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Monte Carlo-Integrated Valuation of Call Options in Leasehold Solid Mineral Property Development in Nigeria

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ABSTRACT: The existing closed-form models for the valuation of call options in solid mineral land use/development; (including the Samuelson-McKean model) often feature deterministic inputs and outputs that inadequately account for the dynamics of risk and uncertainty. Consequently, the objective of this paper is to evaluate the Monte Carlo-integrated valuation of call options in the development and use of leasehold solid mineral landed properties in Nigeria using appropriate mathematical models and simulations. The simulated future value (FV) of the call option in the case of a barite-endowed mineral property was put at ₦138,376,422.92 with a standard deviation of ₦32,235,981.11 and significant at p<0.05. The Monte Carlo simulation indicated 49.41% chance of the mining operator realizing FV of call option between ₦48,115,676.82 and ₦139,887,676.70 in order to justify the commencement of development and production activities on the leasehold solid mineral property with effect from the valuation date. However, for a mining operator with 19.25% cost of capital, FV below the ₦48,115,676.82 threshold might warrant the postponement of development and production activities. The Monte Carlo-integrated valuation provided insights regarding input variables that a mining operator should address with caution, in pursuit of sustainable profit from the mineral land use decision. This dimension of development appraisal provides robust answers to the appropriate timing and operation of mineral land use decisions, based on the assessment of the mining operator's profit under the conditions of risk and uncertainty.

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The extraction of solid minerals from land constitutes an investment decision that should be driven by the analysis of cost and benefits of the mining enterprise (Pagouni *et al*., 2024). Consequently, such decision exhibits the dimension of land use and development (Maus *et al*., 2022; Ratcliffe *et al.*, 2009), besides being an environmentally significant change of land use (The Nigerian Urban and Regional Planning Act, 1992). Owing to the enormity of capital investment involved, this land use decision requires the deployment of appraisal tools comprising discounted cash flow (DCF) valuation of the developer's (mining operator's) profit, as well as real options valuation (ROV) and pricing respectively (Ali and Rafique, 2024; Maier, 2021). According to Geltner *et al.* (2010), the exercise aims to furnish land developers with information regarding the viability and appropriate time to commence a project. The International Mineral Valuations Standards Committee (IMVSC) (2021) developed a global

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standard for the valuation of mineral properties, which recognized successive editions of the International Valuations Standards (IVS) pertaining to the use of Income Approaches to Valuation, among which include the option pricing technique (International Valuations Standards Council, 2022). Inferred from Fraser (1993), Geltner *et al.* (2010), and Parsons (2003) is the fact that investment pricing or appraisal entails a combination of valuation model designed to determine the likely value-in-exchange (sales price) of an option contract on the one hand, and the assessment of the profitability or otherwise of exercising the same option contract on the other hand. Ground works on ROV models applicable to natural resource projects, especially mineral land uses and enterprises abound. In the most part, the discounted cash flow (DCF) techniques featured as the fundamental basis for the deployment of these models (Colwell *et al*., 2003; Drieża*et al*., 2002; Mardones, 1993). Some of these models were featured in the form of deterministic and stochastic decision trees (Brandão *et al*., 2005; Xiaoping and Jie, 2014), while other variants such as the Black-Scholes equation, Samuelson-McKean model, and the Brennan-Schwartz model are closed-form models that were derived from differential equations (Hui *et al*., 2011; Kelly, 1998). The Samuelson-McKean model, which was featured as one of the analytical tools for this study is a closed-form model that treats the development of land and land resources as a call option whose value and expected maximum profitability constitute the basis for optimal timing of project commencement (Hui *et al*., 2011). Underlying this valuation model are the economic principles of anticipated (expected) benefits, highest and best use, and the concept of hope value (Drapikovskyi *et al*., 2020). These principles imply that the actual existence of a solid mineral property is immaterial, because a pre-development (exploration) mineral property as noted by Gandhi and Sarkar (2016) and Lawrence (1994) could be subject to valuation of expected net benefits arising from present and future development decisions in the light of available/foreseeable data. Although the original structure of the Samuelson-McKean model applies to ROV of freehold interest in mineral property, its modification as featured by Ho *et al*. (2009) and Hui *et al*. (2011) makes it applicable to ROV of terminable (leasehold) mineral interests in Nigeria. The property interest held by the mining operator is likened to leasehold, which according to Hepburn (2001) creates contractual- and proprietary interest of limited duration in land. Just like any other leasehold, an interest in solid mineral property is an investment in land which generates earnings for an operator over a limited duration of time, after which it expires. Fraser (1993) demonstrated this investment attribute using a rising income profile that declines gradually after a certain peak is reached, until the receipt of income equals zero upon termination of the lease contract. Costello and Leishman (2011), found the predictive strength of the Samuelson-McKean model to be statistically significant at $p<0.01$, although, outperformed by the Black-Scholes model. Nevertheless, both closed-form ROV models utilize similar input variables except for the differences in the analyses that precede their application, and the seemingly conservative results arising from the Samuelson-McKean model. Slade (2001) recounted the inadequacy of the traditional DCF techniques to handle risk and flexibility in mining enterprises. These inadequacies could be adequately addressed using Monte Carlo simulations (Shivute, 2024). Similarly, the existing closed-form models of ROV are typically deterministic models that exhibit the same limitations as the traditional DCF techniques. However, the integration of Monte Carlo simulation with decision tree analysis as credited to Guj and Chandra (2019) and Yao and Jaafari (2003) addressed the issue of risk and value changes in the light of risk-adjusted input variables for options in mining enterprises. In a related study, Hui *et al*. (2011) applied a Monte Carlo approach to the Samuelson-McKean model of ROV with outcomes indicating strong likelihood of negative returns to justify the rejection of an urban renewal project. Meanwhile, there is a limited knowledge regarding the application of Monte Carlo simulation to the same Samuelson-McKean model when accounting for risk in the valuation of call option in profits from mineral property development/land use decisions. Therefore, the objective of this paper is to evaluate the Monte Carlo-integrated valuation of call options in the development and use of leasehold solid mineral landed properties in Nigeria.

MATERIALS AND METHODS

Methodology: This is an epistemological-realist study that presents the researchers' perspective of an alternative approach of appraising American call option in the development and use of solid mineral-bearing lands; but this time, with recourse to the discounted cash flow (DCF) valuation of the developer's (mining operator's) profit. An American call option in this case is a derivative investment whose underlying asset (solid mineral property) can be transacted or exercised (redeemed) at any time prior to- and including the terminal holding period in contrast with the European call option (Luenberger, 1998; Reilly and Brown, 2002; Syz, 2008). This feature adds to the flexibility of the American call option as a derivative asset that is desirable by investors in exploration mineral properties. The study entailed a combination of exploratory case study- and experimental designs. The exploratory case study featured a hypothetical instance of the valuation of American call option in the mining operator's profit; whereas, the experimental design featured the application of Monte Carlo simulation to the valuation of call options in solid mineral property.

Computer Hardware for the Monte Carlo simulation: Micro-computer with typical configuration in Table 1 was used for the simulation.

Table 1: Minimum hardware configuration

Hardware component	Minimum configuration
Processor (CPU)	Intel® 2.30GHz Corei7-360QM CPU OR Equivalent
System memory	8GB RAM
Hard Drive	500 GB
Optimal screen size	17 inches TFT display
Operating system	Windows 10 Professional, 64 bit

Source: Authors' Simulation experiment, 2024

Computer Software for the Monte Carlo simulation: Installed on the micro-computer and used in connection with the Monte Carlo simulation are the software packages indicated in Table 2.

Table 2: Software packages for the simulation exercise

Software package	Model/specification
Spreadsheet package	Microsoft® Excel® 2007
Monte Carlo simulation software	Oracle® Crystal ball, Fusion Edition 11.1.1.1.00

Source: Authors' Simulation experiment, 2024

The process of Monte Carlo simulation using Oracle® Crystal ball: Featured in Fig. 1 is a flowchart for the Monte Carlo simulation of the call option valuation of the decision to develop and use an exploration mineral property. As a precursor to the Monte Carlo simulation, data for the discounted cash flow (DCF) appraisal of the mining operator's profit were organized in Microsoft® Excel® 2007.

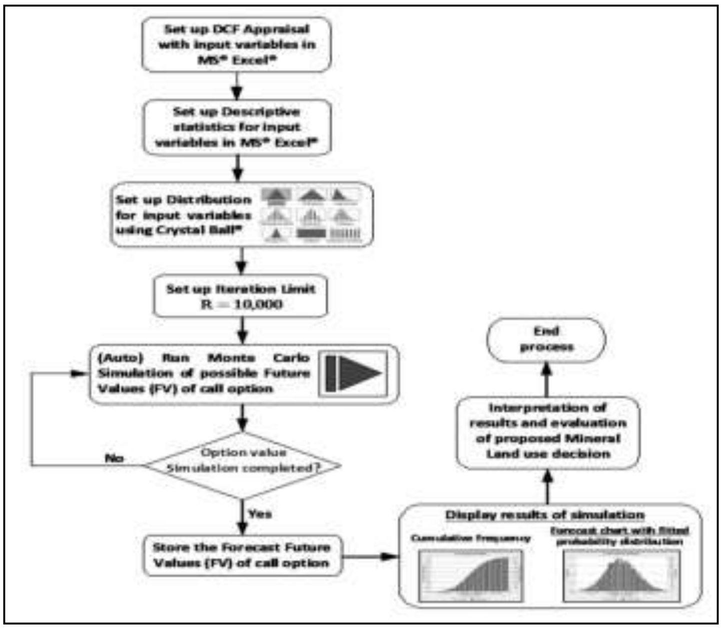

Fig. 1: Flowchart of the Monte Carlo simulation of call option values
Source: Authors' Simulation experiment, 2024

Thereafter, a modification of the DCF variant of residual valuation of land use decision, which in this case, is a solid mineral property development project for which the developer's profit was featured as the target outcome was instantiated in tandem with a similar experimentation carried out by Morley (2002) regarding the financial appraisal of development projects. Succeeding this residual valuation was the determination of the call option value of the exploration (proposed) interest in the solid mineral property, using ideas from the application of the Samuelson-McKean option valuation model to residential and commercial property development projects as demonstrated in similar studies credited to Costello and Leishman (2011), Ho *et al*. (2009) and Hui *et al*. (2011) respectively. Prior to the use of this model is the calculation of preliminary parameters namely the option elasticity (η) and the option hurdle value (rate) (V_H).

The option elasticity is expressed in equation 1 as:

$$\eta = \frac{1}{\sigma^2} \times \left(k - r + \left(\frac{\sigma^2}{2} \right) + \sqrt{\left((r-k) + \left(\frac{\sigma^2}{2} \right) \right)^2 + 2r\sigma^2} \right) \quad (1)$$

In addition, the hurdle value, V_H for the option is expressed in equation 2 as:

$$V_H = \frac{\eta X_t}{\eta - 1} \qquad (2)$$

So that the variables η and V_H are plugged into equation 3 to determine the call option value of leasehold interest in solid mineral property:

$$OV_t = (V_H - X_t) \bullet \left(\frac{S_0}{V_H} \right)^{\eta} \qquad (3)$$

Where the variables in equations 1 to 3 are described to include: η (eta) = Option elasticity, σ (sigma) = Price volatility of mineral property, k = capitalization rate or current rental yield of mineral property, r = Risk-free rate of interest, V_H = Hurdle value, X_t = Exercise (strike) price or Total Development cost (TDC), S_0 = Market value of the underlying asset or gross development value, and OV_t = Call option value. Next was the identification and tabulation of the input variables for the Monte Carlo simulation in MS Excel®, which led to the assignment of normal- and triangular probability distributions respectively (See Table 7) using the Crystal Ball® add-in within the MS Excel® environment as demonstrated by Brown and Matysiak (2000), French and Gabrielli (2005), and Hoesli *et al*. (2006) respectively. The simulation was set to a maximum of 10,000 iterations. By clicking on the Run button, the Crystal Ball simulation engine would check if the input variables and settings for the exercise are complete before the actual Auto-Run of the simulation (Goldman, 2002). The simulation engine shall flag for a review of data and settings in the event of incomplete information. It however enters the Auto-Run mode in the event of complete data and appropriate settings, such that a successive build-up and storage of the forecast Future Values (FVs) of the call option (outcome variable) is automated, leading to a display of the simulation results in charts and tables. Charts for the forecast outcomes were featured as histograms with fitted distribution. Associated with these charts are tables of Worst- and Best case scenarios of Future Values (FVs) of the call option, and the Percentile distribution of the scenarios of the FVs of the call option. These forecast tools were intended to provide factual insights to the viability and timing of a proposed mineral land use and developmental decision.

Data analysis and presentation techniques: This study deployed financial appraisal tools comprising discounted cash flow (DCF) valuation of mining operator's profit, Net Present Value (NPV), Internal Rate of Return (IRR), the Samuelson-McKean model of ROV, Option premium calculations, and What-If (Goal Seek) tool in MS Excel®. Others include specific stochastic (Monte Carlo) tools that were used to analyze the FVs of call option namely - distribution-fitted histogram, descriptive statistics of forecast outcomes, sensitivity analysis, and scenario analysis respectively. Prominent among the data presentation tools for this study include the distribution-fitted histograms and cross tabulations of sensitivity data. Meanwhile, the reported statistics for the Fitted Lognormal- and Forecast (Simulated) Future Values (FVs) of American call options in the solid mineral property development include the mean, median, mode, standard deviation and variance, skewness and kurtosis, coefficient of variation, minimum and maximum FVs, and the standard error of mean respectively, as displayed alongside the certainty chart in Fig. 2.

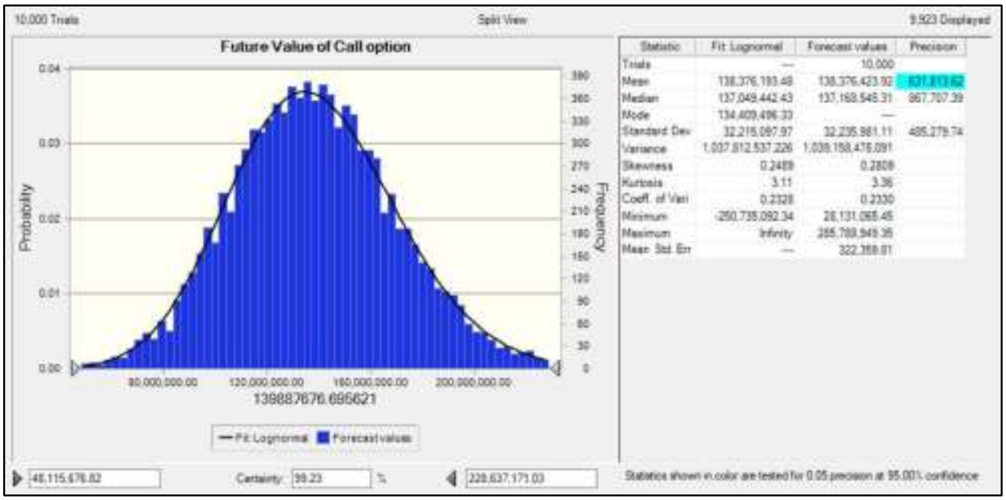

Fig. 2: Monte Carlo forecast and distribution of the future value of call option
Source: *Authors' Simulation experiment, 2024*

RESULTS AND DISCUSSION

Data for DCF valuation of profit from the development of solid mineral property: In tandem with Fig. 1, hypothetical data for the DCF appraisal that precedes the Monte Carlo simulation were organized in Table 3. The appraisal pertains to a mining operator who proposed to acquire a mining lease over 540 hectares of land in Nasarawa State in Nigeria, with a proven reserve of 1,977,500 metric ton of barite that is expected to last for 35 years, given an expected production rate of 56,500 metric ton per annum.

Although predominantly hypothetical, a selection of variables in Table 3 featured realistic inputs obtained from specific sources among which include average yield on 30-year gilts (Central Bank of Nigeria, 2024), cost of finance, maximum loan tenor, and loan processing charges for solid minerals development (Bank of Industry, 2024), and a graded crude barite price that was determined with reference to the international price of pure barite estimated at $1.25 (₦2,044.94) per metric ton (U.S. Geological Survey, 2024). A selection of these data further featured in the Monte Carlo simulation.

Table 3: Hypothetical data for DCF Appraisal of mining operator's profit

Variables	Values
Surface area of mining site	540 hectares
Estimated economic life of Barite deposit	35 years
Expected output of crude barite	56,500 metric ton per annum
Grade of mineral (Based on the economic cut-off grade criterion)	0.85
Market price of existing grade of crude barite	₦1,738.20/metric ton
Expected site restoration cost in 35 years' time	₦5,755,000,000
Spot (Gross development) value of interest in solid mineral property	₦161,767,334.47 (See Table 4)
30-year FGN Bond yield (Gilt yield) per annum	12.50%
Quarterly equivalent of 30-year FGN Bond yield (Gilt yield)	2.99%
Total return on surface rent (Gilt yield + 5% risk premium)	17.50%
Cost of procuring Equipment and Machinery	₦67,700,000
Mine site construction cost including site clearance and preparation	₦42,300,000
Cost of acquiring mining title	₦1,095,000
Professional fees on mine site development	15% of total construction cost
Construction Period	24 months
Defect liability period	3 months
Cost of Finance from the Bank of Industry (BOI)	5% per annum
Cost of Finance from the Bank of Industry (BOI)	1.23% per quarter
Loan processing charges payable to the Bank of Industry (BOI)	0.50% of Loan disbursed
Tenor of Loan	5 years
Mining operator's cost of capital per annum	19.25%
Quarterly equivalent of Mining operator's cost of capital	4.50%

Source: Authors' Simulation experiment, 2024

Discounted Cash Flow (DCF) analysis of Mining operator's profit: The deterministic valuation of the mining operator's profit as featured in Table 4 was with recourse to the quarterly apportionment of the elements of the costs and benefits of the mineral land use decision. The development of the mine site is expected to last for 9 quarters, after which the mining operator can realize the spot (gross development) value of the mineral property.

This spot value was instrumental to the determination of the mining operator's profit. Contrary to most prescribed solutions to DCF techniques of residual valuations, where the developer's cost of capital was treated as being synonymous to the cost of short term finance, these two variables were separately applied in Table 4.

This was aimed at averting the error of suggesting that the mining operator (developer) is receiving a return on the risk of the project equal to that required by the financier of the project, such that no surplus capital is left after paying up the indebtedness. Derived from Table 4 for the purpose of valuing and pricing the call option in the mineral property include the Spot (Gross Development) Value of ₦167,546,150.97; Strike price (TDC) of ₦126,093,140.17; Future Value of mining operator's profit (₦41,453,010.80) and the NPV of mining operator's profit at 4.5% cost of capital per quarter (₦27,894,893.47) shaded in green.

The Internal rate of return (IRR = 28.78% per annum) featured in Table 4 is an indication of the maximum risk tolerance of the mining operator, beyond which the discounting of cash flows shall lead to a loss of capital and invalidation of the call options contract.

Deterministic real option valuation (ROV) using the Samuelson-McKean model: Featured in Table 5 are the input variables for the ROV of the leasehold solid mineral property. These input variables, some of which were derived from Tables 3 and 4 specifically apply to the Samuelson-McKean model of ROV.

Table 4: Discounted Cash Flow (DCF) residual valuation of mining Lessee's/operator's profit in Naira (₦)

Months	Period	Mine site construction cost	Equipment and machinery cost	Loan processing charges	Fees on mine site devt.	Gross Development Value	Cost of acquiring mining title	Net Cash flow	Capital outstanding at beginning of the period	Interest on loan @ 1.23%	Capital Outstanding at the end of the period
0	0						-1,095,000				-1,095,000.00
3	1	-4,230,000	-13,540,000	-88,850	-634,500			-18,493,350.00	-1,095,000.00	-13,438.10	-19,601,788.10
6	2	-4,230,000	-16,248,000	-102,390	-634,500			-21,214,890.00	-19,601,788.10	-240,557.74	-41,057,235.84
9	3	-5,076,000	-17,602,000	-113,390	-761,400			-23,552,790.00	-41,057,235.84	-503,864.02	-65,113,889.86
12	4	-5,076,000	-10,155,000	-76,155	-761,400			-16,068,555.00	-65,113,889.86	-799,092.92	-81,981,537.78
15	5	-6,345,000	-10,155,000	-82,500	-951,750			-17,534,250.00	-81,981,537.78	-1,006,096.65	-100,521,884.43
18	6	-6,345,000	0	-31,725	-951,750			-7,328,475.00	-100,521,884.43	-1,233,628.13	-109,083,987.56
21	7	-5,922,000	0	-29,610	-888,300			-6,839,910.00	-109,083,987.56	-1,338,704.27	-117,262,601.83
24	8	-5,076,000	0	-25,380	-761,400			-5,862,780.00	-117,262,601.83	-1,439,074.14	-124,564,455.97
27	9					167,546,150.9		167,546,150.97	-124,564,455.97	-1,528,684.21	41,453,010.80
								Gross Development Value			167,546,150.97
								Total Development Cost			126,093,140.17
								Mining Operator's Profit (Developer's Profit)			41,453,010.80
								NPV of Mining Operator's Profit @ 4.50%			27,894,893.47
								IRR of project per quarter			6.53%
								IRR of project per annum			28.78%

Source: Authors' Simulation experiment, 2024

Table 5: Input variables for the deterministic valuation of real options

Parameter for preliminary analysis	Input values
Risk free rate of interest (r_a) per annum	12.50%
Risk free rate of interest (r_4) per quarter	2.99%
Mining operator's cost of capital per annum	19.25%
Mining operator's cost of capital per quarter	4.50%
Capitalization rate or net yield of underlying asset (k)	17.50%
Price volatility of mineral property (σ)	2.2577%
Expected Development period of the mine site (in quarters)	9
Spot (Gross Development) Value (S_0)	₦167,546,150.97
Strike Price or Total Development cost (X_t)	₦126,093,140.17

Source: Authors' Simulation experiment, 2024

In the absence of data on price volatility of mineral properties in Nigeria, the standard deviation of published yields on 30-year tenor gilts issued between 22 May 2019 and 19 June 2023 was used as surrogate, since a uniform risk premium of 5% across the time series data for net asset yield determination did not alter the desired result put at 2.2577%. Justifying this preliminary analysis of price volatility are similar studies credited to Costello and Leishman (2011), Ho *et al*. (2009) and Hui *et al*. (2011) respectively. Table 6 featured the three parameters for the real option pricing. The first being option elasticity (η = 1.0477), which was computed using equation 1. The second parameter is the option hurdle rate (V_H = 2,769,268,216.19) that was computed using equation 2, and finally, the Future Value of American Call option in the mineral property (OV_t = ₦139,887,676.70) determined using the Samuelson-McKean model (equation 3). These parameters featured in the two deterministic scenarios for option pricing returning option premia of ₦75,518,767.95 (given a risk-free discount rate of 2.99% per quarter) and ₦66,239,447.17 (given mining operator's cost of capital at 4.50% per quarter). The present value (PV) function was used in both scenarios in Table 6 to determine the option premium at these specific rates of interests, the first being the risk-free rate, and the second being the operator's cost of capital. The present value (PV) function is expressed in the actuarial form in equation 4 as:

$$a = \frac{1}{(1+i)^n} = (1+i)^{-n} \quad (4)$$

Where ***a*** is the present value (today's) equivalent of a known capital sum (being future value, FV) of the mineral property that is being discounted over the expected development period, ***n*** (in quarters) and at a given rate of compound interest, ***i***.

The ₦139,887,676.70 call option value and the ensuing option premium arising from the 4.50% cost of capital are indications that the proposed mineral property development decision might be viable. However, it is appropriate to subject this valuation to risk analysis in a Monte Carlo environment, so that the veracity of these viability indicators could be ascertained.

Table 6: Results of deterministic valuation of real options

Parameter for real option pricing		Output
Option elasticity (η)		1.0477
Option Hurdle rate (V_H)		2,769,268,216.19
Future Value of American Call option (OV_t) (₦)		139,887,676.70
	Scenario 1	₦
	PV of call option @ 2.99% per quarter	107,321,388.25
Less	PV of mining operator's profit @ 2.99% per quarter	31,802,620.30
	Option premium @ 2.99% per quarter Risk-Free Rate	75,518,767.95
	Scenario 2	₦
	PV of call option @ 4.50% per quarter	94,134,340.64
Less	PV of mining operator's profit @ 4.50% per quarter	27,894,893.47
	Option premium @ 4.50% per quarter cost of capital	66,239,447.17

Source*: Authors' Simulation experiment, 2024*

Assigning probability distributions to the input variables of Monte Carlo simulation: Table 7 features the eight principal variables for the Monte Carlo simulation and their assigned distributions. The objective assignment of a probability distribution to an input variable depends on the result of statistical test to determine the best-fit distribution. For instance, a normal distribution is appropriately assigned to an input variable which is drawn from a dataset adjudged to have passed any variant of the normality test comprising the Doornik-Hansen-, Jarque-Bera-, and Shapiro-Wilk W tests; otherwise the objective assignment of other forms of probability distributions could be instantiated using the distributions dialog box found in the Crystal Ball® menu bar.

Table 7: Distributional assumptions for selected input variables

Input	Distribution	Mean (Likeliest)	Standard deviation	Min.	Max.
Market values (₦)					
Price of graded crude barite	Normal	1,738.20	105.00	-	-
Interest rates (%)					
30-Year FGN Bond Yield	Normal	12.50%	2.2577%	-	-
Cost of Finance	Triangular	5.0%	-	4.5%	9.0%
Operator's cost of capital	Triangular	19.25%	-	17.50%	25.00%
Professional fees (%)	Triangular	15.00	-	12.00	17.00
Other costs (₦ '000)					
Acquisition of mining title	Normal	1,095	185	-	-
Mine site construction	Normal	42,300	2,500	-	-
Equipment and Machinery	Normal	67,700	8,800	-	-

Source*: Authors' Simulation experiment, 2024*

For experimental purpose, however, this simulation featured normality of data for five variables in Table 7, defined mainly with recourse to their mean (base case input) values and standard deviations; whereas the triangular distribution was featured for the other three variables namely - cost of finance, mining operator's cost of capital, and professional fees, but defined by the likeliest-, minimum-, and maximum values only. This assignment of probability distributions for the input variables in Table 7 paved the way for the incorporation of Monte Carlo simulation and stochastic risk analysis to the valuation of call option in the solid mineral land use and development decision.

Distribution-fitted Histogram and descriptive statistics of the Monte Carlo simulation: Featured on the left side of Fig. 2 is the certainly chart (histogram) of possible future values (FVs) of the call option, while the right hand side captured the statistics of the forecast- and fitted values. It should be recalled that these forecast FVs pertain to the operator's (developer's) profit in the proposed mining enterprise. The forecast Mean Monte Carlo FV of the call option is ₦138,376,422.92 with a standard deviation of ₦32,235,981.11. This mean FV is significant at $p < 0.05$.

Contrary to the MS Excel® goal seek threshold FV to the tune of ₦41,453,010.80, the Monte Carlo forecast of this threshold FV is ₦48,115,676.82, so that a mining operator with 19.25% cost of capital might decide to defer the development and operation of the mineral enterprise if call option valuation falls below this threshold. Nevertheless, Fig. 2 features a 99.23% confidence interval of the call option to be likely in the money (viable).

It is deduced from Fig. 3 that there is 49.41% chance of the mining operator realizing FV of call option in the range of ₦48,115,676.82 threshold and the deterministic value of ₦139,887,676.70, which constitute viable range of valuations required to justify the commencement of development and use of the barite-endowed landed property on the valuation date.

Furthermore, the adjustment of the certainty forecast chart in Fig. 4 indicates that the risk-seeking mining operator with 19.25% cost of capital has a 46.56% chance of realizing FV of call option in the range: ₦120,000,000 ≤ OV_t ≤ ₦160,000,000. This translates to present values (PVs) of the mining operator's profit in the range: ₦52,856,471.80 ≤ PV_t ≤ ₦79,773,593.55, thereby justifying the development and use of the barite-endowed landed property with effect from the valuation date.

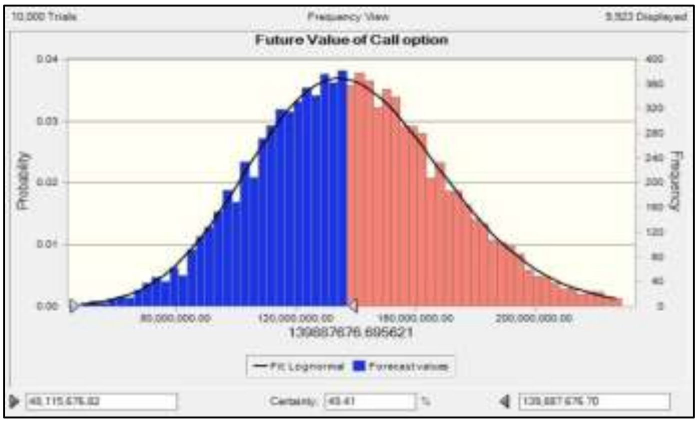

Fig. 3: Confidence interval between mean forecast- and threshold of viable future value of call option
Source: Authors' Simulation experiment, 2024

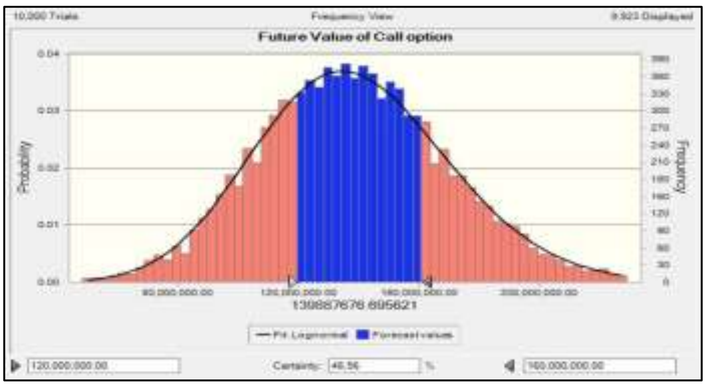

Fig. 4: Confidence interval of the optimistic future values of call option in the mineral property
Source: Authors' Simulation experiment, 2024

Sensitivity of call option value to input variables: Featured in Fig. 5 are correlations of forecast FV of the call option with the Monte Carlo input variables. This forecast FV is positively correlated to the market price of the graded crude barite by 93% and the cost of acquiring mining title by 1% respectively. Secondly, there is a negative correlation between this FV and four input variables, namely - the gilt yield (29%), cost of borrowing (1%), cost of mine site construction (1%), and Equipment and Machinery Cost (1%) respectively.

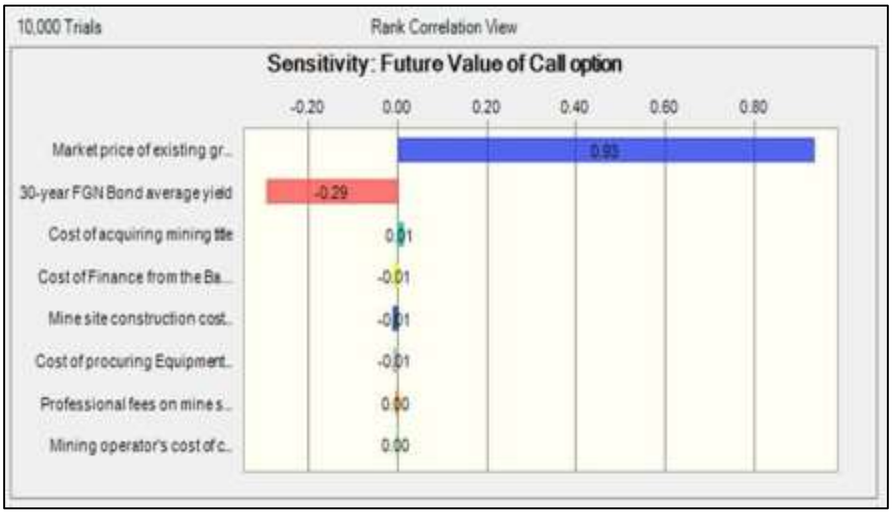

Fig. 5: Rank correlation view of the sensitivity chart of input variables
Source: Authors' Simulation experiment, 2024

Lastly, there is no rank correlation between this FV and the duo of professional fees and mining operator's cost of capital. In other words, the choice of mining operator's cost of capital does not significantly determine the viability of the call option as compared to gilt yield and the market price of the graded barite.

The demonstrated Monte Carlo simulation is insightful to the use of stochastic modelling of call option values to justify decisions regarding when to commence the development and use of mineral-bearing landed property. This insight aligns with results from existing studies credited to Ali and Rafique (2024), Colwell *et al*. (2003), and Drieża *et al*. (2002) respectively. Like in existing studies credited to Colwell *et al*. (2003) and Dreiza *et al* (2002) regarding the interpretation of sensitivity charts from Monte Carlo simulation of real option values, this study applied similar tool to identify the two major input variables that exert significant impact on the value of call option in the mining operator's profit namely - the market price of crude barite and gilt yield respectively. Therefore, a rational mining operator would not take these high-risk variables for granted if sustained viability of the land use decision is desired.

Best- and Worst case scenarios: The Tornado tool in the Crystal Ball simulation engine was used to analyze the "at worst" (downside) and "at best" (upside) future values of call option for the solid mineral land use decision. For this appraisal exercise, the first five significant input variables were featured. With reference to Table 8, the best forecasts of the FV of call option in the mineral property were shaded alongside the values of associated input variables. The shaded input variables churned out FVs of call option above the deterministic valuation put at ₦139,887,676.70 to underscore the importance of negotiating or optimizing these variables in pursuit of a viable mineral land use decision.

Table 8: Worst- and Best case scenarios of call option values from the five significant inputs

S/N	Variable	Future value of call option (₦)		Input	
		Downside	Upside	Downside	Upside
1	Price of graded crude barite	102,278,831.94	177,980,615.95	1,603.64	1,872.76
2	30-year gilt yield	151,240,697.74	123,431,486.23	9.61%	15.39%
3	Equipment and Machinery cost	140,576,720.44	139,277,136.43	56,422,346	78,977,654
4	Cost of mine site construction	140,103,375.08	139,690,990.97	39,096,121	45,503,879
5	Professional fees	139,864,128.03	139,929,697.24	16%	13%

Source: Authors' Simulation experiment, 2024

Percentile of select Best- and Worst case scenarios: By default, the 50th percentile for all the five key variables equals the deterministic FV of ₦139,887,676.70. In the first place, the shaded cells in Table 9 imply that rising market prices of the graded crude barite, which is outside the control of the mining operator is likely to place the call option value within the range of 70th to 90th percentile of the distribution.

Table 9: Select percentile of the scenarios of Future value of call option

S/N	Input variable	Future value of call option (₦)			
		10.0%	30.0%	70.0%	90.0%
1	Price of graded crude barite	102,278,831.94	124,438,043.00	155,427,945.28	177,980,615.95
2	30-year gilt yield	151,240,697.74	145,754,315.44	133,300,771.60	123,431,486.23
3	Equipment and Machinery cost	140,576,720.44	140,164,504.66	139,634,221.22	139,277,136.43
4	Cost of mine site construction	140,103,375.08	139,978,752.99	139,810,056.69	139,690,990.97
5	Professional fees	139,929,697.24	139,908,440.53	139,881,358.31	139,864,128.03

Note: The 50th percentile for all the variables equals the Monte Carlo mean forecast of ₦139,887,676.70
Source: Authors' Simulation experiment, 2024

Also beyond the control of the leasehold mining operator is the possibility of a falling gilt yield, which could occur marginally by 10% to 30%, but with corresponding increase in call option value. Nonetheless, there is 70% to 90% chance that this yield might likely account for FVs of call options below ₦139,887,676.70, which is still above the threshold FV of ₦48,115,676.82. Within the control of the mining operator are the three expenditure elements in Table 9 namely equipment and machinery cost, cost of mine site construction, and professional fees, which when negotiated downwards, might contribute to call option valuations above ₦139,887,676.70, but with 10% to 30% chances of occurrence. Meanwhile, these three input variables indicated FVs below the deterministic call option valuation in the 70th to 90th percentile.

Conclusion: Profitable value of call option in solid mineral property is required to justify the timing and execution of a proposed solid mineral land use decision spanning across the development of mine site and the operation of the mining enterprise. Whereas existing closed-form models for the valuation of this call option often feature deterministic inputs and outputs, the extra dimension of Monte Carlo-integrated valuation, particularly, in the case of the Samuelson-McKean model as featured in this study, is recommended when accounting for the present- or future impact of risk and uncertainty on the developer's (mining operator's) profit. Among the benefits of the Monte Carlo-integrated valuation include proactive identification of risk-laden input variables, including those that are within and beyond the mining operator's control; and flexibility towards negotiating expenditure items that could help free up more capital required to improve the viability of the project.

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REFERENCES

Ali, MA; Rafique, Q (2024). Strategic Integration of Real Options for Enhanced Valuation and Optimization in Mining Project Planning under Uncertainty: A Comprehensive Review. *Int. J. Proj. Mgt*, *6*(2): 26 - 50. DOI: https://doi.org/10.47672/ijpm.2004.

Bank of Industry (2024). Artisanal and Small-scale Miners (ASM) Financing Support Fund. Retrieved 20th September, 2024, from https://futuresoftproject.com/boi/wp-content/uploads/2023/01/Artisanal-and-Small-scale-Miners-ASM-Financing-Support-Fund_.pdf

Brandão, LE; Dyer, JS; Hahn, WJ (2005). Using Binomial Decision Trees to Solve Real-Option Valuation Problems. *Decision Analysis*, *2*(2): 69 - 88. DOI: https://doi.org/10.1287/deca.1050.0040.

Brown, G; Matysiak, G (2000). Real Estate Investment: A Capital Market Approach. Financial Times-Prentice Hall, England, pp. 187

Central Bank of Nigeria (2024). Government Securities - FGN Bonds. Retrieved 5 August, 2024 from https://www.cbn.gov.ng/rates/GovtSecurities.asp

Colwell, D; Henker, T; Ho, J; Fong, K (2003). Real option valuation of Australian gold mines and mining companies. *The J. Alt. Inv., 6*(1): 23 - 38. DOI: https://doi.org/10.3905/jai.2003.319080.

Costello, G; Leishman, C (2011). Using Option Pricing Theory to Value Development Land. Paper presented at the 17th Pacific Rim Real Estate Society Conference, Gold Coast, Australia, 16th - 19th January.

Drapikovskyi, O; Ivanova, I; Renigier-Biłozor, M; Źróbek, S (2020). How to assess the impact of hope on a change in the use of land on market value? *Land Use Pol., 97*: 104746. DOI: https://doi.org/10.1016/j.landusepol.2020.104746.

Drieża, JA; Kicki, J; Saluga, P (2002). Real options in mine project budgeting – Polish mining industry example. In Brebbia, CA (Ed), Risk Analysis III. Southampton, WIT Press, U.K. pp. 27 - 36.

Fraser, WD (1993). Principles of Property Investment and Pricing (2nd ed.). Macmillan, Basingstoke. pp. 124, 153

French, N; Gabrielli, L (2005). Discounted cash flow: accounting for uncertainty. *J. Prop. Inv.& Fin*, 23(1): 75-89. DOI: https://doi.org/10.1108/14635780510575102.

Gandhi, SM; Sarkar, BC (2016). Essentials of Mineral Exploration and Evaluation. Elsevier, Amsterdam. pp.321.

Geltner, DM; Miller, NG; Clayton, J; Eichholtz, P (2010). Commercial Real Estate Analysis and Investments. (2nd ed). Cengage Learning Inc., Australia. pp. 739, 762.

Goldman, LI (2002). Crystal Ball Professional Introduction Tutorial. In Yucesan, E; Chen, C-H; Snowdon, JL; Charnes, JM (Eds). Winter Simulations Conference, (pp. 1539 - 1545), San Diego, USA, 08 - 11 December.

Guj, P; Chandra, A (2019). Comparing different real option valuation approaches as applied to a copper mine. *Res. Pol., 61*: 180 - 189. DOI: https://doi.org/10.1016/j.resourpol.2019.01.020.

Hepburn, S (2001). Principles of Property Law. (2nd ed.). Cavendish Publishing Pty Ltd, Sydney. p. 369

Hoesli, M; Jani, E; Bender, A (2006). Monte Carlo Simulations for Real Estate Valuation. *J. Prop. Inv. Fin.*, 24(2): 102 - 122. https://doi.org/10.1108/14635780610655076

Ho, DKH; Hui, EC; Ibrahim, MFB (2009). Asset Value Enhancement of Singapore's Public Housing Main Upgrading Programme (MUP) Policy: A Real Option Analysis Approach. *Urban Stud., 46*(11): 2329 - 2361. DOI: https://doi.org/10.1177/0042098009342451.

Hui, EC; Ng, IM; Lo, K (2011). Analysis of the Viability of an Urban Renewal Project under a Risk-Based Option Pricing Framework. *J. Urban Plan.& Devt., 137*(2): 101 - 111. DOI: https://doi.org/10.1061/(asce)up.1943-5444.0000047.

International Mineral Valuation Standards Committee. (2021). International Mineral Valuation Standards (IMVAL) Template. (4th ed.). International Mineral Valuation Committee, Colorado. pp. 11.

International Valuation Standards Council. (2022). International Valuation Standards - Effective 31 January 2022. International Valuation Standards Council (IVSC), London. pp. 64 – 66.

Kelly, S (1998). A binomial lattice approach for valuing a mining property IPO. *The Qtly Rev. of Econs. Fin. 38*(3): 693 - 709. DOI: https://doi.org/10.1016/s1062-9769(99)80097-0.

Lawrence, MJ (1994). An Overview of Valuation Methods for Exploration Properties. Paper presented at the Mineral Valuation Methodologies Conference, Sydney, Australia, 27 - 28 October.

Luenberger, DG (1998). Investment Science. Oxford University Press, Oxford, p. 320

Maier, S (2021). Re-evaluating natural resource investments under uncertainty: An alternative to limited traditional approaches. *Annals Op. Res., 299*: 907 - 937. DOI: https://doi.org/10.1007/s10479-020-03569-w.

Mardones, J (1993). Option valuation of real assets. *Res. Pol., 19*(1): 51 - 65. DOI: https://doi.org/10.1016/0301-4207(93)90052-o.

Maus, V; Giljum, S; da Silva, DM; Gutschlhofer, J; da Rosa, RP; Luckeneder, S; Gass, SLB; Lieber, M; McCallum, I (2022). An update on global mining land use. *Sci. Data, 9*(1): 433. DOI: https://doi.org/10.1038/s41597-022-01547-4.

Morley, S (2002). The financial appraisal of development projects. In Guy, S and Henneberry, J (Eds.), Development and Developers: perspectives on property (pp. 73 - 95). E. & F. N. Spon, London.

Pagouni, C; Pavloudakis, F; Kapageridis, I; Yiannakou, A (2024). Transitional and Post-Mining Land Uses: A Global Review of Regulatory Frameworks, Decision-Making Criteria, and Methods. *Land*, *13*(7): 1051. DOI: https://doi.org/10.3390/land13071051.

Parsons, G (Ed). (2003). The Glossary of property terms. Estates Gazette, London. pp. 16.

Ratcliffe, J; Stubbs, M; Keeping, M (2009). Urban Planning and Real Estate Development (3rd ed). Routledge, Oxon. pp. 76 - 77.

Reilly, FK; Brown, KC (2002). Investment Analysis and Portfolio Management. Virginia Association for Investment Management and Research. p. 868

Shivute, AP (2024). Comparative analysis of discounted cash flow and real options techniques on a gold mining project. *Int. J. Min. Mineral. Engr.*, *15*(1): 99 - 109. DOI: https://doi.org/10.1504/IJMME.2024.138731.

Slade, ME (2001). Valuing Managerial Flexibility: An Application of Real-Option Theory to Mining Investments. *J. Env. Econs. Manage. 41*(2): 193 - 233. DOI: https://doi.org/10.1006/jeem.2000.1139.

Syz, JM (2008). Property Derivatives: Pricing, Hedging and Applications. John Wiley & Sons Ltd, England. p. 14

The Nigerian Urban and Regional Planning Act, (1992). Laws of the Federation of Nigeria, CAP. N138, 2004 *Edition*.

U.S. Geological Survey (2024). Barite - Mineral Commodity Summaries. Retrieved 20th September, 2024, from https://pubs.usgs.gov/periodicals/mcs2024/mcs2024-barite.pdf

Xiaoping, H; Jie, C (2014). Randomized Binomial Tree and Pricing of American-Style Options. *Math. Probl. Engr.*, *2014*, 1 - 6. DOI: https://doi.org/10.1155/2014/291737.

Yao, J; Jaafari, A. (2003). Combining Real Options and Decision Tree. *The J. Struct. Fin.*, *9*(3): 53 – 70. DOI: https://doi.org/10.3905/jsf.2003.320320.