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Non-Isothermal flow of Third Grade Fluid with Thermal Radiation through a Porous Medium

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ABSTRACT: A mathematical model is analyzed to study the effect of various physical parameters related to non-isothermal flow of third grade fluid in the presence of thermal radiation through a porous medium. Hence, the objective of this paper is to investigate the combined impacts of magnetic field, viscous dissipation, thermal radiation, varying thermal conductivity and viscosity on non-isothermal flow of a third grade fluid with thermal radiation through a porous medium. The Galerkin weighted residual method was used to solve the resulting non-linear ordinary differential equations numerically. The graphic representation and discussion of the effect of various significant parameters on the flow system are presented. The investigation of this problem leads to the conclusion that the thermal radiation parameter, the variable thermal conductivity and viscosity parameters have a significant impact on the mass flow and the energy transfer phenomena in the system.

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Researchers of recent time have shown a great deal of interest in non-Newtonian fluid flows. The primary driving force behind these fluids' motivation is their utilization in industrial and technological applications. It is impossible to forecast the rheological characteristics of every non-Newtonian fluid with a single constitutive equation. One non-Newtonian fluid model that has a clear advantage over the others is the third grade fluid model, which can predict both shear thickening and normal stress effects.

Fluid can be regarded as a material in nature that deforms continuously under applied shear stress. The most important property of fluid that characterises the flow resistance of simple fluids is its viscosity. It is a measure of the internal fluid's friction which causes resistance to flow. A material's capacity to conduct or transport heat is indicated by its thermal conductivity. High thermal conductivity materials have the ability to quickly transport heat from one place to another. They are employed in heat exchangers and other applications where quick heat transfer is crucial. The electromagnetic wave that the body emits is called thermal radiation. Its impact is crucial for guaranteeing the safety of people and property when handling and processing these kinds of fluids. Meanwhile dynamic porous media analysis is a useful method for resolving a wide range of engineering

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problems including soil-structure connection, disaster engineering, and robotics. Many academics have recently become interested in the study of third grade fluids in porous media due to its practical applications. Hayat *et al.* (2014); Hayat *et al.* (2015); Idowu *et al.* (2024); Laxmi and Shankar (2016); Mureity *et al.* (2013); Ogunsola and Peter (2014); Olajuwon and Baoku (2014); Peter *et al.* (2022); Peter *et al.* (2019). Hence, the objective of this paper is to investigate the combined impacts of magnetic field, viscous dissipation, thermal radiation, varying thermal conductivity and viscosity on non-isothermal flow of a third grade fluid with thermal radiation through a porous medium.

MATERIALS AND METHODS

Application of mass and energy transfer in the presence of thermal radiation in non-Newtonian fluids flow: The effects of mass and energy transfer with effects of variable viscosity and thermal conductivity, viscous dissipation in the presence of thermal radiation in non-Newtonian fluids flow of third grade fluid is comparatively not well-known. Using similarity transformations, the system of PDEs was developed and transformed into ordinary differential equations. The numerical approach of solving the equations is the Galerkin weighted residual method. The software Maple 18 was used to implement the method. The flow's physical characteristics were examined in relation to concentration, temperature and velocity profiles.

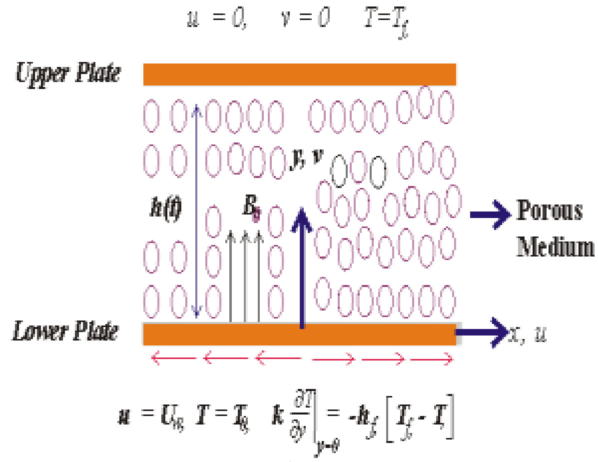

Fig.1: The flow geometry

Mathematical formulation: Through a conduit filled with an isotropic, homogenous porous media, an incompressible, third-grade, variable viscosity, variable thermal conductivity, magnetic field, thermal radiation, and reactive fluid are believed to flow in an unpredictable manner. It is thought that there is asymmetric convective heat exchange between the plate surfaces and the surrounding air because the heat transfer coefficients are not equal. The y-axis is normal to the channel, and the x-axis is parallel to it. According to Idowu *et al.* (2014); Olajuwon and Baoku (2014); Peter *et al.* (2022), the basic equations that explain the motion of an incompressible fluid are in equations 1-14:

$$\frac{\partial w}{\partial t}+w\frac{\partial w}{\partial x}+z\frac{\partial w}{\partial y}-\mu(T)\frac{\partial^2 w}{\partial y^2}-6\beta_3\left(\frac{\partial w}{\partial y}\right)^2\frac{\partial^2 w}{\partial y^2}+\mu(T)\frac{w}{\rho_m K}+\frac{\sigma B_0{}^2 w}{\rho_m}=0 \quad (1)$$

$$\frac{\partial T}{\partial t}+w\frac{\partial T}{\partial x}+z\frac{\partial T}{\partial y}-k(T)\frac{\partial^2 T}{\partial y^2}-\mu(T)\left(\left(\frac{\partial w}{\partial y}\right)^2+\frac{w^2}{(\rho C_P)_m K}\right)+\frac{\partial q_r}{\partial y}-2\beta_3\left(\frac{\partial w}{\partial y}\right)^4 \quad (2)$$

$$-\frac{\sigma B_0{}^2 w^2}{(\rho C_P)_m}=0$$

$$\frac{\partial C}{\partial t}+w\frac{\partial C}{\partial x}+z\frac{\partial C}{\partial y}-D_m\frac{\partial^2 C}{\partial y^2}=0 \quad (3)$$

The boundary conditions are

$$z=0,\quad T=T_0,\quad w=U_w,\quad -k\frac{\partial T}{\partial y}=h_f\left(T_f-T\right),\ C=C_w \text{ at } y=0$$

$$w=0,\quad T=T_f,\quad C=C_\infty \text{ as } y\to\infty \quad (4)$$

where $w=$x-direction, $z=$y-direction, $\rho=$ Fluid density, $\lambda=$Relaxation time, $\mu=$ Viscosity, β_2 and $\alpha_2=$ Non-Newtonian fluids, $\mu_{ef}=$ Effective viscosity, $B_0{}^2=$ Uniform magnetic field, $C_P=$ Specific heat, $T =$ Fluid temperature, $Q =$ Heat released due to exothermic reaction, $q_r =$ Radiation heat flux, $A =$ Arrhenius pre-exponential factor, $E =$ Activation energy, $q_r =$ Radiation heat flux, $D_m =$ Diffusion coefficient of the diffusing species, $T_0 =$ Initial fluid temperature, $T_f =$ Ambient temperature and $h_f =$ Heat transfer coefficient.

Following Roseland approximation for radiation energy

$$\frac{\partial q_r}{\partial y} = -\frac{16T_\infty^{\ 3}\sigma^r}{3k^r}\frac{\partial}{\partial y}\left(\frac{\partial T}{\partial y}\right) \qquad (5)$$

The temperature dependence of viscosities can be expressed as

$$\mu(T) = \mu_0 e^{-\left(\frac{T-T_0}{T_0}\right)} \qquad (6)$$

Temperature dependence of thermal conductivities can be expressed as

$$k(T) = k_0 e^{-\alpha T} \qquad (7)$$

The following similarity variables are introduced into equations (1) – (7).

$$w = w' v_0,\ \ z = z' v_o,\ \ x = x' v_0,\ \ t = \frac{v_0}{U} t',\ \ \theta(\eta) = \frac{T - T_0}{T_f - T_0},\ \ \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty},$$

$$\eta = \sqrt{\frac{U_w}{x v_0}}\, y,\ \ v = -\frac{\partial \Psi}{\partial x} = -\sqrt{\frac{a v_0}{1-ct}}\, f(\eta),\quad u = \frac{\partial \Psi}{\partial y} = U_w f'(\eta),\ \ U_w = \frac{ax}{1-ct},\ \ \tau = \frac{c}{2(1-ct)},$$

$$\beta_n = \frac{2U_w^{\ 5}}{\rho v_0^{\ 3} x^2 U}\beta_3,\quad \mathrm{Re} = \frac{\rho v_0 U}{\mu_0},\quad M_0 = \frac{U_w}{\rho U}\sigma B_0^{\ 2},\quad \Pr = \frac{\rho C_P U v_0}{k U_w},\quad Ec = \frac{\mu U_w^{\ 3}}{\rho C_P v_0 x},$$

$$b_1 = b(T_f - T_0),\quad n_1 = n(T_f - T_0),\quad R_d = \frac{4T_\infty^{\ 3}\sigma^r U_w}{K^r \rho C_P x v_0^{\ 2}},\quad Da = \frac{\rho U_w}{\mu_{ef}} K,\quad Bi = \frac{h_f}{k\left(\frac{U_w}{x v_0}\right)^{\frac{1}{2}}},$$

$$\Psi = \frac{v_0 E l_0 e^{-\frac{E}{RT_0}}}{RT_0^{\ 2} U \rho C_P} Q C_0 A Y,\ Sc = \frac{x l_0 v_0^{\ 2}}{D U_w} \qquad (8)$$

Equations (1) – (8) and dropping the primes transform into

$$(S\eta - \eta f' - f) f'' - \left(\beta_n (f'')^2 - \frac{b_1}{\mathrm{Re}} e^{-b_1\theta}\right) f''' - \left(M_0 - b_1 \frac{e^{-b_1\theta}}{Da\,\mathrm{Re}}\right) f' = 0 \qquad (9)$$

$$(S\eta - \eta f' - f)\theta' - \left(\frac{4}{3}R_d + \frac{n_1}{\Pr} e^{n_1\theta}\right)\theta'' + \frac{e^{n_1\theta}}{\Pr} n_1 (\theta')^2 + \frac{b_1}{\mathrm{Re}} e^{-b_1\theta}\left(Ec - \frac{1}{Da}\right)(f')^2$$
$$+ \beta_n (f'')^4 - M_0 (f')^2 = 0 \qquad (10)$$

$$\phi'(S\eta - \eta f' - f) - \frac{1}{Sc}\phi'' = 0 \quad (11)$$

Transformed boundary condition

$$f(\eta) = 0,\ \theta(\eta) = 0,\ f'(\eta) = 1,\ \theta'(\eta) = -Bi(1-\theta),\ \phi(\eta) = 1 \quad \text{at} \quad \eta = 0,$$
$$f'(\eta) = 0,\ \ \theta(\eta) = 1,\ \ \phi(\eta) = 0 \quad \text{as} \quad \eta \to \infty \qquad (12)$$

Method of Solution: The non-linear equations (9) – (11) alongside transformed boundary conditions as presented in equation (12) were solved numerically using Galerkin weighted residual method as follows:

$$\text{Let } f = \sum_{i=0}^{2} A_i e^{\left(-\frac{i}{5}\right)y},\ \theta = \sum_{i=0}^{2} B_i e^{\left(-\frac{i}{5}\right)y},\ \phi = \sum_{i=0}^{2} B_i e^{\left(-\frac{i}{5}\right)y} \quad (13)$$

We employ the following parameter values:

$$\tau = 0.1, b_1 = n_1 = 2.0, \mathrm{Re} = \mathrm{Pr} = Ec = Sc = 1.5, \beta_n = 1.6, R_d = 1.4, Da = M_0 = 1.8 \qquad (14)$$

These will be the default values in this work.

RESULTS AND DISCUSION

The findings of the numerical computation carried out for variation in physical parameters are graphically exhibited in Figures 2 – 15. Figures 2 – 4 illustrate how the unsteadiness parameter τ affects the velocity, temperature and concentration of the fluids.

It is clear from the graphs that increasing the unsteadiness parameter causes the velocity, temperature and concentration profiles of the flow to decline. The dimensionless velocity and temperature profiles for various values of variable viscosity parameter b_1 are displayed in Figures 5 – 6. It is observed from the graphs that when b_1 increases, the velocity and temperature profiles rise.

Figures 7 – 8 illustrate how the Reynolds number Re affects the velocity and temperature fields. It is cleared from the graphs that when the Reynolds number rises, the velocity profile rises and the temperature profile falls.

Figures 9 – 15 illustrates how the velocity and temperature profiles were affected by the third grade parameter β_n, Darcy number Da, magnetic field parameter M_0, variable thermal conductivity parameter n_1 and radiation parameter R_d.

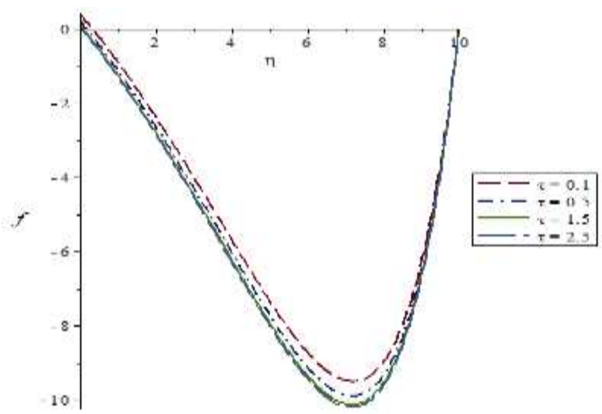

Fig. 2: Unsteadiness parameter τ on velocity profile

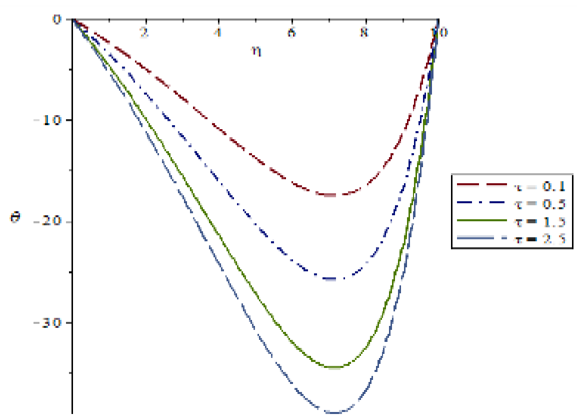

Fig. 3: Parameter τ on temperature Profile

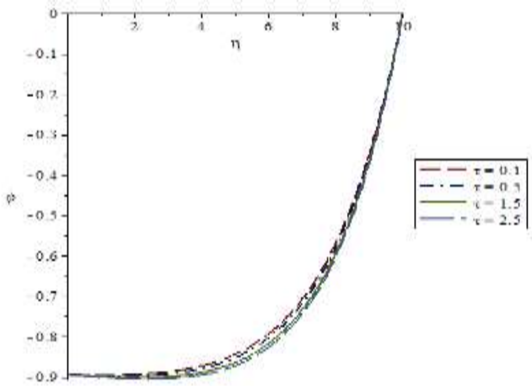

Fig. 4: Parameter τ on concentration Profile

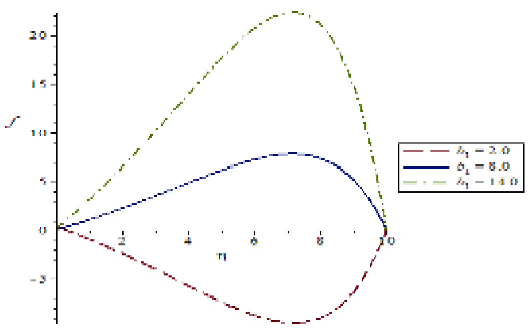

Fig. 5: Variable viscosity b_1 on velocity profile

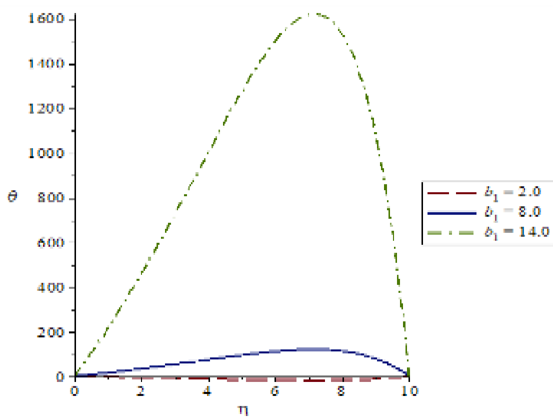

Fig. 6: Parameter b_1 on temperature Profile

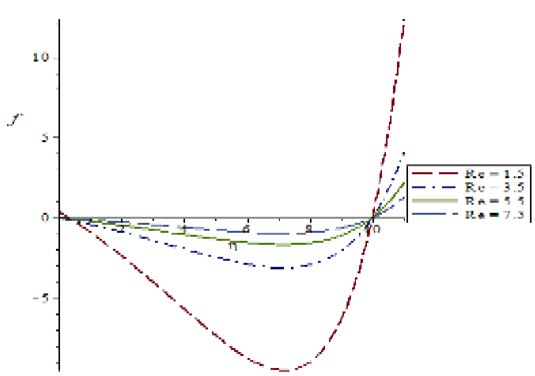

Fig. 7: Effect of Re on velocity profile

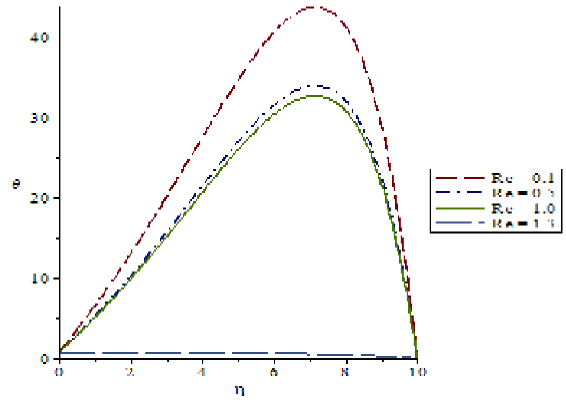

Fig. 8: Effect of Re on temperature profile

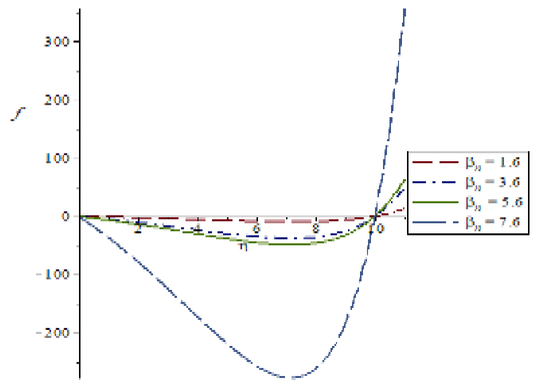

Fig. 9: Parameter β_n on velocity profile

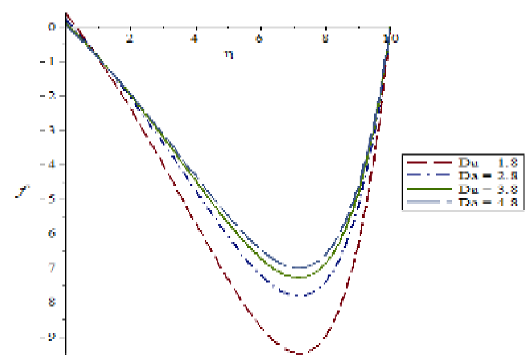

Fig. 10: Effect of Da on velocity profile

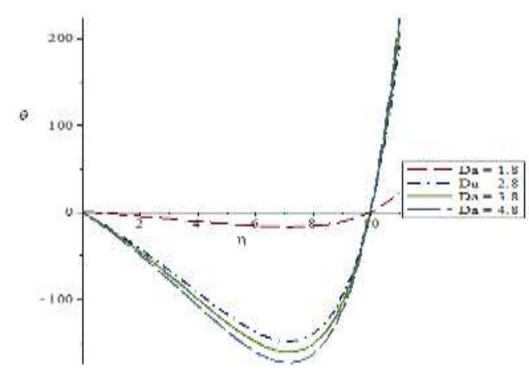

Fig. 11: Darcy number Da on temperature profile

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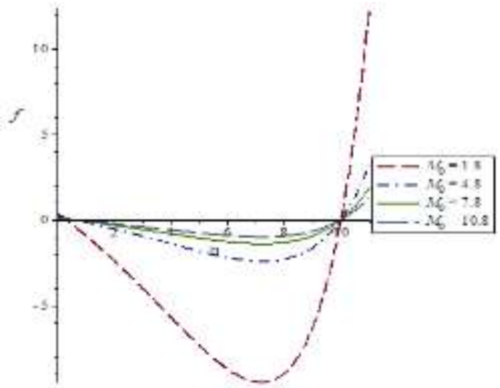

Fig. 12: Effect of M_0 on velocity profile

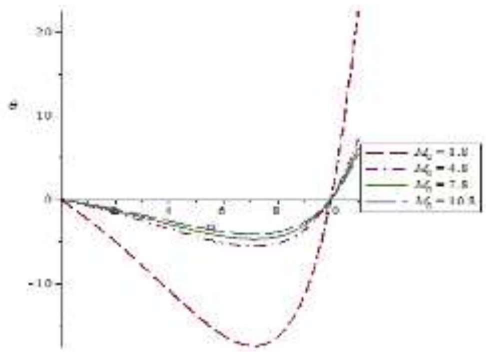

Fig.13: Effect of M_0 on temperature profile

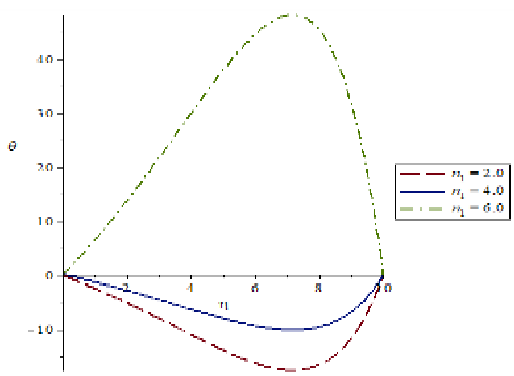

Fig. 14: Parameter n_1 on temperature profile

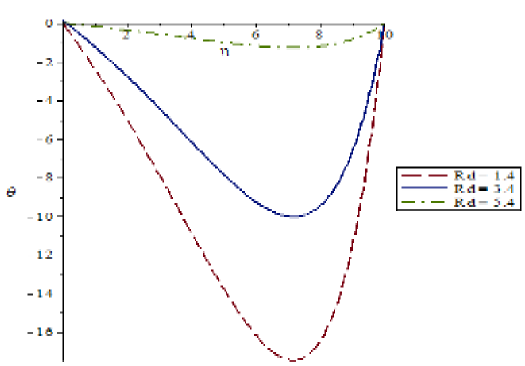

Fig. 15: Effect of R_d on temperature profile

Generally speaking, the Lorentz force—the opposite force to the flow—develops with a rise in the magnetic field parameter. This force tends to increase the thickness of the thermal boundary layer while decreasing the momentum boundary layer. Conclusively, it is worth mentioning that all the profiles of the flow presented in Figures 2 to 15 win over the boundary conditions as presented in equation (12), and thus supports the numerical results obtained.

Conclusion: The physical parameters that characterize the fluids with variable viscosity, variable thermal conductivity and thermal radiation through a porous medium have considerable effects on both the mass and energy transfer of the flow system. The study inferred that third grade fluid affect both the velocity as well as the temperature distribution of the flow. Most of crude oils exhibit non-Newtonian characteristics and thus represent a good example of the application of our modeled problem. Many technical and scientific procedures can benefit more from this work.

Declaration of Conflict of Interest: The authors declare no conflict of interest.

Data Availability Statement: Data are available upon request from the corresponding author.

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