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Impact of Variable Thermal Conductivity and Viscosity on Powell-Eyring Fluid in the Presence Of Thermal Radiation through a Porous Medium

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ABSTRACT: This work uses a mathematical model to investigate the impact of physical factors on the non-isothermal flow of Powell-Eyring fluid with variations in thermal conductivity and viscosity through a porous medium. The governing equations defining the flow, mass, and energy transfer issue are converted into nonlinear ordinary differential equations via selected transformation variables, and the resultant problem is numerically solved using the Galerkin weighted residual technique. This approach is implemented with Maple 18 program. The examination of the findings revealed that the radiation parameter, variations of thermal conductivity, and viscosity characteristics had a substantial impact on the flow system. This presentation includes a visual picture and explanation of how different physical characteristics affect the flow system.

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Variable thermal conductivity refers to a material's capacity to modify its ability to transfer heat in reaction to particular external conditions. This can be accomplished in a variety of ways, including adding an electric or magnetic field, changing the material's composition, or modifying its physical state.

Materials having variable thermal conductivity have uses in thermal management, energy storage, and electrical devices. Academics are increasingly studying non-Newtonian fluids in porous media for practical applications (Oyelami and Dada, 2016; Peter *et al.*, 2019; Gbadeyan and Dada, 2013; Ara *et al.*, 2014; Banerjee *et al.*, 2018; Parmar and Jain, 2019). In general, viscosity is determined by a fluid's condition, such as temperature, pressure, and rate of deformation. In certain circumstances, however, the dependency on these qualities is insignificant. Furthermore, thermal radiation is the electromagnetic radiation released by a body according to its temperature.

Previous studies and research overlooked the combined impacts of variable thermal conductivity, variable viscosity, thermal radiation, magnetic field, and viscous dissipation. As a result, this work used the Powell-Eyring model to evaluate the combined impacts of variations of thermal conductivity and viscosity, thermal radiation, magnetic field, and viscous dissipation on the non-isothermal flow of non-Newtonian fluid in a porous medium.

MATERIALS AND METHODS

Mathematical Formulation: Recognized as an incompressible, non-Newtonian viscous, electrically conducting fluid surrounded by two parallel stationary plates. The channel is filled with a saturated porous

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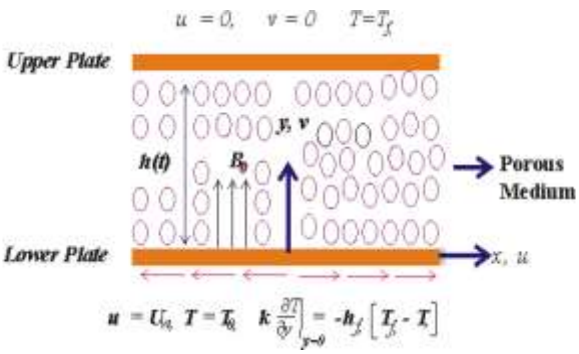
media, with the flow considered to be erratic. The Powell-Eyring model describes the viscous behavior of a non-Newtonian flow, which is responsible for shear. A uniform magnetic field is applied, and the y-axis runs parallel to the plate in that direction. The x-axis moves vertically upward along the plate.



$$A = -pl + \tau \qquad (1)$$

Fig. 1: The flow geometry

Where extra stress tensor τ_{ij} is given by

$$\tau_{ij} = \mu \frac{\partial u_i}{\partial x_j} + \frac{1}{\beta}\sinh^{-1}\left(\frac{1}{B}\frac{\partial u_i}{\partial x_j}\right) \qquad (2)$$

Where $\partial u / \partial x$ is the velocity gradient, μ is the coefficient of dynamic viscosity, β and B characterizes the Powell-Eyring model.

Taking the first and second order approximation of the hyperbolic sine function, the stress tensor for Powell-Eyring model becomes

$$\tau_{ij} = \mu \frac{\partial u_i}{\partial x_j} + \frac{1}{\beta B}\frac{\partial u_i}{\partial x_j} - \frac{1}{6\beta B}\left(\frac{1}{B}\frac{\partial u_i}{\partial x_j}\right)^3 \qquad (3)$$

The governing equations of the flow field and thermal radiation can be stated in dimensional form in accordance with the boundary layer assumptions, as

$$\rho\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu(T)\frac{\partial u}{\partial y} + \frac{1}{\beta B}\frac{\partial u}{\partial y}\right) - \frac{1}{2\beta B^3}\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2} - \frac{\mu_{ef}}{K}u - \sigma {B_0}^2 u \qquad (4)$$

$$\rho C_P\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial y}\left(k(T)\frac{\partial T}{\partial y}\right) - \frac{\partial q_r}{\partial y} - \mu(T)\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\mu_{ef}}{K}u^2 + \sigma {B_0}^2 u^2 + QC_0 AYe^{-\frac{E}{RT}} \qquad (5)$$

with the following boundary conditions:

$$\begin{aligned} &v = 0, \quad T = T_0 \quad \text{at } y = 0 \\ &u = \frac{1}{\rho_0 \upsilon}\frac{\partial \sigma}{\partial T}\frac{\partial T}{\partial y}, \qquad -k\frac{\partial T}{\partial y} = h_F(T_F - T) \qquad \text{at } y = 0 \\ &u = 0, \quad T = T_F, \quad \text{as } y \to \infty \end{aligned} \qquad (6)$$

Where

$u =$ x-direction, $v =$ y-direction, $\rho =$ density, $\mu =$ Newtonian, $\mu_{ef} =$ Effective viscosity, $K =$ Permeability, $k =$ Thermal conductivity, $\sigma =$ Electrical conductivity, ${B_0}^2 =$ magnetic field, $C_P =$ Specific heat, $T =$ Fluid temperature, $Q =$ Heat released due to exothermic reaction, $q_r =$ Radiation heat flux, $A =$ Arrhenius pre-exponential factor, $E =$ Activation energy, $T_0 =$ Initial fluid temperature, $T_F =$ Ambient temperature and $h_F =$ Heat transfer coefficient.

Following Roseland approximation for radiation energy in an optically thick fluid, the radiant heat flux of the fluid is expressed as follows:

$$q_r = -\frac{4\sigma^r}{3k^r}\frac{\partial T^4}{\partial y} \qquad (7)$$

$$\frac{\partial q_r}{\partial y} = \frac{16T_\infty{}^3\sigma^r}{3k^r}\frac{\partial}{\partial y}\left(\frac{\partial T}{\partial y}\right) \qquad (8)$$

The temperature-dependent shifts of viscosity and thermal conductivity can be stated as, respectively:

$$\mu(T) = \mu_0 e^{-\left(\frac{T-T_0}{T_0}\right)} \quad \text{and}$$

$$k(T) = k_0 e^{-\alpha T} \qquad (9)$$

The continuity equation (1) is automatically satisfied by defining a stream function $\Psi(x, y)$ such that

$$u = \frac{\partial\Psi}{\partial y}, \qquad v = -\frac{\partial\Psi}{\partial x} \qquad (10)$$

In order to transform equations (4) and (9) into a set of ordinary differential equations, the following similarity variables are introduced.

$$u' = \frac{u}{v_o},\ y' = \frac{y}{v_o},\ x' = \frac{x}{v_o},\ t' = \frac{U}{v_o}t,\ \theta(\eta) = \frac{T - T_0}{T_F - T_0},\ \varepsilon = \frac{RT_0}{E}, \eta = \sqrt{\frac{U_w}{xv_0}}y,$$

$$v = -\frac{\partial\Psi}{\partial x} = -\sqrt{\frac{av_0}{1-ct}}f(\eta), u = \frac{\partial\Psi}{\partial y} = U_w f'(\eta), U_w = \frac{ax}{1-ct}, S = \frac{c}{2(1-ct)}, \qquad \xi = \frac{1}{2\beta B^3},$$

$$\mathrm{Re} = \frac{\rho U v_0}{\mu},\quad Da = \frac{\rho U}{\mu_{ef}}K,\quad M_0 = \frac{v_0 U_w}{\rho U}\sigma B_0{}^2,\quad \mathrm{Pr} = \frac{\rho C_P U v_0}{k_0},\quad Ec = \frac{Ev_0{}^2}{\rho C_P U R T_0{}^2},$$

$$R_d = \frac{4T_\infty{}^3\sigma^r}{K^r\rho C_P v_0{}^2},\quad n_1 = \frac{\alpha R T_0{}^2}{E}, \psi = \frac{Ev_0 e^{-\frac{E}{RT_0}}}{RT_0{}^2 U\rho C_P}QC_0AY, Ma = \frac{1}{\rho_0\upsilon U_w}\frac{\partial\sigma}{\partial T}\left(\frac{U_w}{xv_0}\right)^{\frac{1}{2}}(T_f - T_0),$$

$$Bi = \frac{h_f}{k\left(\frac{U_w}{xv_0}\right)^{\frac{1}{2}}}. \qquad (11)$$

Where v_o is a reference length scale, U is a reference velocity, θ is a dimensionless temperature, Re is the Reynolds number, M_0 is the magnetic field parameter, Ec is the Eckert number, R_d radiation parameter, n_1 thermal conductivity parameter, Ma Marangoni number, Bi Biot number and ψ is the Frank-Kamenetskii parameter.

In view of equation (11), the equations (4) and (9) and dropping the primes transform into

$$(S\eta - \eta f' - f)f'' = \left(\frac{1}{\mathrm{Re}}\left(e^{-b\theta} + \xi\right) - \xi(f'')^2\right)f''' - \frac{b\theta' e^{-b\theta}}{\mathrm{Re}}f'' - \left(\frac{1}{Da} + M_0\right)f' \qquad (12)$$

$$(S\eta-\eta f'-f)\theta'=\left(\frac{e^{-n_1\theta}}{\Pr}+\frac{4}{3}R_d\right)\theta''-\frac{e^{-n_1\theta}}{\Pr}n_1(\theta')^2-\frac{e^{-\varepsilon\theta}}{\mathrm{Re}}Ec\left(f''\right)^2+\frac{Ec}{Da}\left(f'\right)^2 \qquad (13)$$

$$+EcM_0\left(f'\right)^2+\psi e^{\frac{\theta}{1+\varepsilon\theta}}$$

The following are the transformed boundary conditions:

$$f(\eta)=0,\ \theta(\eta)=0,\ f'(\eta)=Ma\theta',\ \theta'(\eta)=-Bi(1-\theta) \quad \text{at} \quad \eta=0,$$
$$f'(\eta)=0,\ \theta(\eta)=1 \quad \text{as} \quad \eta\to\infty \qquad (14)$$

3 Method of Solution

Let $$f=\sum_{i=0}^{2}A_i e^{\left(-\frac{i}{5}\right)y}, \quad \theta=\sum_{i=0}^{2}B_i e^{\left(-\frac{i}{5}\right)y} \qquad (15)$$

We employ the following parameter values:

$S=0.1, \varepsilon=0.3, \mathrm{Re}=0.5, \xi=\psi=0.7, Da=1.8, M_0=0.8, b=n_1=0.6, \Pr=1.5, R_d=0.4, Ec=1.2$

RESULT AND DISCUSSION

The results are presented in figures 2–13. The impacts of the unsteadiness parameter are displayed in Figures 2–3. It is self-evident that increasing the unsteadiness parameter S causes the decrease in the velocity and temperature profiles.

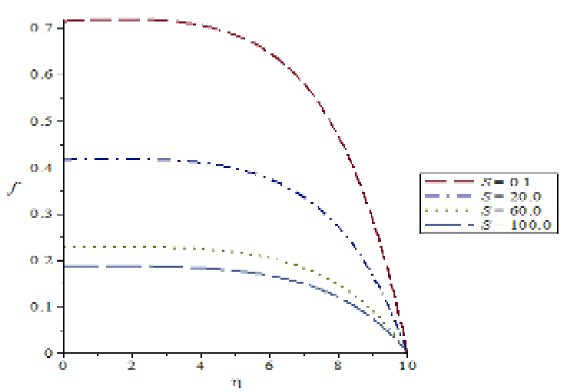


Fig. 2.Velocity profile with S parameter

Figures 4–5 depict various viscosity variation parameter values. It is seen that the temperature and velocity profiles drop when the viscosity variation parameter b increases. Figures 6–7 reveals the Reynolds number Re impacts on the velocity field. It is clear that when Reynolds number rises, temperature and velocity profiles drop. Figure 8 depict the effect of Powell-Eyring parameter. It is observed that when the Powell-Eyring parameter ξ increases, the velocity profile also increases.

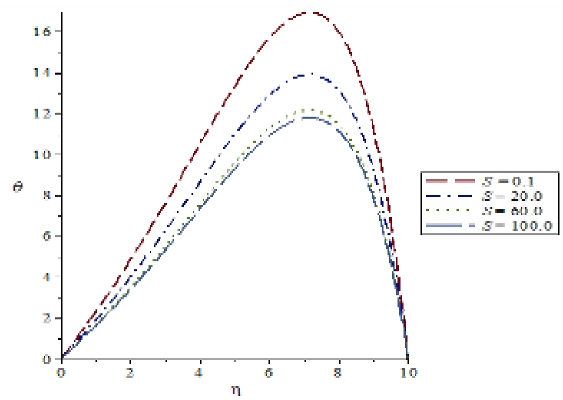


Fig. 3. Temperature profile with S parameter

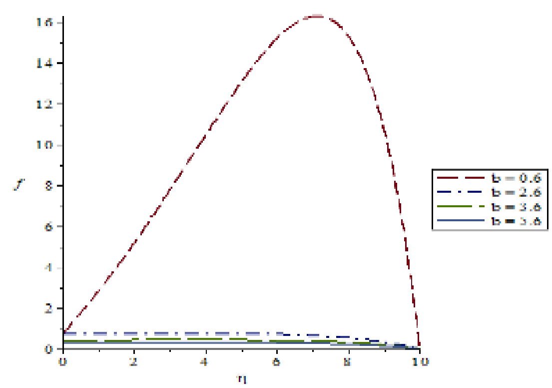


Fig. 4. Velocity profile with b parameter

The velocity profile as a function of a varied thermal conductivity value is shown in Figure 9. A numerical analysis of the Powell-Eyring Fluid in a non-isothermal flow of a porous media has been performed. The results clearly demonstrate the impacts of the unsteadiness parameter, viscosity variation parameter, Reynolds number, Darcy number, magnetic field parameter, thermal conductivity variation parameter, Prandtl number, radiation parameter, Ecket number, and Frank-Kamenetskii parameter on flow, mass, and energy transfer. The following conclusions are made: It is found that raising the variable viscosity parameter causes a drop in the velocity and temperature profiles.

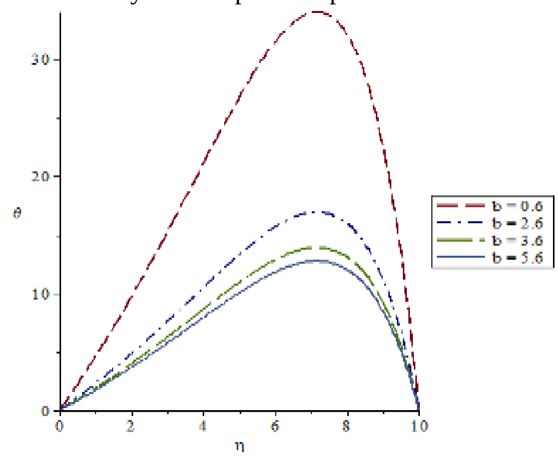


Fig. 5. Temperature profile with b parameter

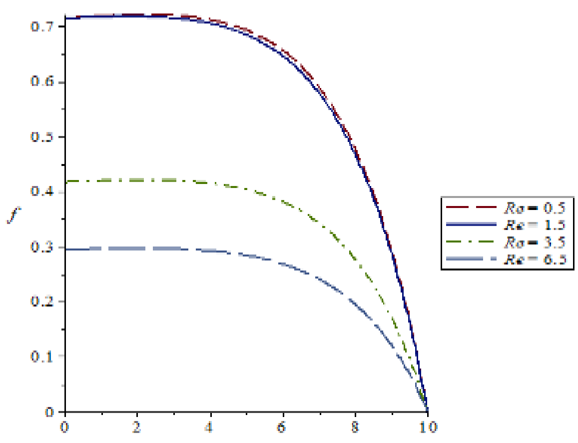


Fig. 6. Reynolds number Re on velocity profile

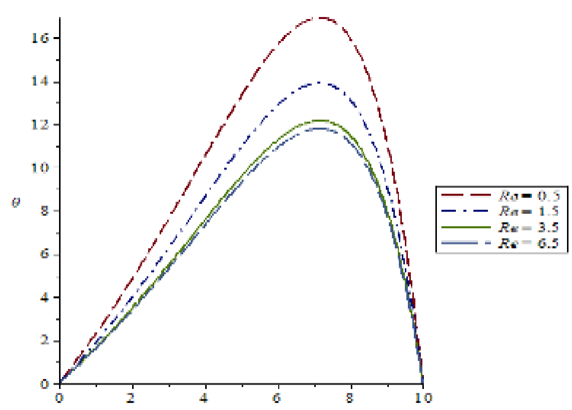


Fig. 7 Reynolds number Re on velocity profile

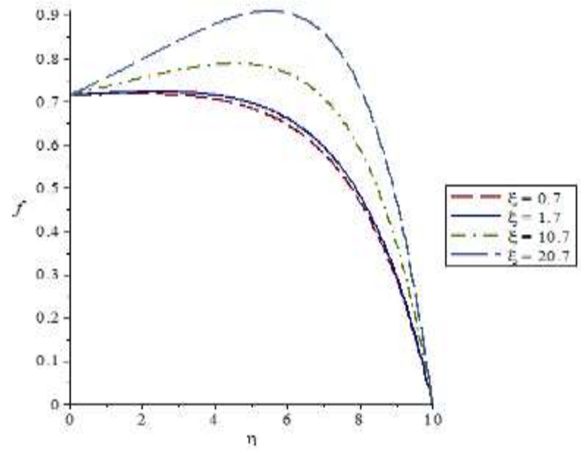


Fig.8. Powell-Eyring parameter ξ on velocity profile

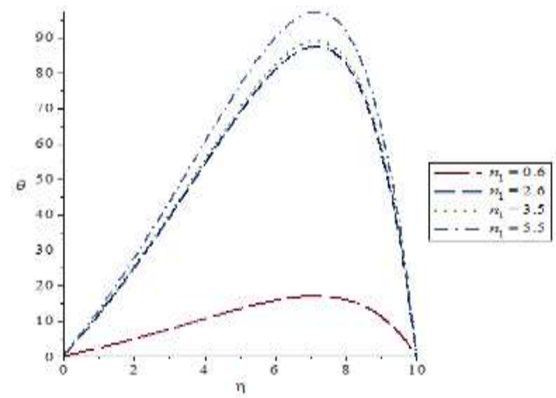


Fig. 9. Parameter n_1 on temperature profile

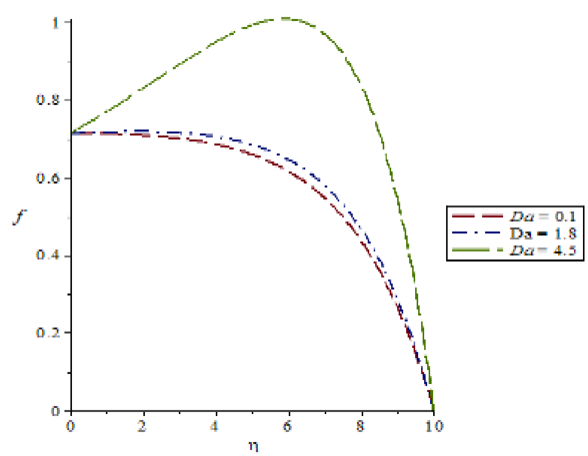


Fig. 10. Darcy number Da on temperature profile

The temperature profile appears to grow as the variable thermal conductivity parameter increases. As the Prandtl number and radiation parameter grow, so do the temperature profiles. It is clear that increasing the Ecket number improves the temperature profile while increasing the Frank-Kamenetskii parameter reduces it.

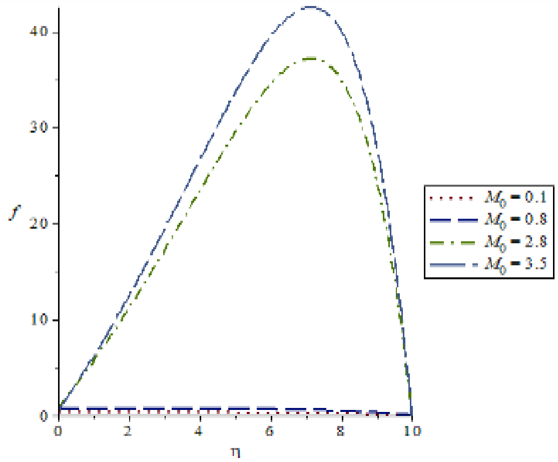


Fig. 11. magnetic field parameter M_0 on velocity profile

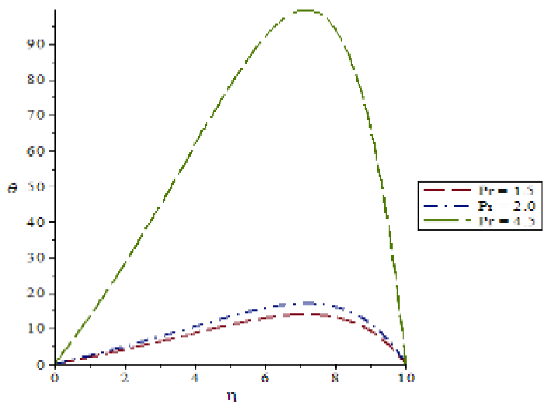


Fig. 12. Prandtl number $\Pr$ on temperature profile

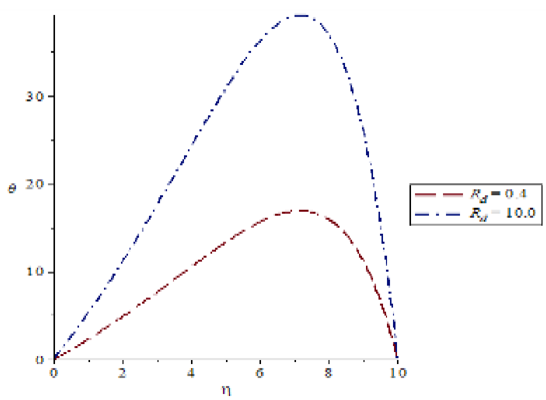


Fig.13. R_d on temperature profile

Conclusion: This work has several practical applications, including petroleum drilling, food processing, polymer and slurry manufacture. More crucially, the flow model of the problem illustrates the oil well and the rapid recovery of oil from it. This topic has significant implications for production, vehicle engines, safety, and material handling.

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