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Properties and Potentials of Gumbel Power Function Distribution to Rainfall and Wind Speed Datasets

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ABSTRACT: The objective of this paper is to present the properties and potentials of Gumbel Power function (GuPF) distribution to rainfall and wind speed datasets using the T-X methodology. The density and hazard rate function of the GuPF distribution are unimodal and increasing respectively. Statistical properties of the new distribution such as quantile, moments, and probability weighted moments (PWMs), order statistics and entropy are derived. The Maximum likelihood estimation method is used to estimate the parameters of the proposed model. The superiority of GuPF distribution over other distributions with the same baseline is illustrated using two environmental datasets.

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The power function distribution is a very flexible parametric distribution which has been applied in modelling data from actuarial science, biological studies and reliability analysis. Through inverse transformation, the power function can be derived from the Pareto distribution Bursa and Ozel (2017). The cumulative density function (cdf) and probability density function (pdf) of a two-parameter power function are given by

$$F(x)=\left(\frac{x}{c}\right)^{d} \quad (1)$$

$$f(x)=\left(\frac{d}{c}\right)\left(\frac{x}{c}\right)^{d-1} \quad 0<x<c,\ d>0 \quad (2)$$

Where d and c are shape and scale parameters respectively. Due to its tractability, power function has attracted the interest of many researchers who have attempted to extend it through the addition of shape parameters. Some extensions of power function distribution include Exponentiated Weibull-Power function Amal *et al*. (2017), Exponentiated Weibull power function Hassan and Nassar (2017) Exponentaited Kumaraswamy-Power function Bursa and Ozel (2017), Kumaraswamy-Power Abdul-Moniem, (2017), Transmuted Power function Haq *et al*. (2016), Log-Weighted Power function Mandouh and Mohamed (2020), Another generalized Transmuted Power function Nwezza and Uwadi (2021), Weibull-power function Tahir *et al* (2016), Transmuted Topp-Leone power function Hassan *et al*. (2021), exponentiated generalized power function Hassan and Nassar (2020) and New cubic transmuted power function distribution Haq *et al* (2023). The transformed transformer (T-X) family of distribution was introduced by Alzaatreh *et al*. (2013) as a method

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of generating probability distributions. Many authors have used the T-X framework to propose different probability distributions. See Tomy *et al* (2019) for a review on the T-X family of distributions. A random variable X is said to be generated from the T-X family of distribution if the cdf has the form given in (3)

$$G(x)=\int_{-\infty}^{W(F(x))} r(t)dt = R\left(W\left(F(x)\right)\right) \qquad (3)$$

T is a random variable defined on the interval $[a,b]$, $-\infty \le a < b \le \infty$ with cdf and pdf $R(t)$ and $r(t)$ respectively and $F(x)$ is the cdf of the baseline distribution. The function $W(F(x))$ acts as a "transformer" and different choices of $W(.)$ will result to different T-X distributions and the form $W(.)$ takes depends on the support of $r(t)$. For details of different definitions of $W(.)$ based on the support of the random variable T see Alzaatreh *et al.*, (2013). This paper proposes Gumbel power function (GuPF) distribution using the T-X methodology, studies its properties and explores the potentiality of the proposed distribution using rainfall and wind speed datasets.

MATERIALS AND METHODS

Let T be a random variable from Gumbel distribution with cdf and pdf given by

$$R(t)=\exp\left(-\exp-\left(\frac{t-\varepsilon}{\alpha}\right)\right) \qquad (4)$$

and

$$r(t)=\frac{1}{\alpha}\exp\left(-\left(\frac{t-\varepsilon}{\alpha}\right)\exp\left(-\exp-\left(\frac{t-\varepsilon}{\alpha}\right)\right)\right), -\infty < t < \infty, \alpha > 0, -\infty < \varepsilon < \infty$$

(5)

Define $W(.)$ as the logit of the cdf of the baseline distribution $F(x)$. The cdf of the proposed T-X family of distribution can be expressed as

$$G(x)=\int_{-\infty}^{\log\left(\frac{F(x)}{1-F(x)}\right)} r(t)dt = R\left(\log\left(\frac{F(x)}{1-F(x)}\right)\right) \qquad (6)$$

Substituting (1) in (6) and simplifying yields the cdf of GuPF distribution.

$$G(x)=\int_{0}^{\log\left(\frac{\left(\frac{x}{c}\right)^d}{1-\left(\frac{x}{c}\right)^d}\right)} r(t)dt = R\left(\log\left(\frac{x^d}{c^d-x^d}\right)\right) = \exp\left(-B\left(\frac{x^d}{c^d-x^d}\right)^{-\frac{1}{\alpha}}\right)$$

(7)

Where $B=\exp\left(\frac{\varepsilon}{\alpha}\right)$. The pdf of GuPF distribution is obtained from derivative of (7) as

$$g(x)=\frac{dBc^d x^{-\left(\frac{d}{\alpha}+1\right)}}{\alpha\left(c^d-x^d\right)^{-\left(\frac{1}{\alpha}+1\right)}}\exp\left\{-B\left(\frac{x^d}{c^d-x^d}\right)^{-\frac{1}{\alpha}}\right\}$$

(8)

The hazard rate function $h(x)$ of GuPF distribution is given by

$$h(x)=\frac{dBc^d x^{-\left(\frac{d}{\alpha}+1\right)}\exp\left\{-B\left(\frac{x^d}{c^d-x^d}\right)^{-\frac{1}{\alpha}}\right\}}{\alpha\left(c^d-x^d\right)^{-\left(\frac{1}{\alpha}+1\right)}\left[1-\exp\left(-B\left(\frac{x^d}{c^d-x^d}\right)^{-\frac{1}{\alpha}}\right)\right]}$$

The plots of pdf and $h(x)$ of GuPF distribution are given in Figure 1. The plots indicate that the pdf of GuPF distribution is unimodal and skewed to the right while the $h(x)$ is increasing.

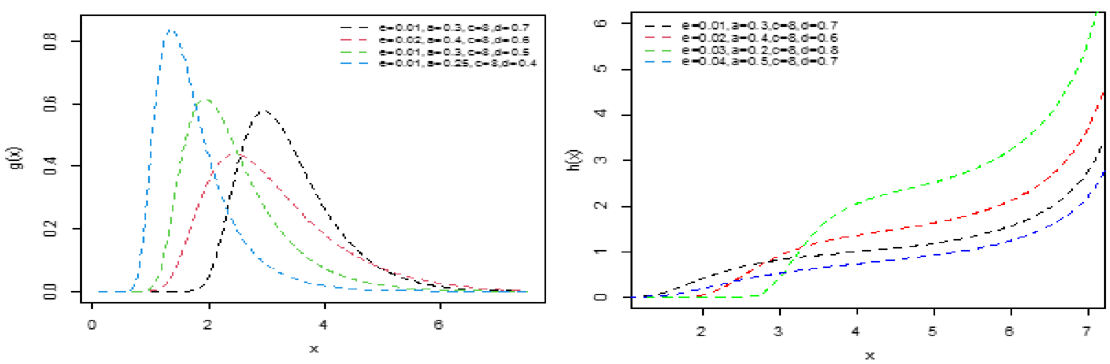

Fig 1: Pdf plots of GuPF distribution (left), Plots of $h(x)$ for GuPF distribution (right)

Statistical Properties: Derivation of some statistical properties of GuPF distribution are provided in this section.

Quantile Function: Let a random variable X have a GuPF distribution. The quantile function of GuPF distribution say $Q(u)$ of X is derived directly from (7) as $X = G^{-1}(u)$ and is given as

$$Q(u) = c\left[1+\left(-\frac{1}{B}\log(u)\right)^{\alpha}\right]^{-\frac{1}{d}} \quad (9)$$

Where u is uniformly distributed between 0 and 1 and $G^{-1}(.)$ is the inverse function of $G(.)$. The lower quartile, the median and upper quartile of the GuPF distribution are derived by respectively substituting for $u = 0.25, 0.5$ and 0.75 in (9).

Useful Expansions: A useful re-representation of pdf and cdf GuPF distribution are derived in this subsection. The expression for GuPF distribution as given (8) can be re-written as

$$g(x) = \frac{Bd\left(\frac{x}{c}\right)^{-1}}{\alpha c\left(1-\left(\frac{x}{c}\right)^{d}\right)}\left(\frac{\left(\frac{x}{c}\right)^{d}}{1-\left(\frac{x}{c}\right)^{d}}\right)^{-\frac{1}{\alpha}}\exp\left(-B\left(\frac{\left(\frac{x}{c}\right)^{d}}{1-\left(\frac{x}{c}\right)^{d}}\right)^{-\frac{1}{\alpha}}\right)$$

(10)

Applying the expansion of the exponential function and the binomial expansion of the form

$$\exp(-z) = \sum_{i=1}^{\infty}\frac{(-1)^{i} z^{i}}{i!} \quad |z|\ \alpha > 0$$ and

$$(1-z)^{\alpha} = \sum_{j=1}^{\infty}(-1)^{j}\binom{\alpha}{j}z^{j},$$ (10) becomes

$$g(x) = \sum_{i,j=0}^{\infty}\eta_{ij} f_{d\left[j-\frac{1}{\alpha}(i+1)\right]}(x) \quad (11)$$

where

$$\eta_{ij} = \frac{-1^{i+j}B^{i+1}}{\alpha i!}\left[j-\frac{i}{\alpha}(i+1)\right]^{-1}\binom{\frac{1}{\alpha}(i+1)-1}{j}$$

$f_{d\left[j-\frac{1}{\alpha}(i+1)\right]}(x)$ is a power function distribution with power parameter $d\left[j-\frac{1}{\alpha}(i+1)\right]$ and scale parameter c. Thus GuPF distribution can be expressed as an infinite linear combination of power function with shape parameter $d\left[j-\frac{1}{\alpha}(i+1)\right]$.

Furthermore, considering the expansion of $G^{h}(x)$ for $h > 0$ we have

$$G^{h}(x) = \left\{\exp\left(-B\left(\frac{\left(\frac{x}{c}\right)^{d}}{1-\left(\frac{x}{c}\right)^{d}}\right)^{-\frac{1}{\alpha}}\right)\right\}^{h} \quad (12)$$

Using the expansion of the exponential function and the general binomial formula in (12) we have

$$G^{h}(x) = \sum_{ij=0}^{\infty}\eta^{*}_{ij}F_{d\left(i-\frac{i}{\alpha}\right)}(x) \quad (13)$$

Where $\eta^{*}_{ij} = \frac{-1^{i+j}}{i!}(Bh)^{i}\binom{\frac{i}{\alpha}}{j}$

While the re-representation given (11) is very useful in deriving the moments and incomplete moments of the GuPF distribution (13) is handy in deriving the Probability weighted moments (PWM) and Order statistics of the GuPF distribution.

Moments and Incomplete Moments: Non-central Moments: The non-central moments are very important and necessary in any statistical analysis and applications. Hence we derive the rth non-central moment of GuPF distribution. Given that X is distributed as (8) the rth moment is derived as follows.

$$\mu_r = \int x^{r} g(x)\,dx \quad (14)$$

Using (11) in (14), we have

$$\mu_r = \sum_{ij=0}^{\infty} n_{ij}\int_0^c x^{r} f_{d\left[j-\frac{1}{\alpha}(i+1)\right]}(x)\,dx$$

$$u_r = \sum_{ij=0}^{\infty}\eta_{ij}\int_0^c x^{r}\frac{d\left[j-\frac{1}{\alpha}(i+1)\right]}{c}\left(\frac{x}{c}\right)^{d\left[j-\frac{1}{\alpha}(i+1)\right]-1}$$

$$\mu_r = dc^{r}\sum_{ij=0}^{\infty}\left(\frac{\left[j-\frac{1}{\alpha}(i+1)\right]}{r+d\left[j-\frac{1}{\alpha}(i+1)\right]}\right) \quad (15)$$

The first four non-central moments of GuPF distribution can be obtained by setting $r = 1, 2, 3, 4$ in (15). The moment generating function (mgf) of a random variable is given as

$$E\left(e^{tx}\right) = M_X(t) = \sum_{r=0}^{\infty} \frac{t^r}{r!} E(X)$$

Following the same line of reasoning used in arriving at (15), the mgf of GuPF distribution is given by

$$M_X(t) = \sum_{r=0}^{\infty}\sum_{ij=0}^{\infty} \frac{t^r}{r!} dc^r \sum_{ij=0}^{\infty} \left(\frac{\left[j - \frac{1}{\alpha}(i+1) \right]}{r + d\left[j - \frac{1}{\alpha}(i+1) \right]} \right) \quad (16)$$

Incomplete Moment: The incomplete moments say $\varphi_r(z)$ of GuPF distribution is obtained using (11) as

$$\varphi_r(z) = \int_0^z x^r f_{d\left[j - \frac{1}{\alpha}(i+1) \right]} dx$$

$$\varphi_r(z) = \int_0^z x^r \frac{d\left[j - \frac{1}{\alpha}(i+1) \right]}{c} \left(\frac{x}{c} \right)^{d\left[j - \frac{1}{\alpha}(i+1) \right] - 1} dx$$

$$\varphi_r(z) = \sum_{ij=0}^{\infty} \frac{\eta_{ij} d\left[j - \frac{1}{\alpha}(i+1) \right] z^{r + d\left[j - \frac{1}{\alpha}(i+1) \right]}}{r + d\left[j - \frac{1}{\alpha}(i+1) \right] c^{r + d\left[j - \frac{1}{\alpha}(i+1) \right]}} \quad (17)$$

Applications of (17) can be found in mean deviation from the mean and inequality curves such as the Bonferenoni curves and Lorenze curves.

Probability Weighted Moments: A class of moments called the probability weighted moments (PWMs), was proposed by Greenwood *et al*. (1979). PWMs can be used to obtain estimators of parameters and quantiles of distributions which can be expressed in inverse forms Hassan *et al*. (2017). The PWMs of a random variable X $\tau_{r,s}$ is defined by

$$\tau_{r,s} = E\left[X^r G(x)^s \right] = \int x^r g(x) G(x)^s \, dx \quad (18)$$

Substituting (11) and (13) into (18) above and replacing h with s the PWMs of GuPF distribution is derived as follows

$$\tau_{r,s} = \sum_{i,j=0}^{\infty} \sum_{k,m=0}^{\infty} \eta_{ij} \eta^*_{km} \int_0^c x^r \left(\frac{d\left[j - \frac{1}{\alpha}(i+1) \right]}{c} \right) \left(\frac{x}{c} \right)^{d\left[j - \frac{1}{\alpha}(i+1) + \left(k - \frac{m}{\alpha} \right) \right] - 1} dx$$

$$\tau_{r,s} = c^r \sum_{i,j=0}^{\infty} \sum_{k,m=0}^{\infty} \eta_{ij} \eta^*_{km} \frac{d\left[j - \frac{1}{\alpha}(i+1) \right]}{d\left[j - \frac{1}{\alpha}(i+1) + \left(k - \frac{m}{\alpha} \right) \right]} \quad (19)$$

Renyi Entropy: The entropy of a random variable X is a measure of variation of uncertainty. It has applications in physics, engineering and economics. According to Renyi (1961), the Renyi entropy is defined by

$$I_R(x) = \frac{1}{1-\sigma} \log \left\{ \int_{-\infty}^{\infty} g^\sigma(x) dx \right\} \sigma > 0 \;\; \sigma \neq 0 \quad (20)$$

By using $g(x)$ as defined in 10, in 20 we have that

$$g^\sigma(x) = \frac{(Bd)^\sigma \left(\frac{x}{c} \right)^{-\sigma}}{(\alpha c)^\sigma \left(1 - \left(\frac{x}{c} \right)^d \right)^\sigma} \left(\frac{\left(\frac{x}{c} \right)^d}{1 - \left(\frac{x}{c} \right)^d} \right)^{-\frac{\sigma}{\alpha}} \exp\left(-B\sigma \left(\frac{\left(\frac{x}{c} \right)^d}{1 - \left(\frac{x}{c} \right)^d} \right)^{-\frac{1}{\alpha}} \right) \quad (21)$$

Applying the expansion of exponential and general binomial expansion functions to (21) and simplifying we have

$$g^\sigma(x) = \sum_{i,j=0}^{\infty} V_{ij} \left(\frac{x}{c} \right)^{d\left[j - \frac{1}{\alpha}(\sigma + i) \right] - \sigma} \quad (22)$$

Where $V_{ij} = \frac{d}{\alpha c}(-1)^i B^{(\sigma+i)}\sigma^i \binom{\sigma - \frac{1}{\alpha}(\sigma+i) + j - 1}{j}$

Substituting (22) in (20) and integrating we have

$$I_R(x) = \frac{1}{1-\sigma}\log\left\{c\sum_{i,j=0}^{\infty} V_{ij}\left(d\left[j - \frac{1}{\alpha}(\sigma+1)\right] - \sigma + 1\right)^{-1}\right\} \quad \sigma > 0 \; \sigma \neq 0$$

The q-entropy is defined by

$$H_q(x) = \frac{1}{1-q}\log\left(1 - \int_{-\infty}^{\infty} g^q(x)\,dx\right) \quad q > 0 \; q \neq 0 \qquad (23)$$

Thus the q-entropy of GuPF distribution is given by

$$H_q(x) = \frac{1}{1-q}\log\left\{1 - c\sum_{i,j=0}^{\infty} V_{ij}\left(d\left[j - \frac{1}{\alpha}(q+1)\right] - q + 1\right)^{-1}\right\} \quad q > 0 \; q \neq 0 \qquad (24)$$

Order Statistics

Let $X_1, X_2, X_3, \cdots X_n$ be independent and identically distributed random variables having a continuous distribution function $G(x)$. Let $X_{(1)} < X_{(2)} < X_{(3)} < \cdots < X_{(n)}$ be the corresponding ordered sample. The density of the rth order statistics, for $r = 1, \cdots, n$ is given by

$$g_{r:n}(x) = \frac{1}{B(r, n-r+1)}\sum_{v=0}^{n-r}(-1)^v \binom{n-r}{v} g(x) G^{v+r-1}(x) \qquad (25)$$

where $B(.,.)$ is a beta function. The density of rth order statistic for GuPF distribution is easily derived by substituting (11) and (13) in (25) and replacing h with $v + r - 1$ and simplifying we have

$$g_{r:n}(x) = \frac{\sum_{v=0}^{n-r}\sum_{ij=0}^{\infty}\sum_{km=0}^{\infty}(-1)^v \eta_{ij}\eta_{km}^{*}\binom{n-r}{v}}{B(r, n-r+1)}\left(\frac{d\left[j - \frac{1}{\alpha}(i+1)\right]}{c}\right)\left(\frac{x}{c}\right)^{d\left\{\left[j-\frac{1}{\alpha}(i+1)\right]+\left(k-\frac{m}{\alpha}\right)\right\}-1}$$

Hence the rth order statistics of GuPF distribution can be expressed as

$$g_{r:n}(x) = \frac{\sum_{v=0}^{n-r}\sum_{ijkm=0}^{\infty}\varpi_{ijkm}}{B(r, n-r+1)} f_{d^*}(x) \qquad (26)$$

Where $\varpi_{ijkm} = (-1)^v \binom{n-r}{v}\eta_{ij}\eta_{km}^{*}\frac{d\left[j - \frac{1}{\alpha}(i+1)\right]}{d\left\{\left[j - \frac{1}{\alpha}(i+1) + \left(k - \frac{m}{\alpha}\right)\right]\right\}}$,

$d^* = d\left\{\left[j - \frac{1}{\alpha}(i+1) + \left(k - \frac{m}{\alpha}\right)\right]\right\}$ and $f_{d^*}(x)$ is the pdf of the baseline distribution with power parameter d^*.

Estimation: In this section, the method of maximum likelihood (MLE) is employed in obtaining the ML estimates of parameters of GuPF distribution. Let $X_1, X_2, X_3, \cdots X_n$ be a random sample from GuPF distribution with a set of parameters $\Theta = (\varepsilon, c, d, \alpha)$. The loglikelihood function l for the vector of the parameter $\Theta = (\varepsilon, c, d, \alpha)$ is given as

$$l = n\log d + n\log B + nd\log c - n\log\alpha - \left(\frac{d}{\alpha}+1\right)\sum_{i=1}^{n}\log x_i + \left(\frac{1}{\alpha}+1\right)\sum_{i=1}^{n}\log\left(c^d - x_i^d\right)$$
$$-B\sum_{i=1}^{n}\left(\frac{x_i^d}{c^d - x_i^d}\right)^{-\frac{1}{\alpha}} \tag{27}$$

The associated score function $U(\Theta) = \left(\frac{\partial l}{\partial \varepsilon}, \frac{\partial l}{\partial c}, \frac{\partial l}{\partial d}, \frac{\partial l}{\partial \alpha}\right)$ has its elements given by

$$\frac{\partial l}{\partial \varepsilon} = \frac{n}{\alpha} - \frac{1}{\alpha}B\sum_{i=1}^{n}\left(\frac{x_i^d}{c^d - x_i^d}\right)^{-\frac{1}{\alpha}}$$

$$\frac{\partial l}{\partial \alpha} = -\frac{n}{\alpha} + \frac{d}{\alpha^2}\sum_{i=1}^{n}\log x_i - \frac{1}{\alpha^2}\sum_{i=1}^{n}\log\left(c^d - x_i^d\right) - \frac{B}{\alpha^2}\sum_{i=1}^{n}\left(\frac{x_i^d}{c^d - x_i^d}\right)^{-\frac{1}{\alpha}}\ln\left(\frac{x_i^d}{c^d - x_i^d}\right)$$

$$\frac{\partial l}{\partial c} = \frac{nd}{c} + dc^{d-1}\left(\frac{1}{\alpha}+1\right)\sum_{i=1}^{n}\left(c^d - x_i^d\right)^{-1} - \frac{Bdc^{d-1}}{\alpha}\sum_{i=1}^{n}\left(\frac{x_i^d}{c^d - x_i^d}\right)^{-\frac{1}{\alpha}-1}\left(\frac{x_i^d}{\left(c^d - x_i^d\right)^2}\right)$$

and

$$\frac{\partial l}{\partial d} = \frac{n}{d} + n\log c - \frac{1}{\alpha}\sum_{i=1}^{n}\log x_i + \left(\frac{1}{\alpha}+1\right)\sum_{i=1}^{n}\frac{c^d\ln c - x_i^d\ln x_i}{\left(c^d - x_i^d\right)}$$
$$+\frac{Bc^d}{\alpha}\sum_{i=1}^{n}\left(\frac{x_i^d}{c^d - x_i^d}\right)^{-\frac{1}{\alpha}-1}\frac{x_i^d\left(\ln x_i - \ln c\right)}{\left(c^d - x_i^d\right)^2}$$

Setting each element of the score function to zero and solving the resulting system of equations will yield the MLEs $\hat{\Theta} = \left(\hat{\varepsilon}, \hat{c}, \hat{d}, \hat{\alpha}\right)$ of $\Theta = (\varepsilon, c, d, \alpha)$. These systems of equations cannot be solved analytically hence numerical optimization methods such as Newton-Raphson's algorithm are normally used in solving such systems of equations.

RESULTS AND DISCUSSIONS

We illustrate with two different datasets the potentials of the proposed distribution. The first dataset consists of the annual maximum daily precipitation mm at Busan, Korea for the 1904-2011 period. This dataset was used by Mansoor *et al*. (2016). The data are: 24.8, 140.9, 54.1, 153.5, 47.9, 165.5, 68.5, 153.1, 254.7, 175.3, 87.6, 150.6 , 147.9, 354.7, 128.5, 150.4, 119.2, 69.7, 185.1, 153.4, 121.7, 99.3, 126.9, 150.1, 149.1, 143, 125.2, 97.2, 79.3, 125.8, 101, 89.8, 54.6, 283.9, 94.3, 165.4, 48.3, 69.2, 147.1, 114.2, 159.4, 114.9, 58.5, 76.6, 20.7, 107.1, 244.5, 126, 122.2, 219.9, 153.2, 145.3, 101.9, 135.3, 103.1,74.7, 174, 126, 144.9, 226.3, 96.2, 149.3, 122.3, 164.8, 188.6, 273.2, 61.2, 84.3, 130.5, 96.2, 155.8, 194.6, 92, 131, 137, 106.8, 131.6, 268.2, 124.5, 147.8, 294.6, 101.6, 103.1, 274.51,40.2,153.3, 91.8, 79.4, 149.2, 168.6, 127.7, 332.8, 261.6, 122.9, 273.4, 178, 177, 108.5, 115, 241, 76, 127.5, 190, 259.5, 301.5. The second dataset is on average monthly wind speed in km/h collected from AE-FUNAI metrological centre from 2014-2016. The data is as follows: 9.2, 9.15, 11.15, 10.04, 7.64, 9.73, 12.89, 10.21, 8.24, 8.43, 7.46, 6.55, 11.17, 13.55, 11.23, 9.77, 9.85, 10.97, 10.09, 9.28, 8.28, 8.02, 9.68, 9.25, 13.21, 10.65, 13.21, 12.92, 11.99, 12.68, 10.72, 11.41, 10.9, 9.13, 8.97, 10.7.

Figures 2 and 3 depict the box plot, Total time to test (TTT) plot, kernel density plot and violin plot of the rainfall and wind speed data to check for outliers, shape of the hazard rate function, and nature of datasets. The boxplot for the first dataset shows that it has outliers while that of the second dataset has no outliers. The TTT plots for both datasets are concave implying that the datasets have an increasing hrf hence justifying the use of GuPF distribution in fitting both datasets. The kennel density plot shows that both datasets are asymmetric.

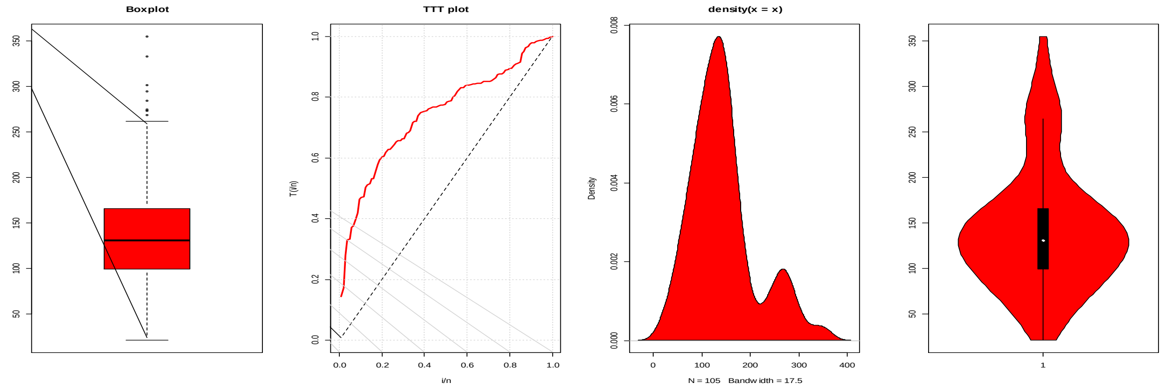

Fig 2: Box, TTT, kennel density and violin plots for the first dataset

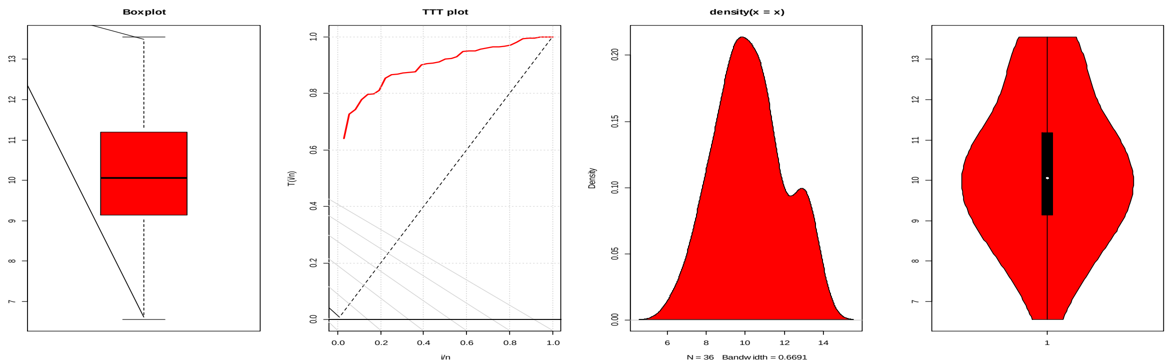

Fig 3: Box, TTT, kennel density and violin plots for the second dataset

The performance of the proposed distribution is compared with another generalized transmuted power (AGTPF) distribution Nwezza and Uwadi (2021), Transmuted Power function (TPF) distribution Haq *et al*. (2016) and Power function (PF) distribution Menoconi and Barry (1996) which are of the same baseline with GuPF distribution. The ML estimates and standard errors in parenthesis for the first and second datasets are respectively given in Tables 1 and 3 while the goodness of fit statistics for GuPF distribution and other competing models with the same baseline distribution are given in Tables 2 and 4 respectively. The goodness of fit statistics considered are Komogorov-Simrov (KS), Cramer-Von-Mises (CV), Anderson Darling (AD), Akaike information criterion(AIC), Bayesian Information Criterion (BIC) and log-likelihood. Generally, the smaller the values of these goodness of fit statistics the better the model. It is evident from Tables 2 and 4 that GuPF distribution has the least value of all goodness of fit statistics considered hence it is adjudged the best among the four competing models. A visual comparison of the fits of the two datasets is given in Figures 4 and 5. The fitted GuPF, AGTPF, TPF, and PF densities and histogram for the first and second datasets suggest that the fit of the GuPF distribution performs better than the other competing models with the same baseline distribution. Also the fitted cdf of GuPF distribution closely fits the empirical cdfs of the two datasets better than other densities considered.

Table 1: Estimates and Standard errors (SE) for first datasets

Distribution	Parameters			
GuPF(ε,α,d,c)	-0.153(0.70)	0.742(0.15)	0.654(0.35)	max(x)=354.7
AGTPF(θ,λ,d,c)	0.859(0.12)	1.738(0.32)	1.797(0.17)	max(x)=354.7
TPF(λ,d,c)	0.952(0.04)	1.412(0.11)		max(x)=354.7
PF(d,c)	0.976(0.10)			max(x)=354.7

Table 2: Goodness of fit Statistics for the first dataset

Distribution	KS	CV	AD	AIC	BIC	-loglik
GuPF	0.126	0.384	2.063	1183.3	1191.3	588.68
AGTPF	0.164	0.581	3.006	1191.1	1199.0	592.54
TPF	0.202	1.134	5.660	1198.7	1204.3	597.35
PF	0.299	2.623	12.873	1234.9	1237.5	616.45

Table 3: Estimates and Standard errors (SE) for the second dataset

Distribution	Parameters			
GuPF(ε,α,d,c)	-2.367(1.64)	1.481(0.60)	6.750(3.83)	max(x)=13.55
AGTPF(θ,λ,d,c)	0.592(0.31)	1.490(0.50)	5.207(0.89)	max(x)=13.55
TPF(λ,d,c)	0.833(0.20)	4.602(0.67)		max(x)=13.55
PF(d,c)	3.372(0.56)			max(x)=13.55

Table 4: Goodness of fit Statistics for the second dataset

Distribution	KS	CV	AD	AIC	BIC	-loglik
GuPF	0.097	0.046	0.279	141.86	146.6	67.93
AGTPF	0.139	0.151	0.908	150.32	155.08	72.16
TPF	0.169	0.215	1.125	149.53	152.7	72.76
PF	0.248	0.496	2.394	152.84	154.42	75.42

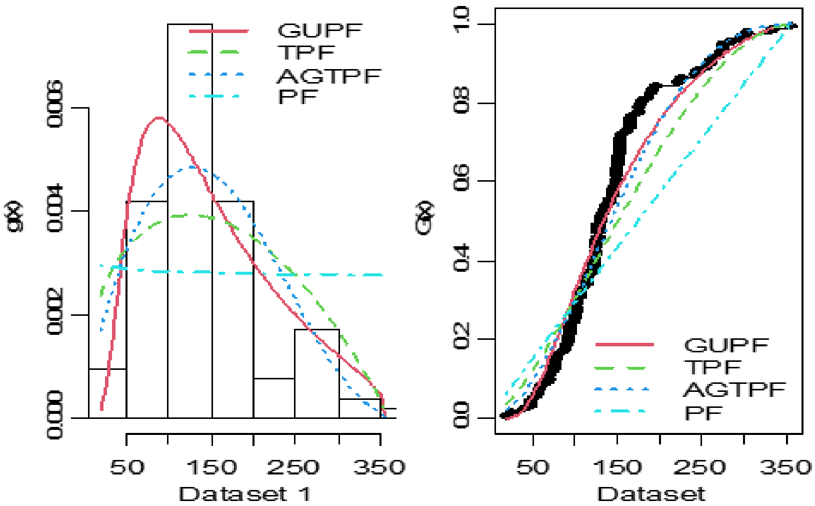

Fig 4: Estimated pdfs left and cdfs right for the first dataset

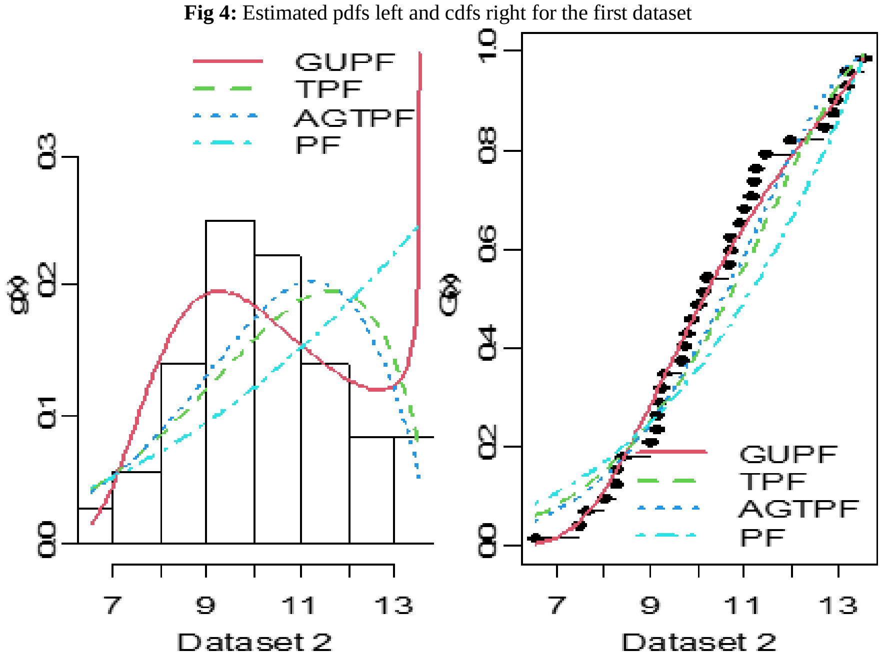

Fig 5: Fitted pdfs (left) and fitted cdfs (right) for second dataset

Statistical properties such as the moments, PWMs, entropy and order statistics of GuPF distribution were obtained. Estimates of the parameters of the GuPF distribution were derived through the method of maximum likelihood. Two environmental datasets namely rainfall and wind speed data were used to fit GuPF distribution alongside AGTPF, TPF and PF distributions. Evidence from the goodness of fit statistics and other plots meant for visual comparison proved that GuPF distribution fits the two datasets better than other competing distributions with the same baseline.

Conclusion: We proposed a new distribution called the Gumbel power function (GuPF) distribution in this article. It was shown that the pdf of the new distribution can be expressed as an infinite linear combination of the baseline distribution.

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