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Analysis of Covariance of Models of Sudoku Squares with no Treatment Effects on Concomitant Variable

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ABSTRACT: Inclusion of concomitant variable(s) in an analysis of variance (ANOVA) model is an indication that the model is of ANCOVA model provided that there is correlation between the concomitant and response variable. This study employ numerical illustrations of the analysis of covariance of models of Sudoku square with no treatment effects on concomitant variable- Result of the illustration, showed that error variance from the ANCOVA Sudoku square models reduced 12.7761 to 5.5152 for model I; 13.6898 to 6.4690 for model II; 15.7926 to 4.8160 for model III and 16.5152 to 4.9226 for model IV respectively in which the concomitant variable had justified its inclusion in the models. The results showed that adjusted treatment effect is similar across the four ANCOVA models, the correlation coefficient for each of the ANCOVA model is highly positively and the error mean square obtained for ANCOVA models are less than the values of error mean square obtained for ANOVA models of Sudoku square.

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In a designed experiment the tendency to detect difference among treatments increases as the size of the experimental errors reduces, a well-planned and good experiment seeks to make sure that all possibilities in minimizing error is put in place. Apart from experimentation, a good data analysis equally helps in controlling experimental error. However, where blocking alone may not accomplished the full control of experimental error, the choice of data analysis may help greatly. The inclusion of one or more concomitant variable whose relationship with the response variable are known. Analysis of covariance can reduce the variability among experimental units by adjusting their values to a common value of the concomitant variable. In ANCOVA, it is a requirement that the relationship of the response variable with concomitant variable is known earlier to avoid using wrong data analysis. If for instance, there is no correlation between the concomitant variable and the response, there is no reason using ANCOVA method. Cox and McCullagh (1987) gave six different ways analysis of covariance (ANCOVA) can be meaningfully explained. These meanings have different situation where the analysis could be applied and useful. One of such that ANCOVA as the sum of squares of dependent variable y is partitioned into components, the presence of decomposition of the sum of products of two variables y and x. Though, these variable could be extended to more than two variables. ANCOVA as a numerical method of improving the precision of a comparative experiment by adjusting concomitant variable

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measured before application of treatments. Fisher (1934) stated that ANCOVA ‘’ combine the advantages and reconcile the requirements of regression and analysis of variance (ANOVA). Cochran (1957) stated that the use of ANCOVA in an agricultural experiment as a way of increasing precision and also identified some challenges and misuse of applications involved such as treatment effect affecting or having interaction with the concomitant variable to a certain level, covariance adjustment resulted on a different meanings which no longer removing the component of experimental error, this situation distorts the nature of treatment effect that is being measured (Tracz *et al.*, 2005; Nimon and Henson, 2010). Grün and Leisch (2008) proposed the concomitant variable models for the component weights that allow to allocate the data into the mixture components through other variables called concomitant. This extension can provide both more precise parameter estimates and better components identification. Several researchers have presented their works on Sudoku square design, authors like, Lorch, (2009), Subramani and Ponnuswamy (2009), Subramani, (2012), Ramon *et al.* (2012), Mahdian and Mahmoodian (2015), Danbaba and Dauran (2016), Shehu and Danbaba (2018), Shehu and Danbaba (2018a) and Shehu *et al.* (2023). This study extends the work of Shehu and Danbaba (2018), the work showed the inclusion of concomitant variable into each of the four Sudoku square models, method of least square was used to derive the estimators for sum of squares and cross products. The extension of the paper will be in area of analytical procedures of obtaining the least square estimates of the sums of squares and products, test of significance of the adjusted treatment effect, obtaining error mean squares for ANCOVA and ANOVA Sudoku square models, comparison of the two error mean squares and correlation coefficients between the concomitant variable and dependent variable. Hence, the objective of this paper is to employ numerical illustration of the analysis of covariance of models of Sudoku squares with no treatment effects on concomitant variable.

MATERIAL AND METHOD

We review the ANCOVA Sudoku square models proposed by Shehu and Danbaba (2018) as follows.

ANCOVA Sudoku Square Model I

$$Y_{ij(k,l,p,q)} = \mu + \alpha_i + \beta_j + \tau_k + C_p + \gamma_l + s_q + \beta^*\left(x_{ij(k,l,p,q)} - \bar{x}_{...}\right) + e_{i,j(k,l,p,q)} \quad (1)$$

where $i,j = 1 \cdots m \quad k,l,p,q = 1 \cdots m^2$

ANCOVA Sudoku Square Model II

$$Y_{ij(k,l,p,q)} = \mu + \alpha_i + \beta_j + \tau_k + \gamma(\alpha)_{l(i)} + C(\beta)_{p(j)} + s_q + \beta^*\left(x_{ij()} - \bar{x}_{...}\right) + e_{i,j(k,l,p,q)} \quad (2)$$

where $i,j\, l,p = 1 \cdots m \quad k,q = 1 \cdots m^2$

ANCOVA Sudoku Square Model III

$$Y_{ij(k,l,p,q,r)} = \mu + \alpha_i + \beta_j + \tau_k + \gamma_l + c_p + s(\alpha)_{q(i)} + \pi(\beta)_{r(j)} + \beta^*\left(x_{ij()} - \bar{x}_{...}\right) + e_{i,j(k,l,p,q,r)} \quad (3)$$

where $i,j,q,r = 1 \cdots m \quad k,l,p = 1 \cdots m^2$

ANCOVA Sudoku Square Model IV

$$Y_{ij(k,l,p,q,r)} = \mu + \alpha_i + \beta_j + \tau_k + \gamma(\alpha)_{l(i)} + c(\beta)_{p(j)} + s(\alpha)_{q(i)} + \pi(\beta)_{r(j)} + \beta^*\left(x_{ij()} - \bar{x}_{...}\right) + e_{i,j(k,l,p,q,r)} \quad (4)$$

Where $i,j,l,p,q,r = 1 \cdots m \;\; and \;\; k = 1 \cdots m^2$

$\mu =$ general mean
$\alpha_i = ith$ Row block effect
$\beta_j = jth$ Column block effect
$\tau_k = kth$ treatment effect
s_q =qth square effect
$\gamma(\alpha)_{l(i)} = l$th Row effect nested in ith row block effect
$c(\beta)_{p(j)}=$ pth Column effect nested in jth column block effect
$s(\alpha)_{q(i)} = qth$ Horizontal square effect nested in ith Row block effect
$\pi(\beta)_{r(j)} = rth$ vertical square effect nested in the jth column block effect
$\beta^* =$ is the slope of the regression
$e_{i,j(k,l,p,q,r)}$ = is the error component assumed to have mean zero and constant variance σ^2.
$x_{ij(k,l,p,q,r)}$ is the concomitant variable
$\bar{x}_{...}$ is the overall mean of the concomitant variable (see Shehu and Danbaba, 2018 for details)

In the study, we assume that e_{ijk} is normally identically, distributed with zero mean and constant variance σ^2, we also assume that there is linear relationship between y_{ijk} and x_{ijk} and regression coefficient for each treatment are identical, the treatment effect sum to zero and the concomitant variable is not affected by the treatment.

The sum of squares and cross-products for various effects have be derived for each of the proposed ANCOVA Sudoku square models (see Shehu and Danbaba, 2018 for details), analysis of covariance will be carried out and test of hypothesis will also be carried out on each of the models.

Test of Significance: we test the null hypothesis $H_0\colon \tau_0 = 0$ and compute

$$F_o = \frac{SS_r - SS_f}{\text{degree of freedom of difference}} \Big/ \left(\frac{SS_f}{\text{degree of freedom of full model}}\right) \quad (5)$$

we test the hypothesis of no regression coefficient : $H_0: \beta_0 = 0$ and compute the statistics

$$F_0 = ((Q_{xy})^2 / Q_{xy}) / MS$$

If $F_0 > F_{\alpha, degree\ of\ freedom\ of\ difference, degree\ of\ freedom\ of\ full\ model}$ we reject the null hypothesis.

$$\hat{\sigma}^2_{ANOVA} = MSEror_{ANOVA} \quad (6)$$

(Shehu and Dauran, 2020, Shehu and Danababa, 2018)

Equation (6) is modified for the ANCOVA models for Sudoku squares as follows

$$\hat{\sigma}^2_{ANCOVA} = MSEror_{ANCOVASMD} \quad (7)$$

Correlation Coefficient Between Response and Concomitant Variables: The ANOVA models equation will also be used, the reason for the ANOVA is that, there exist relationship between its ANCOVA which was described by Maxwell andDelancy (2004); Rencher and Schaalje (2007). To do this, there is need to remove the effect of regression from the models (1-4), therefore the relationship that exists between ANOVA and ANCOVA can be expressed as follows.

$$\rho = \pm\sqrt{1 - \frac{\hat{\sigma}^2_{ANCOVA}}{\hat{\sigma}^2_{ANOVA}}} \quad (8)$$

ρ is the population correlation coefficent between the response and the covariate, which satisfy the inequality $-1 \le \rho \le +1$, $\hat{\sigma}^2_{ANOVA}$ is the error variance estimate from ANOVA, $\hat{\sigma}^2_{ANCOVA}$ is the error variance estimate from ANCOVA. The value of ρ gives amount of linear relationship that exists between response $y_{ij(klpq)}$ and concomitant variable $x_{ij(klpq)}$.

Equation (9) is the fitted value of y, which can be estimated using the estimates of various terms in the equation. The value of $\hat{y}$ is important in the estimation of error.

$$\hat{y}_{i,j(k,l,p,q)} = \bar{y}_{i\cdots} + \bar{y}_{.j(.)} + \bar{y}_{..(k)} + \bar{y}_{l..} + \bar{y}_{p..} + \bar{y}_{q\ldots} - 5\bar{y}_{\ldots}$$

$$-\frac{Q_{xy}}{Q_{xx}}\left[x_{ij(k,l,p,q)} + \bar{x}_{i..} + \bar{x}_{.j.} + \bar{x}_{k..} + \bar{x}_{l..} + \bar{x}_{p..} + \bar{x}_{q..} - 7\bar{x}_{\ldots}\right] \quad (9)$$

Estimated residual $\hat{e}_{i,j(k,l,p,q)} = y_{ij(k,l,p,q)} - \hat{y}_{i,j(k,l,p,q)}$. (10)

$y_{ij(k,l,p,q)}$ can be obtained from the table 1

Data for the Numerical Illustration: The Table 1 is a hypothetical data, y component of the data was adapted from the work of Subramani and Ponnuswamy (2009) while x component was improvised. The data are used for illustration, which illustrates analytical procedures of analysis of covariance of Sudoku square design for the four proposed ANCOVA models. The numerical values of sum of squares and cross products were obtained. The values in each of the cells in the table 1 is of the form (x, y) x represents concomitant variable while y represents response variable.

Table 1: Hypothetical data for the analysis of covariance

A (4,15)	B (5,11)	C (3,16)	D (6,21)	E (5,22)	F (7,21)	G(6,14)	H(6,19)	I (5,15)
D (7,18)	E (9,23)	F (6,20)	G (8,18)	B(7,18)	I (6,23)	A (6,1)	B (5,22)	C (7,15)
G (5,15)	H (3,10)	I (7,22)	A (4,15)	B(7,18)	C (8,19)	D(9,24)	E (2,15)	F (6,16)
B (5,17)	C (6,23)	A (8, 18)	E (3,15)	F (6,16)	D(10,1)	H(5,15)	I (5,12)	G (8,16)
E (6,20)	F (9,25)	D (5,12)	H(9,26)	I (6,21)	G(3 ,17)	B(10,18)	C(7,25)	A (5,19)
H (4,21)	I (5,17)	G (6,18)	B (8,22)	C (5,16)	A (7,23)	E (5,22)	F (6,19)	D (7,25)
C (3,12)	A(6,25)	B(7,20)	F(2,19)	D(4,22)	E(5,16)	I(8,19)	G(3,13)	H(5,18)
F(10,21)	D(9,22)	E10,13)	I(6,22)	G(6,21)	H(9,22)	C(4,20)	A(6,19)	B(7,14)
I(4,14)	G(6,17)	H(2,11)	C(7,16)	A(9,18)	B(10,23)	F(5,15)	D(9,23)	E(7,16)

RESULTS AND DISCUSSION

The first thing to do before the analysis using ANCOVA method is to do the scatter plot of the concomitant variable x and response variable y , this is just to have information about relationship between the variables. Figure 1 show that the variables are correlated, that is, there is a relationship between the concomitant variable and response variable. We can go ahead with the ANCOVA method of analysis.

*Analytical Procedure of obtaining sum of squares***:** Least square estimators of the models (equations 1-4) and their respective estimates are obtained as follows (See Shehu and Danbaba, 2018 for details). From the table 1 number of rowblock is equal to number of columnblock that is, m = 3.

Sums of Squares for y for various sources of variations are obtained as follow

$$V_{yy} = \sum_{i=1}^{m^2}\sum_{j=1}^{m^2} y_{i,jk,l,p,q)}^2 - \frac{y_{....}^2}{m^4} = 1150$$

$$U_{yy}^{i} = \frac{1}{m^3}\sum_{i=1}^{3} y_{i..}^2 - \frac{y_{....}^2}{m^4} = 25.4074$$

$$U_{yy}^{j} = \frac{1}{m^3}\sum_{j=1}^{3} y_{.j.}^2 - \frac{y_{....}^2}{m^4} = 69.5556$$

$$U_{yy}^{k} = \frac{1}{m^2}\textstyle\sum_{j=1}^{9} y_{..k}^2 - \frac{y_{....}^2}{m^4} = 105.778$$

$$U_{yy}^{l} = \frac{1}{m^2}\sum_{j=1}^{9} y_{..l}^2 - \frac{y_{....}^2}{m^4} = 152.4444$$

$$U_{yy}^{p} = \frac{1}{m^2}\sum_{p=1}^{9} y_{..p}^2 - \frac{y_{....}^2}{m^4} = 123.5556$$

$$U_{yy}^{q} = \frac{1}{m^2}\sum_{q=1}^{9} y_{..q}^2 - \frac{y_{....}^2}{m^4} = 111.1111$$

$$U_{yy} = U_{yy}^{i} + U_{yy}^{j} + U_{yy}^{k} + U_{yy}^{l} + U_{yy}^{p} + U_{yy}^{q}$$
$$25.4074 + 69.5556 + 105.778 + 152.4444 + 123.5556 + 111.1111 = 587.8521$$

$$Q_{yy} = V_{yy} - U_{yy} = 1150 - 587.8521 = 562.1479$$

Sums of Squares for x for various source of variations are obtained as follow

$$V_{xx} = \sum_{i=1}^{m^2}\sum_{j=1}^{m^2} x_{i,jk,l,p,q)}^2 - \frac{x_{....}^2}{m^4} = 330.7654$$

$$U_{xx}^{i} = \frac{1}{m^3}\sum_{i=1}^{m} x_{i..}^2 - \frac{x_{....}^2}{m^4} = 2.9876$$

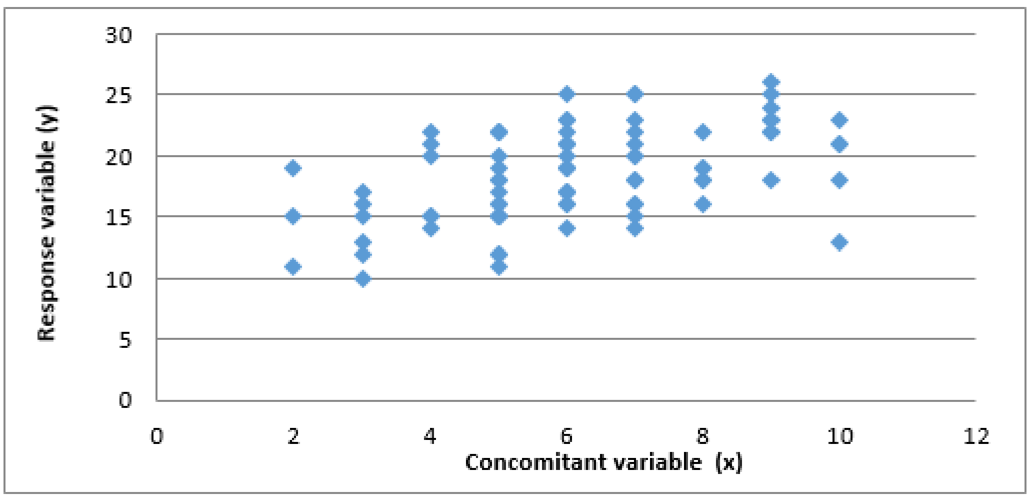

Fig 1: Scattered plot of concomitant variable x and response variable y

$$U_{xx}^{j} = \frac{1}{m^3}\sum_{j=1}^{m} x_{i..}^2 - \frac{x_{....}^2}{m^4} = 1.9506$$

$$U_{xx}^{k} = \frac{1}{m^2}\sum_{k=1}^{m^2} x_{k..}^2 - \frac{x_{....}^2}{m^4} = 28.3209$$

$$U_{xx}^{l} = \frac{1}{m^2}\sum_{l=1}^{m^2} x_{l..}^2 - \frac{x_{....}^2}{m^4} = 48.7651$$

$$U_{xx}^{p} = \frac{1}{m^2}\sum_{q=1}^{m^2} x_{p..}^2 - \frac{x_{....}^2}{m^4} = 23.8762$$

$$U_{xx}^{q} = \frac{1}{m^2}\sum_{q=1}^{m^2} x_{q..}^2 - \frac{x_{....}^2}{m^4} = 6.9876$$

$$U_{xx} = U_{xx}^{i} + U_{xx}^{j} + U_{xx}^{k} + U_{xx}^{l} + U_{xx}^{p} + U_{xx}^{q}$$
$$2.9876 + 1.9506 + 28.3209 + 48.7651 + 23.8762 + 6.9876 = 112.8881$$

$$Q_{xx} = V_{xx} - U_{xx} = 330.7654 - 112.8881 = 217.8775$$

Sums of cross Products between x *and* y

$$V_{xy} = \sum_{i=1}^{m^2}\sum_{j=1}^{m^2} x_{ij}y_{i,j} - \frac{x_{...}y_{...}}{m^4} = 338.5556$$

$$U_{xy}^{i} = \frac{1}{m^3}\sum_{j=1}^{3} x_{i..}y_{i..} - \frac{x_{...}y_{...}}{m^4} = 5.7041$$

$$U_{xy}^{j} = \frac{1}{m^3}\sum_{j=1}^{3} x_{.j.}y_{.j.} - \frac{x_{...}y_{...}}{m^4} = 11.5186$$

$$U_{xy}^{k} = \frac{1}{m^2}\sum_{k=1}^{m^2} x_{k..}y_{k..} - \frac{x_{...}y_{...}}{m^4} = 49.6667$$
$$U_{xy}^{l} = \frac{1}{m^2}\sum_{l=1}^{m^2} x_{l..}y_{l..} - \frac{x_{...}y_{...}}{m^4} = 33.3333$$
$$U_{xy}^{p} = \frac{1}{m^2}\sum_{l=1}^{m^2} x_{p..}y_{p..} - \frac{x_{...}y_{...}}{m^4} = 26.00$$
$$U_{xy}^{q} = \frac{1}{m^2}\sum_{l=1}^{m^2} x_{q..}y_{q..} - \frac{x_{...}y_{...}}{m^4} = 19.5556$$
$$U_{xy} = U_{xy}^{i} + U_{xy}^{j} + U_{xy}^{k} + U_{xy}^{l} + U_{xy}^{p} + U_{xy}^{q}$$
$$= 5.7041 + 11.5186 + 49.6667 + 33.3333 + 26.0000 + 19.5556 = 145.7783$$
$$Q_{xy} = V_{xy} - U_{xy}$$
$$338.5556 - 145.7783 = 192.777$$

Estimation of sum of squares and products for the reduced ANCOVA model I is as follows

$$U_{yy}^{1} = 25.4074 + 69.5556 + 152.44444 + 123.5556 + 1111.1111 = 482.0741$$
$$U_{xy}^{1} = 5.7041 + 11.51856 + 33.3333 + 26.00 + 19.5556 = 96.1116$$
$$U_{xx}^{1} = 2.9876 + 1.9506 + 48.7651 + 23.8762 + 6.9876 = 84.5671$$

The regression coeffient for the model I can be obtained as follows

$$\hat{\beta}^{*} = \frac{Q_{xy}}{Q_{xx}} = \frac{192.777}{217.8775} = 0.8848$$

The estimated residual of the model can be obtained for each observation in the data set given and can be used as diagnostic checking of the covariance model. From equation (10), we have the estimated residual

$$\hat{e}_{i,j(k,l,p,q)} = y_{ij(k,l,p,q)} - \hat{y}_{i,j(k,l,p,q)}.$$

From table 1, we have

$$y_{1,1(1)} = 15 \quad x_{1,1(1)} = 4$$
$$\hat{e}_{i,j(k,l,p,q)} = 15 - \left[\frac{153}{9} + \frac{154}{9} + \frac{150}{9} + \frac{149}{9} + \frac{476}{27} + \frac{484}{27} - \frac{5(1494)}{81} - 0.8848(4 + \frac{161}{27} + \frac{158}{27} + \frac{47}{9} + \frac{48}{9} + \frac{50}{9} + \frac{46}{9} - 7(\frac{496}{81})\right]$$
$$= 15 - 15.5878 = -0.5878$$

The error sum of square for model I is given as

$$SSE = Q_{yy} - \frac{Q_{xy}^2}{Q_{xx}}$$
$$= 562.1479 - \frac{(192.7773)^2}{217.8775} = 391.5791$$

With $m^2(m^2-1) - 1 = 3^2(3^2-1) - 1 = 71$ degrees of freedom.

The experimental error variance is estimated as

$$MSE = \frac{SSE}{m^2(m^2-1)-1} = \frac{391.5791}{71} = 5.5152$$

We find, the Sum of square error for the reducecd model equation

$$SSe = V_{yy} - U_{yy}^{1} - \frac{(V_{xy} - U_{xy}^{1})^2}{V_{xx} - U_{xx}^{1}}$$
$$= 1150 - 482.0741 - \frac{(338.5556 - 96.1116)^2}{330.7654 - 84.5671} = 429.179$$

With $(m^4 - 2) = (3^4 - 2) = 79$ degrees of freedom

The appropriate sum of square treatment

$$TSS = SSe - SSE = 429.179 - 391.5791 = 37.5999$$

With the $m^2 - 1 = 3^2 - 1 = 8$ degrees of freedom.

To test the hypothesi$H_0: \tau_1 = \tau_2 = \cdots = \tau_k = 0$

$$F_0 = \frac{TSS/(m^2-1)}{MSE} = \frac{37.5999/(3^2-1)}{5.5152} = 0.8522$$

Comparing this to $F_{0.05,8,71} = 2.0717$ from F-statistical table.

Therefore we have no cause to reject the null hypothesis as $F_0 < F_{0.05,8,71}$

To test the hypothesi $H_0: \beta^{*} = 0$

$$F_0 = \frac{(Q_{xy})^2/Q_{xx}}{MSE} = \frac{(192.7773)^2/217.8775}{5.5152} = 30.94$$

Comparing this to $F_{0.05,1,71} = 3.9758$ from F-statistical table as $F_0 > F_{0.05,1,71}$
We reject the hypothesis that $\beta^{*} = 0$

This can be summarized in the table 2,

Table 2: ANCOVA of Sudoku Square design model I of order 9

Source of Variation	sum of square	degrees of freedom	mean square	F_0
Regression	170.6472	1	170.6472	30.94
Treatments	37.5999	8	4.6999	0.8522
Error	391.5791	71	5.5152	
Total		80		

Table 2 revealed that regression coefficient is significant at $\alpha = 0.05$ while adjusted treatment effect is not significant at $\alpha = 0.05$. The same procedure is repeated for models II-IV and the summary are presented in the tables 3-5

Table 3: ANCOVA of Sudoku Square design model II of order 9

Source of Variation	sum of square	degrees of freedom	mean square	F_0
Regression	197.8080	1	197.808	30. 5778
Treatments	52.1448	8	6.5181	1.0076
Error	459.3021	71	6.4690	
Total		80		

Table 3 revealed that regression coefficient is significant at $\alpha = 0.05$ while adjusted treatment effect is not significant at $\alpha = 0.05$

Table 4: ANCOVA of Sudoku square design model III of order 9

Source of Variation	sum of square	degrees of freedom	mean square	F_0
Regression	204.066	1	204.066	42.3725
Treatments	77.4763	8	9.6845	2.0109
Error	341.9337	71	4.8160	
Total		80		

Table 4 revealed that regression coefficient is significant at alpha level of 0.05 while adjusted treatment effect is not significant

Table 5: ANCOVA of Sudoku square design model IV of order 9

Source of Variation	sum of square	degrees of freedom	mean square	F_0
Regression	195.2327	1	195.2327	39.6604
Treatments	35.5782	8	4.4473	0.90344
Error	349.5069	71	4.9226	
Total		80		

Table 5 revealed that regression coefficient is significant at alpha level of 0.05 while adjusted treatment effect is not significant

Correlation Coefficient of Sudoku Square models: The error variances for the ANCOVA Sudoku models I-IV have been obtained from the Tables 2-5 and error variances for ANOVA Sudoku models Type I- IV were obtained in Subramani and Ponnuswamy (2009). Using equation (8) we obtain the correlation coefficient for the Sudoku square design models and which reveals association that exist between ANOVA Sudoku model and ANCOVA Sudoku model as shown in Table 6.

Table 6: Correlation Coefficient between response variable and concomitant variable of ANCOVA Sudoku Square design models

Model	$\hat{\sigma}^2_{ANCOVA}$	$\hat{\sigma}^2_{ANOVA}$	ρ
I	5.5152	12.7761	0.7539
II	6.4690	13.6898	0.7262
III	4.8160	15.7926	0.8337
IV	4.9226	16.5152	0.8378

From the table 6 coefficient of correlation is positive across the four models, that is, the correlation between dependent variable $y_{i,j(lkpq)}$ and concomitant variable $x_{i,j(lkpq)}$ for the models are positively correlated it is an indication of good concomitant variables used for the study and also this result was confirmed in figure 1. The concomitant variable used was measured not assigned as in the case of using ANCOVA method to estimating missing values. The four ANCOVA Sudoku square models we have used, we assumed, does not include interaction between the treatment and concomitant variable and also, that treatment and concomitant variable are statistically independent, because if the independence assumption is violated, the estimation of some parameters will be affected, especially treatment effect. Maxwell and Delaney (2004) had discussed four cases that lack of independence can occur and they proffered possible solutions. From the result of the illustration, Tables 2-5 showed that error variance from each of the ANCOVA Sudoku Square Models I, II, III and IV were reduced to a certain levels in which the concomitant variable had justified its inclusion in the models. For instance, ANCOVA Sudoku Model I has error variance 5.5152 while its error variance under ANOVA model was 12.7761. The difference between the two variances can be seen as significant. Similarly, error variances for the other ANCOVA Sudoku Models have reduced significantly. The reduction in the error variance for the four ANCOVA Sudoku Models made it possible to obtain the correlation coefficients for the respective models. Had it been error variances from ANCOVA models were less than that of ANOVA models, using the statistic in equation 7, we would have had a complicated case because the value of $\frac{\hat{\sigma}^2_{ANCOVA}}{\hat{\sigma}^2_{ANOVA}}$ would be greater than one. However, the result under the square root or radical would have been negative which eventually ended in a complex value of correlation coefficient ρ in which ρ ought not to be a complex value. The results of correlation coefficients which are shown in Table 6 revealed that coefficients for the respective ANCOVA model are highly positively correlated and none of the coefficients is less than 0.6.

Conclusion: The study revealed the analytical procedures of analysis of covariance of models of Sudoku squares, significance test on treatment effect and coefficient of correlation between the response and concomitant variable for each of the models were carried out. We may conclude that the concomitant variable used was good and has imparted positively to the ANCOVA Sudoku models in terms of experimental error reduction. The estimated mean square error using ANCOVA models is significantly less than estimated mean square error using ANOVA models of Sudoku square.

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