\infty f(t)e^{-s^\alpha t}dt$

$Emad - Falih\ Transform$: $EF\{f(t)\} = \frac{1}{s}\int_0^\infty f(t)e^{-s^2t}dt$

$SEE\ Transform$: $S\{f(t)\} = \frac{1}{s^n}\int_0^\infty f(t)e^{-st}dt$

$Iman\ Transform$: $I\{f(t)\} = \frac{1}{s^2}\int_0^\infty f(t)e^{-s^2t}dt$

$R - Transform$: $R\{f(t)\} = s\int_0^\infty f(ut)e^{-\frac{t}{s}}dt$

$Rohit\ Transform$: $R\{f(t)\} = s^3\int_0^\infty f(t)e^{-st}dt$

$Gupta\ Transform$: $R\{f(t)\} = \frac{1}{s^3}\int_0^\infty f(t)e^{-st}dt$

Relation between the new transform and other transforms

In this section, we discuss about the relation of the new transform (1) with other Laplace-type transforms.

In view of (1) and the definitions of the above transforms which are given in section 3, we have

If $u = 1, p(s) = 1$, and $q(s) = s$, then this new transform gives Laplace transform.

If $p(s) = 1$, and $q(s) = 1$, then this new transform gives Sumudu transform.

If $p(s) = 1$, and $q(s) = s$, then this new transform gives Natural transform.

If $u = 1, p(s) = 1$, and $q(s) = \frac{1}{s}$, then this new transform gives Elzaki transform.

If $u = 1, p(s) = s$, and $q(s) = s$, then this new transform gives Mahgoub transform.

If $u = 1, p(s) = 1$, and $q(s) = \frac{1}{s}$, then this new transform gives Kamal transform.

If $u = 1, p(s) = s^2$, and $q(s) = s$, then this new transform gives Mohand transform.

If $u = 1, p(s) = \frac{1}{s^2}$, and $q(s) = \frac{1}{s}$, then this new transform gives Sawi transform.
If $p(s) = 1$, and $q(s) = \frac{1}{s}$, then this new transform gives Abaoub-Shkheam transform.
If $u = 1, p(s) = \frac{1}{s}$, and $q(s) = s$, then this new transform gives Aboodh transform.
If $u = 1, p(s) = s$, and $q(s) = s^2$, then this new transform gives HY transform.
If $u = 1, p(s) = s^2$, and $q(s) = \frac{1}{s}$, then this new transform gives Anuj transform.
If $u = 1, p(s) = s^3$, and $q(s) = \frac{1}{s^2}$, then this new transform gives Kharrat-Toma transform.
If $u = 1$, then this new transform gives Jafari transform.
If $u = 1, p(s) = s^{\alpha}$, and $q(s) = \frac{1}{s}$, then this new transform gives G-transform.
If $p(s) = s$, and $q(s) = s$, then this new transform gives Formable transform.
If $u = 1, p(s) = \frac{1}{s}$, and $q(s) = s^{\alpha}$, then this new transform gives Soham transform.
If $u = 1, p(s) = \frac{1}{s^2}$, and $q(s) = s$, then this new transform gives Emad-Sara transform.
If $u = 1, p(s) = \frac{1}{s}$, and $q(s) = s$, then this new transform gives NEE transform.
If $p(s) = \frac{1}{s^2}$, and $q(s) = \frac{1}{s}$, then this new transform gives Y-transform.
If $u = 1, p(s) = \frac{1}{s}$, and $q(s) = \frac{1}{s^2}$, then this new transform gives Kushare and Fundo transform.
If $u = 1, p(s) = \frac{1}{s^{\beta}}$, and $q(s) = s^{\alpha}$, then this new transform gives Sadik transform.
If $u = 1, p(s) = s$, and $q(s) = s^{\alpha}$, then this new transform gives Kushare transform.
If $u = 1, p(s) = \frac{1}{s}$, and $q(s) = s^2$, then this new transform gives Emad-Falih transform.
If $u = 1, p(s) = \frac{1}{s^n}$, and $q(s) = s$, then this new transform gives SEE transform.
If $u = 1, p(s) = \frac{1}{s^2}$, and $q(s) = s^2$, then this new transform gives Iman transform.
If $p(s) = s$, and $q(s) = \frac{1}{s}$, then this new transform gives R-transform.
If $u = 1, p(s) = s^3$, and $q(s) = s$, then this new transform gives Rohit transform.
If $u = 1, p(s) = \frac{1}{s^3}$, and $q(s) = s$, then this new transform gives Gupta transform.

Iyeme-Okpo transform of some functions
(i) Let $f(t) = a$, where a is a constant, then by the definition given in (1) we have:

$$D\{a\} = p(s)\int_0^{\infty} ae^{-q(s)t}\,dt$$
$$= ap(s)\int_0^{\infty} e^{-q(s)t}\,dt$$
$$= -\frac{ap(s)}{q(s)}\left[e^{-q(s)t}\right]_0^{\infty}$$
$$= \frac{ap(s)}{q(s)}$$
$$\Rightarrow D\{a\} = \frac{ap(s)}{q(s)} \quad (2)$$

(ii) Let $f(t) = t$, then
$$D\{t\} = p(s)\int_0^{\infty} ute^{-q(s)t}\,dt$$
Using integration by parts, we get
$$D\{t\} = \frac{up(s)}{q(s)}\left[-\frac{1}{q(s)}e^{-q(s)t}\right]_0^{\infty}$$
$$= \frac{up(s)}{[q(s)]^2}$$
$$\Rightarrow D\{a\} = \frac{up(s)}{[q(s)]^2} \quad (3)$$

(iii) Let $f(t) = t^2$, then
$$D\{t^2\} = p(s)\int_0^{\infty} ut^2e^{-q(s)t}\,dt$$
$$= \frac{2u^2p(s)}{[q(s)]^3}$$
$$D\{t^2\} = \frac{2u^2p(s)}{[q(s)]^3} \quad (4)$$

(iv) In general, if $n > 0$ is an integer, then
$$D\{t^n\} = p(s)\int_0^{\infty} ut^ne^{-q(s)t}\,dt$$
$$= \frac{n!\,u^np(s)}{[q(s)]^{n+1}}$$
$$\Rightarrow D\{t^n\} = \frac{n!\,u^np(s)}{[q(s)]^{n+1}} \quad (5)$$

(v) Let $f(t) = e^{at}$, then
$$D\{e^{at}\} = p(s)\int_0^{\infty} e^{-aut}e^{-q(s)t}\,dt$$
$$= \frac{p(s)}{q(s) - au}$$
$$D\{e^{at}\} = \frac{p(s)}{q(s) - au} \quad (6)$$

(vi) Let $f(t) = sinat$, then
$$D\{sinat\} = p(s)\int_0^{\infty} sinaute^{-q(s)t}\,dt$$

$$= \frac{aup(s)}{[q(s)]^2 + [au]^2}$$

$$D\{sinat\} = \frac{aup(s)}{[q(s)]^2 + [au]^2} \quad (7)$$

(vii) Let $f(t) = cosat$, then

$$D\{cosat\} = p(s) \int_0^\infty cosaute^{-q(s)t}\, dt$$

$$= \frac{p(s)q(s)}{[q(s)]^2 + [au]^2}$$

$$D\{cosat\} = \frac{p(s)q(s)}{[q(s)]^2 + [au]^2} \quad (8)$$

(viii) Let $f(t) = sinhat$, then

$$D\{sinhat\} = p(s) \int_0^\infty sinhaute^{-q(s)t}\, dt$$

$$= \frac{aup(s)}{[q(s)]^2 - [au]^2}$$

$$D\{sinhat\} = \frac{aup(s)}{[q(s)]^2 - [au]^2} \quad (9)$$

(ix) Let $f(t) = coshat$, then

$$D\{coshat\} = p(s) \int_0^\infty coshaute^{-q(s)t}\, dt$$

$$= \frac{p(s)q(s)}{[q(s)]^2 - [au]^2}$$

$$D\{coshat\} = \frac{p(s)q(s)}{[q(s)]^2 - [au]^2} \quad (10)$$

Conclusion: In this paper, we introduce a general Laplace-type integral transform called the Iyeme-Okpo transform. We have shown this new integral generalizes all existing Laplace-type integral transforms. Also, it is interesting to know that new variants of Laplace-type integral transforms can be obtained from this integral transform by making a suitable choice for u, $p(s)$, and $q(s)$.

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Data Availability Statement: Data are available upon request from the corresponding author.

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