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Dynamic Response of Axial Forced Rayleigh Non-uniform Beam Under Accelerating Distributed Load

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ABSTRACT: This research examines the dynamic response of axial forced Rayleigh non-uniform beam under accelerating distributed load. The governing partial differential equation of order four and the transverse deflection in which the simply supported Rayleigh beam of finite length L under accelerating distributed load was considered. The effects of axial force values, shear modulus, foundation modulus, load speed and gyrating radius in connection with the length of the span of a Rayleigh beam were investigated. The fourth order partial differential equation in equation was reduced to a second order differential equation using the Fourier sine finite integral transformation method. Starting with the definition of the Fourier sine finite for$0 \leq x \leq L$, the simplified second order differential equation is solved using the Laplace transform. Numerical results were presented and it was observed that for all values of various axial force, the deflection is constant from the origin, but there is a minor difference on the positive side. Also, the result showed that for all values of shear modulus, the deflection remains constant. It was further observed that the midpoint deflection of the Rayleigh beam increases as load speeds, foundation modulus and gyrating radius increases. Finally, the results obtained are validated and are found to compare favorable well with those in the open literature.

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The problem of analyzing the response of a single elastic beam traversed by a moving load at a given velocity has drawn significant attention from researchers since the 19th century. This interest stems from its relevance to various applications, including machinery processes, guide-way systems, and the design of railway bridges (Abiala, 2010; Inglis, 2015). The dynamic behavior of beams is governed by both linear and nonlinear differential equations in spatial and temporal domains. Consequently, it is crucial to develop analytical and numerical techniques to solve these partial differential equations and study the beams' behavior. The dynamic analysis of structures supported by elastic foundations, such as highway pavements, structural foundations, and railway tracks, holds significant practical importance. Such studies are foundational in applied physics, applied mathematics, and engineering disciplines. For instance, investigations into these foundations aid in understanding the behavior of plates, such as those used in roadways or runways. These plates, often made of concrete or reinforced concrete, rest on

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foundations that can be approximated by the Winkler foundation model (Gbadeyan and Dada, 2007; Hayir, 2010; Gbadeyan and Agboola, 2012; Usman *et al.*, 2018; Gbadeyan *et al*., 2019).

Jimoh *et al.* (2018) investigated the dynamic behavior of a non-uniformly prestressed thick beam subjected to a distributed moving load with variable velocity. The analysis utilized a Galerkin-based approach, incorporating a series representation of Heaviside functions to reformulate the governing equations. These reformulated equations were subsequently solved using a combination of Stubble's asymptotic method and Laplace transformation techniques, integrated with convolution theory. Displacement responses for moving distributed force and mass models were computed over different time intervals and graphically represented. The study revealed that the moving force model does not provide an upper limit for accurately addressing the moving distributed mass problem, highlighting the importance of including the inertia term for precise evaluation of structural responses to distributed moving loads (Usman and Hammed, 2007; Usman *et al.*, 2021). Furthermore, the results indicated that increasing parameters such as axial force (N), shear modulus (G), and foundation stiffness (K) significantly reduces the response amplitudes of non-uniformly prestressed thick beams. The validity of the proposed method was confirmed by comparing the dynamic responses of a simply supported Timoshenko beam obtained through the proposed approach and the frequency-domain spectral element method (SEM) under two different velocities (Isede and Gbadeyan, 2013; Usman *et al.*, 2015)). The comparison showed strong agreement between the two methods, affirming the accuracy of the approach. Notably, the critical speed for a beam subjected to a moving distributed force was found to be higher than that for a moving distributed mass at the same natural frequency, resulting in earlier resonance in the case of the moving distributed mass.

Similarly, Usman *et al*. (2020) explored the vibrational response of beams under the influence of moving force and mass. The governing fourth-order partial differential equation was solved using the Finite Fourier Sine Transform in conjunction with the method of undetermined coefficients. Their findings revealed that, for a moving mass, the response amplitude increases as the load mass increases. However, for a moving force, the response amplitude remains unaffected by variations in load mass. Additionally, the study found that the response amplitude is generally higher in the moving force scenario compared to the moving mass case. Consequently, the objective of this paper is to examine the dynamic response of axial forced Rayleigh non-uniform beam under accelerating distributed load (Oni and Ayankop-Andi, 2017; Usman *et al.*, 2018). Hence, the objective of this paper is to examine the dynamic response of axial forced Rayleigh non-uniform beam under accelerating distributed load.

MATERIALS AND METHODS

Problem formulation: The governing equation of partial differential equation for the transverse deflection of the simply supported axial forced Rayleigh beam under accelerating distributed load is given by:

$$\frac{\partial^2}{\partial x^2}\left(EI(x)\frac{\partial^2 Y(x,t)}{\partial x^2}\right) + \rho A(x)\frac{\partial^2 Y(x,t)}{\partial t^2} + k(x)Y(x,t) - G(x)\frac{\partial^2 Y(x,t)}{\partial x^2} - r^2\frac{\partial^4 Y(x,t)}{\partial x^2 \partial t^2} + N\frac{\partial^2 Y(x,t)}{\partial x^2} = P(x,t) \quad (1)$$

where, $P(x,t)$ is accelerating distributed moving load defined by,

$$P(x,t) = Q_1 H(x - ct) + Q_2 H\big(x - (x_0 + ct)\big) \quad (2)$$

Hence, substituting (2) into (1) , the governing equation can be rewritten as

$$\frac{\partial^2}{\partial x^2}\left(EI(x)\frac{\partial^2 Y(x,t)}{\partial x^2}\right) + \rho A(x)\frac{\partial^2 Y(x,t)}{\partial t^2} + k(x)Y(x,t) - G(x)\frac{\partial^2 Y(x,t)}{\partial x^2} - r^2\frac{\partial^4 Y(x,t)}{\partial x^2 \partial t^2} + N\frac{\partial^2 Y(x,t)}{\partial x^2} = M_1 g H(x - ct) + M_2 g H\big(x - (x_0 + ct)\big) \quad (3)$$

The boundary conditions for a simply supported beam associated with the equation are:

$$Y(x,t) = \frac{\partial^2 Y(x,t)}{\partial x^2} = 0 \ \text{at} \ x = 0 \ \text{and} \ x = L \quad (4)$$

Since the displacement and the bending moment vanish at a simply supported end, the initial conditions are

$$Y(x,t) = \frac{\partial Y(x,t)}{\partial t} = 0 \text{ at } t = 0 \qquad (5)$$

Method of Solution: The fourth order partial differential equation in equation (3) is reduced to a second order differential equation using the Fourier sine finite integral transformation method. Starting with the definition of the Fourier sine finite for $0 \le x \le L$, the simplified second order differential equation is solved using the Laplace transform.

$$Y_n(t) = \int_0^L Y_n(x,t) \sin \frac{n\pi x}{L} dx \qquad (6)$$

With its inverse defined as
$$Y_n(t) = \frac{2}{L} \sum_{n=1}^{\infty} Y_n(t) \sin \frac{n\pi x}{L} \quad (7)$$

Invoking equation (6) on equation (3) and using the transformation,
$$\alpha = \left(\frac{n\pi}{L}\right)$$
we have,
$$\alpha_n^4 EI(x) Y_n(t) + \alpha_n^3 EI'(x) Y_n(t) \cos \frac{n\pi x}{L} - \alpha_n^2 EI''(x) Y_n(t) + k(x) Y_n(t) - \alpha_n^2 G(x) Y_n(t) - r^2 \alpha_n^2 Y_n''(t) + \alpha_n^2 N Y_n(t) + \rho A(x) Y''(t) = \frac{M_1 g}{n\pi} (\cos \alpha_o ct - \cos n\pi) - \frac{M_2 g}{n\pi} (\cos \alpha_n (x_0 + ct) - \cos n\pi) \ (8)$$

$$\left(\alpha_n^4 EI(x) - \frac{4}{L} \alpha_n^3 EI'(x) \cos \frac{n\pi x}{L} - \alpha_n^2 EI''(x) + k(x) - \alpha_n^2 G(x) + \alpha_n^2 N\right) Y(t) - r^2 \alpha_n^2 Y''(t) + \rho A(x) Y_n''(t) = \frac{M_1 g}{n\pi} \cos \alpha_n ct - \frac{M_2 g}{n\pi} \cos \alpha_n (x_0 + ct) + \frac{g}{n\pi} \cos n\pi (M_1 - M_2) \quad (9)$$

Further simplification of equation (9), the following are introduced
$$A_n = \alpha_n^4 EI(x) - \frac{4}{L} \alpha^3 EI'(x) \cos \frac{n\pi x}{L} - \alpha EI''(x) + k(x) - \alpha^2 G(x) + \alpha^2 N$$
$$B_n = r^2 \alpha_n^2 + \rho A(x)$$
$$C_n = \frac{A_n}{B_n}, \quad D_n = \frac{M_1 g}{n\pi B_n}, \quad E_n = \frac{M_2 g}{n\pi B_n}, \quad F_n = \frac{g}{n\pi} \cos n\pi (M_1 - M_2)$$
$$Y_n''(t) + C_n Y_n(t) = D_n \cos \alpha_n ct - E_n \cos \alpha_n (x_o + ct) + F_n$$
$$Y_n''(t) + C_n Y_n(t) = D_n \cos \alpha_n ct - E_n \cos(\alpha_n x_o + \alpha_n ct) + F_n \quad (10)$$

Using trigonometry identity
$$Y_n''(t) + C_n Y_n(t) = D_n \cos \alpha_n ct - E_n (\cos \alpha_n x_0 \cos \alpha_n ct - \sin \alpha_n x_0 \sin \alpha_n ct) + F_n$$

Using Laplace transformation to equation (10)
$$s^2 \bar{Y}_n(t) + C_n \bar{Y}_n(t) = \frac{D_n s}{s^2 + (\alpha_n C)^2} + \frac{E_n \alpha_n C}{s^2 + (\alpha_n C)^2} \sin \alpha_n x_0 - \frac{D_n s}{s^2 + (\alpha_n C)^2} - \cos \alpha_n x_0 + F_n$$

$$\bar{Y}_n(t) = \frac{D_n s^2 + E_n s^2 \cos(\alpha_n x_0) + E_n \alpha_n C s \sin(\alpha_n x_0) + F_n (s^2 + (\alpha_n C)^2)}{(s^2 + C_n)(\alpha_n^2 C^2 + s^2) s} \quad (11)$$

Resolving into partial fraction
$$\frac{D_n s^2}{(s^2 + C_n)(\alpha_n^2 C^2 + s^2)^2} = \frac{D_n s}{(\alpha_n^2 C^2 + s^2)(C_n - \alpha_n^2 C^2)} - \frac{D_n s}{(s^2 + C_n)(C_n - \alpha_n^2 C^2)}$$

$$\frac{E_n s^2\cos(\alpha_n x_0)}{(s^2+C_n)(\alpha_n^2C^2+s^2)s} = \frac{E_n s\cos(\alpha_n x_0)}{(s^2+\alpha_n^2C_n^2)(C_n-\alpha_n^2C^2)} - \frac{E_n s\cos(\alpha_n x_0)}{(s^2+C_n)(C_n-\alpha_n^2C^2)} \quad (12)$$

$$\frac{E_n\alpha_n Cs\sin(\alpha_n x_0)}{(s^2+C_n)(\alpha_n^2C^2+s^2)s} = \frac{E_n\alpha_n Cs\sin(\alpha_n x_0)}{(s^2+\alpha_n^2C_n^2)(C_n-\alpha_n^2C^2)} - \frac{E_n\alpha_n Cs\sin(\alpha_n x_0)}{(s^2+C_n)(C_n-\alpha_n^2C^2)} \quad (13)$$

$$\frac{F_n(s^2+\alpha^2C^2)}{(s^2+C_n)(\alpha_n^2C^2+s^2)s} = \frac{F_n}{s(s^2+C_n)} = \frac{F}{C_n s} - \frac{Fs}{C_n(S^2+C_n)} \quad (14)$$

Finding the Laplace inverse of (12) to (14)

$$L^{-1}\left(\frac{D_n s^2}{(s^2+C_n)(\alpha_n^2C^2+s^2)s}\right) = L^{-1}\left(\frac{D_n s}{(\alpha_n^2C^2+s^2)(C_n-\alpha_n^2C^2)}\right) - L^{-1}\left(\frac{D_n s}{(s^2+C_n)(C_n-\alpha_n^2C^2)}\right) = \frac{D_n\cos\alpha_n ct)}{C_n-\alpha^2C^2} - \frac{D_n\cos\sqrt{C_n}\,t}{C_n-\alpha^2C^2} \quad (15)$$

$$L^{-1}\left(\frac{E_n s^2\cos(\alpha_n x_0)}{(s^2+C_n)(\alpha_n^2C^2+s^2)s}\right) = L^{-1}\left(\frac{E_n s\cos(\alpha_n x_0)}{(s^2+\alpha_n^2C_n^2)(C_n-\alpha_n^2C^2)}\right) - L^{-1}\left(\frac{E_n s\cos(\alpha_n x_0)}{(s^2+C_n)(C_n-\alpha_n^2C^2)}\right) = \frac{E_n\cos(\alpha_n x_0)\cos\alpha_n ct}{(C_n-\alpha_n^2C^2)} - \frac{E_n\cos(\alpha_n x_0)\cos\sqrt{C_n}\,t}{(C_n-\alpha_n^2C^2)} \quad (16)$$

$$L^{-1}\left(\frac{E_n\alpha_n Cs\sin(\alpha_n x_0)}{(s^2+C_n)(\alpha_n^2C^2+s^2)s}\right) = L^{-1}\left(\frac{E_n\alpha_n Cs\sin(\alpha_n x_0)}{(s^2+\alpha_n^2C_n^2)(C_n-\alpha_n^2C^2)}\right) - L^{-1}\left(\frac{E_n\alpha_n Cs\sin(\alpha_n x_0)}{(s^2+C_n)(C_n-\alpha_n^2C^2)}\right) = \frac{E_n\sin(\alpha_n x_0)\sin\alpha_n ct}{(C_n-\alpha_n^2C^2)} - \frac{E_n\alpha_n C\sin(\alpha_n x_0)\sin\sqrt{C_n}t}{\sqrt{C_n}(C_n-\alpha_n^2C^2)} \quad (17)$$

$$L^{-1}\left(\frac{F_n}{s(s^2+C_n)}\right) = L^{-1}\left(\frac{F}{C_n s} - \frac{Fs}{C_n(S^2+C_n)}\right) = \frac{F_n}{C_n} - \frac{F_n\cos\sqrt{C_n}t}{C_n} \quad (18)$$

So that

$$Y_n(t) = \frac{D_n\cos\alpha_n ct)}{C_n-\alpha^2C^2} - \frac{D_n\cos\sqrt{C_n}\,t}{C_n-\alpha^2C^2} + \frac{E_n\cos(\alpha_n x_0)\cos\alpha_n ct}{(C_n-\alpha_n^2C^2)} - \frac{E_n\cos(\alpha_n x_0)\cos\sqrt{C_n}\,t}{(C_n-\alpha_n^2C^2)} + \frac{E_n\sin(\alpha_n x_0)\sin\alpha_n ct}{(C_n-\alpha_n^2C^2)} - \frac{E_n\alpha_n C\sin(\alpha_n x_0)\sin\sqrt{C_n}t}{\sqrt{C_n}(C_n-\alpha_n^2C^2)} + \frac{F_n}{C_n} - \frac{F_n\cos\sqrt{C_n}t}{C_n} \quad (19)$$

$$Y_n(t) = I_n\cos(\alpha_n ct) + J_n\sin(\alpha_n ct) + k_n\cos\sqrt{C_n}t + L_n\sin\sqrt{C_n}t + M_n$$
(20)

Where $I_n = \frac{D_n - E_n\cos(\alpha_n x_0)}{(C_n-\alpha_n^2C^2)}$ (21)

$J_n = \frac{E_n\sin(\alpha_n x_0)}{(C_n-\alpha_n^2C^2)}$
(22)

$K_n = \frac{-C_nD_n-(C_n-\alpha_n^2C^2)F_n + C_nE_n\cos(\alpha_n x_0)}{C_n(C_n-\alpha_n^2C^2)}$ (23)

$L_n = \frac{\sqrt{C_n}\alpha_n C\,E_n\sin(\alpha_n x_0)}{C_n(C_n-\alpha_n^2C^2)}$
(24)

$M_n = \frac{F_n}{C_n}$
(25)

Hence,

$Y(x,t) = \frac{2}{L}\sum_{n=0}^{\infty} Y_n(t)\sin\alpha_n x$ (26)

Numerical Analysis: On the basis of the analytical solution presented above, computations ensure was fashioned out by using Maple tools. The computation was carried out using a typical 10m long with parameters EI = 265432Nm^2 and m = 9.15kg

RESULTS AND DISCUSSION

Fig. 1 shows the deflection of Rayleigh beam at the midpoint against time at various values of axial force. Result shows that the deflection is constant from the origin for all values of N but there is a slight difference at the positive side. Fig. 2 shows the deflection of Rayleigh beam at midpoint against time at various values of shear modulus. Result revealed that the deflection is constant for all values of G_0.

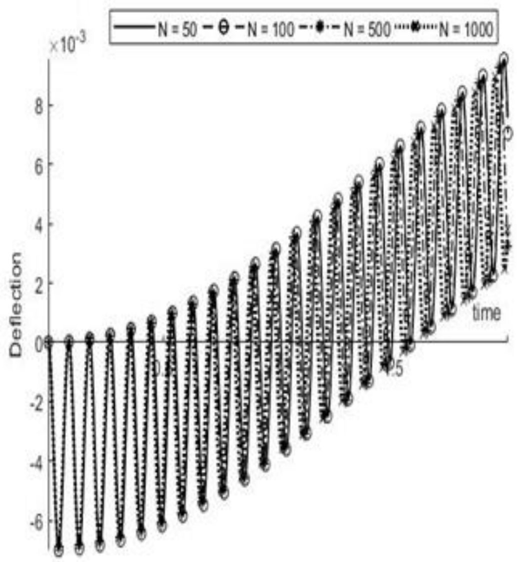

Fig. 1: Deflection of beam at various values of N(axial force)
Source: Authors, 2023

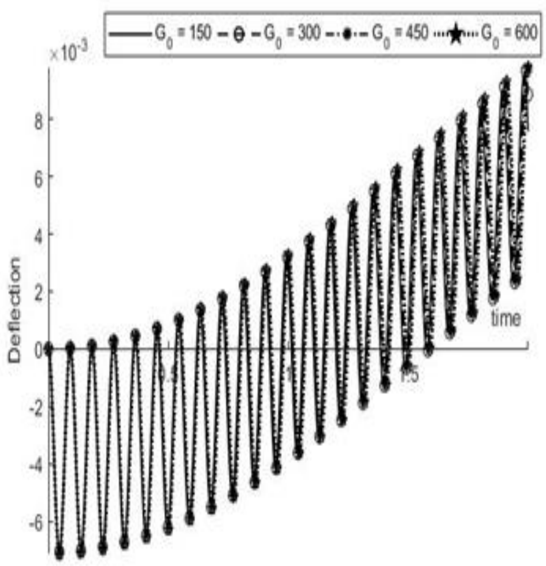

Fig. 2: Deflection of beam at various values of $\boldsymbol{G_0}$(shear modulus)
Source: Authors, 2023

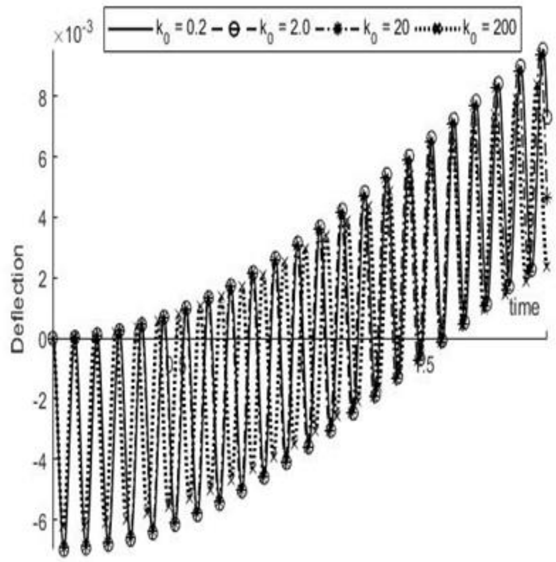

Fig. 3: Deflection of beam at various values of $\boldsymbol{k_0}$(foundation modulus)
Source: Authors, 2023

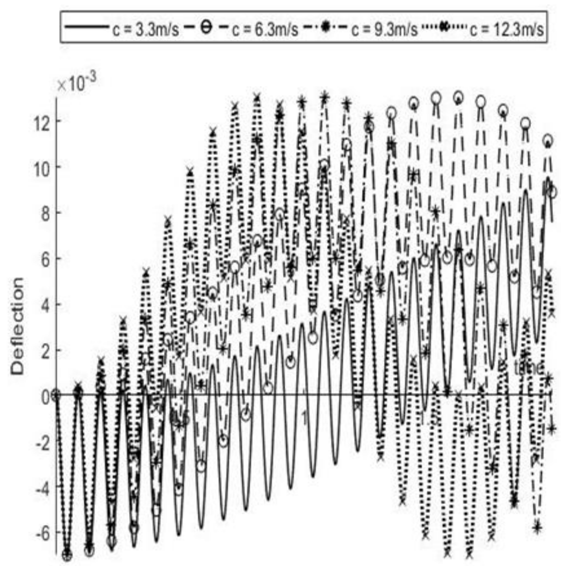

Fig. 4: Deflection of beam at various values of c(load speed)
Source: Authors, 2023

Figure 3 shows the deflection of Rayleigh beam at the midpoint against time at various values of foundation modulus. Result shows that the deflection is a slight difference as k_0 increases. Figure 4 shows the deflection of Rayleigh beam at the midpoint against time at various values of speed of the load. Result shows that the deflection increases as c increases. Figure 5 shows the deflection of Rayleigh beam at the midpoint against time at various values of square of the radius of gyration. Results show that the deflection increases as r^2 increases but decreases in amplitude as r^2 increases.

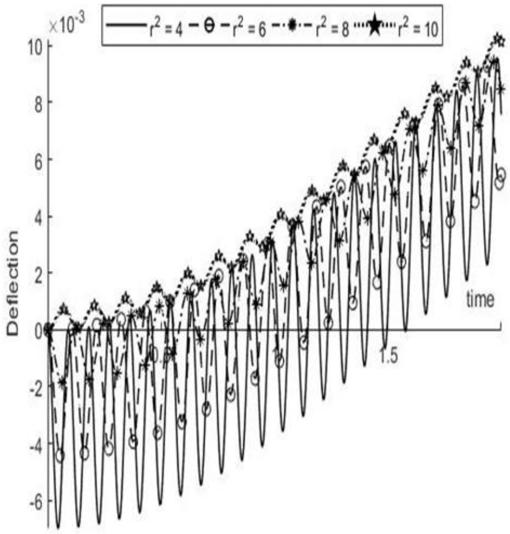

Fig. 5: Deflection of beam at various values of r^2(gyrating radius)
Source: Authors, 2023

Conclusion: The dynamic behavior of an axially forced Rayleigh non-uniform beam subjected to an accelerating distributed load is analyzed. The findings of this study provide valuable insights into potential variations in the parameters influencing the vibration characteristics of engineering structures. In engineering, understanding the vibrational behavior of mechanical and structural systems is crucial for ensuring the safe design, construction, and operation of various machines and structures. Failures in many mechanical and structural components are often attributed to vibration-related issues. For instance, blade and disk failures in steam and gas turbines, as well as structural failures in aircraft, are frequently linked to vibrations and the resulting fatigue.

Declaration of Conflict of Interest: The authors declare no conflict of interest.

Data Availability Statement: Data are available upon request from the first author or corresponding author

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