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Effect of Coefficient of Viscous Damping on Dynamic Analysis of Euler-Bernoulli Beam Resting On Elastic Foundation Using Integral Numerical Method

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ABSTRACT: In this paper, the effect of coefficient of viscous damping on the dynamic analysis of Euler-Bernoulli beam resting on elastic foundation was investigated using Integral-Numerical method which reduces to an ordinary differential equation with series representation of Heaviside function. The dynamic responses of the beam in terms of normalized deflection and bending moment has been investigated for different velocity ratios under moving load and moving mass conditions. Generally, closed-form solution to the generalized mathematical model for prismatic beam was computed by means of symbolic programming approach through MAPLE 18. Results obtain revealed that the presence of an elastic foundation and the provision of sufficient reinforcement in beams and beam-like structure reduces vibration intensity and ensure safe passage of load and prolong the beam life.

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Keywords: axial force; elastic foundation; foundation stiffness; integral-numerical method; moving load.

Beam is an important mechanical element that is also used as a simplest and accurate model for analyzing a complex engineering component structures like turbine and compressor rotor blades, airplane wings, robot arms, spacecraft antennae, structure of buildings, bridges and vibrating drilling can be modeled as a beam. The study of dynamical behavior of structures such as beams and plates, under the action of moving loads has attracted the attention of several researchers in Engineering, Applied Physics and Applied Mathematics. Notable among such researchers are Kolousek (1961), Eisenberger and Clastornik (1987), Sadiku (1987) and Leipholz (1987) and so on. In this moving load bearing problem, the influence of the load mass is very important because the position of the load changes continuously. Extensive work has been done on this class of dynamical problems when the structural members have uniform cross-sections.

Recently, a number of researchers have made great efforts in studying the dynamics of structures subjected to moving loads, including Usman (2019), Gbadeyan and Oni (2018), Jimoh (2021), Savin (2019). 2001), Ogunbamike (2012). The flexural motions of elastically supported beams sitting on winkler elastic foundations with stiffness variation

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were really taken into consideration by Oni and Awodola (2010). The technique was based on the generalized Galerkin's method and integral transformations and the beam was assumed to have uniform cross section. In all of these, considerations have been hunted to cases of uniform beams and where non-uniform beams are considered, they are considered only for classical boundary conditions.

There is also the work of Oni (1996), who examined the response of a thin, heterogeneous beam placed on a constant elastic foundation to some moving mass. To solve the problem, he used Galerkin's versatile technique to produce a complex quaternary partial differential equation with variable and singular coefficients at a set of ordinary differential equations. The set of ordinary differential equations was later simplified and solved using modified asymptotic method of struble. Although noteworthy, this work was merely based on a beam with the conventional simply supported end circumstances. Other studies on non-uniform beam include Mehmet (2014), Oni and Awodola (2011), Oni and Omolofe (2011).

We remark that most of the studies in this area have been treated only for classical boundary conditions. Nevertheless, for practical applications in many cases, it is more realistic to consider non-classical boundary conditions because the ideal boundary condition can seldomly be realized. Bridge-vehicle interaction is the most common problem in moving load analysis and it has been the vast area of research. If the speed of the vehicle is very low, it could not be treated as moving load problem because, at low speed it behaves as a static load condition.

Traditional methods can be used to rectify this issue. If the vehicle is moving at constant speed then it will be treated as moving load problem. By using mathematical and computational analysis, this issue can be resolved. Vibration of structure arises due to motion of vehicles, earthquake, flow of stream and winds.

There are various factors needed to be considered for design safety purpose such as mass of the moving body and the structure, inertia of moving mass of structure due to eccentric load. Hence, the objective of this paper was to investigate the effect of coefficient of viscous damping on the dynamic analysis of Euler-Bernoulli Beam resting on elastic foundation.

MATERIALS AND METHODS

Mathematical Formulation: In this section, the equation of motion for Euler-Bernoulli beam lying on a two-parameter Pasternak foundation and subjected to a moving load or mass, is depicted in Figure.1 below. The resulting vibrational behavior of this system is described by the following Partial differential equation.

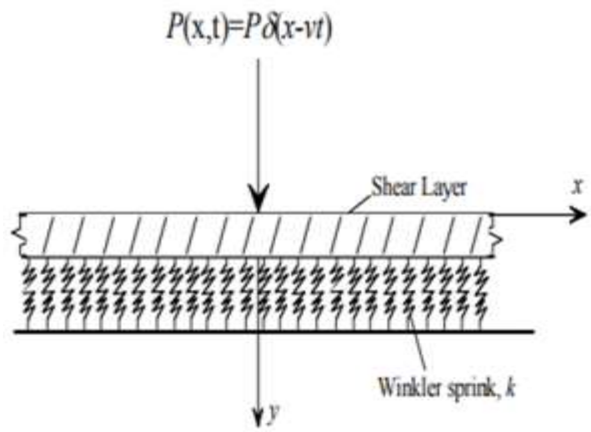

Fig 1: Beam on Pasternak foundation subjected to moving load.
Source: scholarsmine.mst.edu

$$EIL_{xxxx}(x,t) + mL_{tt}(x,t) + cL_t(x,t) - k_1 L_{xx}(x,t) + kL(x,t) = P(x,t) \quad (1)$$

Where

$$\mathrm{P(x,t)} = \rho\delta(x - vt) \quad (2)$$

Where $x \quad (0 \leq x \leq L)$ is the distance along the beam

t is the time in second

$I(x)$ is the moment of inertia of the beam cross section at a distance x

$\mu(x)$ is the external axial load acting on the beam cross section at a distance x

$L(x,t)$ is the beam lateral displacement

Convective acceleration operator $\mathrm{L_{tt}}$ is given as:

$$L_{tt} = L_{tt}(x,t) + 2vL_{tx}(x,t) + v^2 L_{xx}(x,t) \quad (3)$$

For simply supported beam of finite length L, the boundary conditions may be described Mathematically as

$$L(0,t) = L(L,t) = 0 \quad (4)$$

$$L'(0,t) = L''(L,t) = 0 \quad (5)$$

The supported beam is considered to be initially at rest. Hence the corresponding initial boundary conditions is

$$L(x,0) = L_{tt}(x,0) = 0 \quad (6)$$

Method of Solution: The overall equation of the problem under consideration is the partial derivative of the fourth order. To solve the problem, techniques called integral numerical methods are used to reduce fourth-order partial differential equations with variable and singular coefficients to a series of second-order ordinary differential equations. From equation (1) becomes,

$$EIL_{xxxx}(x,t) + mL_{tt}(x,t) + cL_t(x,t) - k_1L_{xx}(x,t) + kL(x,t) = \rho\delta(x - vt) \quad (7)$$

$$L_{xxxx}(x,t) = \sum_{n=1}^{\infty} X_n^{iv}(x)T_n(t) \quad (8)$$

$$L_{xx}(x,t) = \sum_{n=1}^{\infty} X_n^{11}(x)T_n(t) \quad (9)$$

$$L(x,t) = \sum_{n=1}^{\infty} X_n(x)T_n(t) \quad (10)$$

$$L_{tt}(x,t) = \sum_{n=1}^{\infty} X_n(x)\ddot{T}_n(t) \quad (11)$$

$$L_t(x,t) = \sum_{n=1}^{\infty} X_n(x)\dot{T}_n(t) \quad (12)$$

$$EI\sum_{n=1}^{\infty} X_n^{iv}(x)T_n(t) + m\sum_{n=1}^{\infty} X_n(x)\ddot{T}_n(t) + c\sum_{n=1}^{\infty} X_n(x)\dot{T}_n(t) - k_1\sum_{n=1}^{\infty} X_n^{11}(x)T_n(t) + k\sum_{n=1}^{\infty} X_n(x)T_n(t) = \rho\delta(x - vt) \quad (13)$$

For free vibration

$$\sum_{n=1}^{\infty} X_n^{iv}(x)T_n(t) = \mu\omega_n^2 X_n(x)T_n(t) \quad (14)$$

$$EI\mu\omega_n^2 X_n(x)T_n(t) + m\sum_{n=1}^{\infty} X_n(x)\ddot{T}_n(t) + c\sum_{n=1}^{\infty} X_n(x)\dot{T}_n(t) - k_1\sum_{n=1}^{\infty} X_n^{11}(x)T_n(t) + k\sum_{n=1}^{\infty} X_n(x)T_n(t) = \rho\delta(x - vt) \quad (15)$$

$$m\ddot{T}_n(t) + c\dot{T}_n(t) + EI\mu\omega_n^2 T_n(t) + kT_n(t) + k_1\int_0^{L_\alpha} \frac{n^2\pi^2}{L_\alpha}\sin\frac{n\pi x}{L_\alpha}\sin\frac{k\pi x}{L_\alpha}T_n(t)dx$$

$$= \int_0^{L_\alpha} \rho\,\delta(x - vt)\sin\frac{k\pi x}{L_\alpha}dx \quad (16)$$

RESULTS AND DISCUSSION

The differential equation for beam motion is a non-homogeneous partial differential equation of order 4 with coefficients of variation and heterogeneity. The solution of the dynamic beam problem is obtained, representing the displacement response of the beam, using the integral numerical method. Therefore, a numerical illustration of the results obtained from this analysis is represented by graphed curves. The graphs reveal that for the stiffness of the fixed foundation, as the axial force values change more and more, the magnitude of the deflection will decrease accordingly. In addition, for different values of the fixed axial force of the foundation stiffness, the horizontal deflection of the beam decreases as the value of the foundation stiffness increases. Therefore, for higher values of foundation stiffness, the stability and reliability of the structural design is guaranteed.

Table 1: Time against Deflection at c=0, c=1, c=2

TIME(s)	$c = 0Nsm^{-2}$	$c = 1Nsm^{-2}$	$c = 2Nsm^{-2}$
0	0	0	0
1	0.0554376	0.0510953	0.0473501
2	0.0199872	0.0246042	0.0278671
3	0.0332862	0.0315279	0.0306974
4	0.0389097	0.0366096	0.0350753
5	0.0207978	0.0265076	0.0293215
6	0.0440338	0.0367032	0.0338069
7	0.0221181	0.0290222	0.0311997
8	0.0383328	0.0334531	0.0322817
9	0.0297576	0.0318412	0.0321160
10	0.0309432	0.0315690	0.0318820

Table 2: Time against Deflection at $c = 0, c = 1, c = 2$.

TIME(s)	$c = 0Nsm^{-2}$	$c = 1Nsm^{-2}$	$c = 2Nsm^{-2}$
0	0	0	0
1	3.90990	3.90990	3.90990
2	9.49352	9.49352	9.49352
3	7.96824	7.96824	7.96824
4	1.63485	1.63485	1.63485
5	0.29167	0.29167	0.29167
6	5.96105	5.96105	5.96105
7	9.73752	9.73752	9.73752
8	5.67085	5.67085	5.67085
9	0.04237	0.04237	0.04237
10	1.57370	1.57370	1.57370

Table 3: Time against Deflection at c=0 Nsm^{-2}, c=1 Nsm^{-2}, c=2 Nsm^{-2}

TIME(s)	$c = 0Nsm^{-2}$	$c = 1Nsm^{-2}$	$c = 2Nsm^{-2}$
0	0	0	0
1	0.0473501	0.0510953	0.0554376
2	0.0278671	0.0246042	0.0199872
3	0.0306974	0.0315279	0.0332862
4	0.0350753	0.0366096	0.0389097
5	0.0293215	0.0265076	0.0207978
6	0.0338069	0.0367032	0.0440338
7	0.0311997	0.0290222	0.0221181
8	0.0322817	0.0334531	0.0383328
9	0.0321160	0.0318412	0.0297576
10	0.0318820	0.0315690	0.0309432

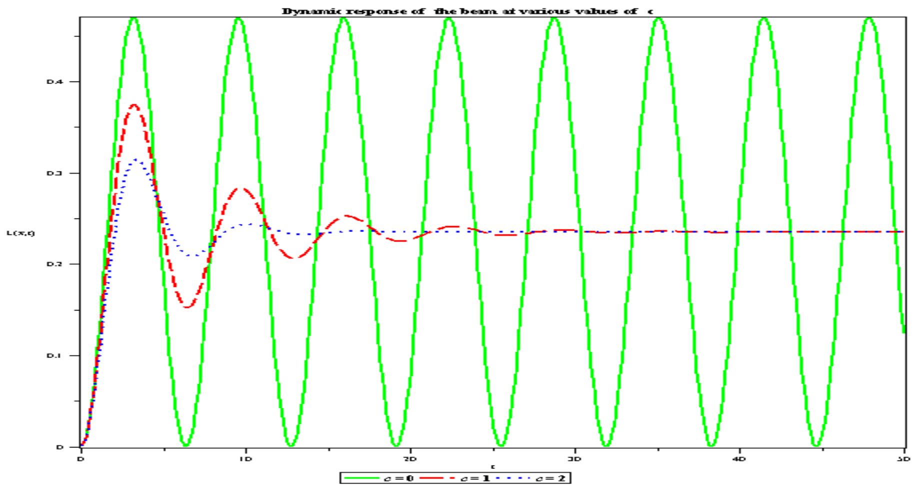

Fig 2: Dynamic response of the beam at various values of c for $m = 3kgm^{-1}$, $\mu = 1kgm^{-1}$, $I = 3kgm^2$, $\omega_n^2 = 1rads^{-1}$, , $E = 1Nm^{-2}$, $L = 1m$, $c = 1Nsm^{-2}$, $g = 10ms^{-2}$, $w_1 = 0.1rads^{-1}$, $k = 1pa$

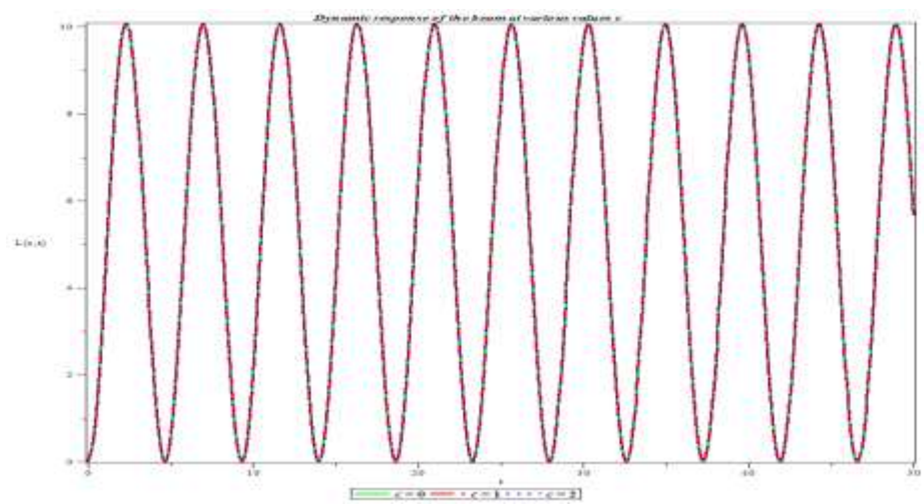

Fig 3: Dynamic response of the beam at various values of c for $m = 3kgm^{-1}$, $\omega_n^2 = 1rads^{-1}$, $\mu = 1kgm^{-1}$, $I = 3kgm^2$, $E = 1Nm^{-2}$, $L = 1m$, $g = 10ms^{-2}$, $w_1 = 0.1rads^{-1}$, $k = 1pa$

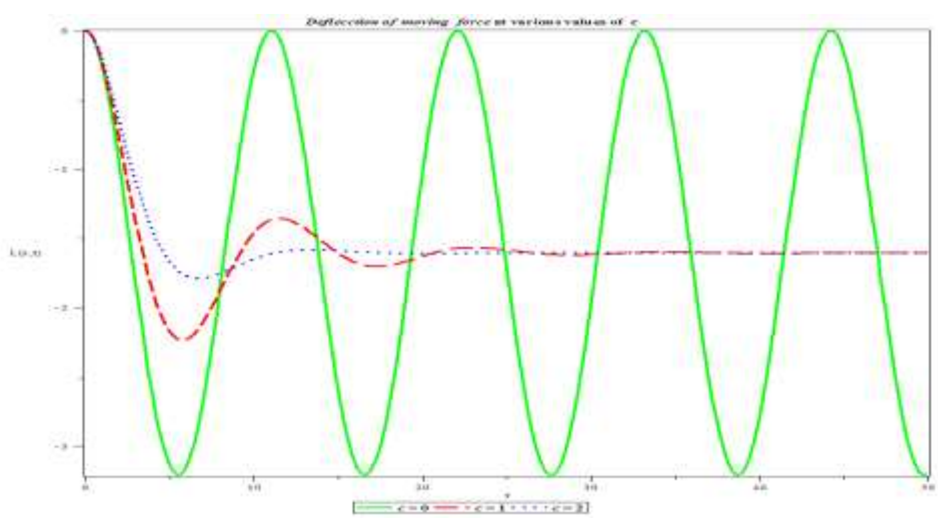

Fig 4: Graph of moving mass and moving load for $m = 3kgm^{-1}$, $v = 3.3ms^{-1}$, $\mu = 1kgm^{-1}$, $L = 1m$, $E = 1Nm^{-2}$, $g = 10ms^{-2}$, $w_1 = 0.1rads^{-1}$, $\omega_n^2 = 1rads^{-1}$

Conclusion: The solution of the dynamic beam problem is obtained, representing the displacement response of the beam, using the integral numerical method. Therefore, a numerical illustration of the results obtained from this analysis is represented by graphed curves. The graphs reveal that for the stiffness of the fixed foundation, as the axial force values increases, the magnitude of the deflection will decrease accordingly. In addition, for different values of the fixed axial force of the foundation stiffness, the horizontal deflection of the beam decreases as the value of the foundation stiffness increases.

REFERENCES

Eisenberger, M; Clastornik, J (1987). Vibrations and buckling of a beam on a variable Winkler elastic foundation. *J. Sound. Vibe 115*(2), 233-241.

Jimoh, SA (2021). On Modal-Asymptotic Analysis to prestressed thick beam on Bi-Parametric foundation subjected to moving loads. *Achiev. J. Sci. Res. 3(2), 28-46.*

Koloušek, V (1961). Dynamics of civil engineering structures—Part II—Continuous beams and frame systems. *Prague (in Czech)*.

Mehmet, A (2014). Free vibration analysis of Beams Considering Different Geometric Characteristics and Boundary Conditions. *Inter. J. Mech. Applicat. 4(3), 94-100.*

Ogunbamike, OK (2012). Response of Timoshenko beams on winkler foundation subjected to dynamic load. *Inter. J. Sci. Tech Res*. 1(8), 48-52.

Oni, ST; Awodola, TO (2010). Dynamic response under a moving load of an elastically supported non-prismatic Bernoulli-Euler beam on variable elastic foundation. *Lat. Am. J. Solids. Struct.* 3-20.

Oni, ST; Awodola, TO (2011). Dynamic behaviour under moving concentrated masses of simply supported rectangular plates resting on variable Winkler elastic foundation. *Lat. Am. J. Solids. Struct. 8*, 373-392.

Oni, ST; Omolofe, B (2011). Dynamic response of prestressed Rayleigh beam resting on elastic foundation and subjected to masses traveling at varying velocity.

Sadiku, S; Leipholz, HE (1987). On the dynamics of elastic systems with moving concentrated masses. *Ingenieur-archiv*. 57(3), 223-242.

Savin, E (2001). Dynamics amplification factor and response spectrum for the evaluation of vibrations of beam under successive moving loads. *J. Sound. Vibe*. 248(2), 267-288.

Usman, MA (2019). Series solution of Euler-Bernoulli beam subjected to concentrated load. *Is. Univ. Multidis. J.* 6(4), 178-186.