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Discrete-Time Markov Chain Application to Population Growth Control in Lagos Metropolis, Nigeria

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ABSTRACT: The study considered the birth gap with four states, length of breastfeeding with four states and type of contraceptives used with five states, by mothers with at least three conceptions in Lagos Metropolis, Nigeria. Data was obtained from the population using a self-designed and administered questionnaire. Results showed that the steady state probability of birth gap was highest at state 4 which is > 24 months implying that 3 out of five women on the long – run will space their births at more than 2 years which ultimately will lead to lower child birth, improved mother's welfare and healthier children. Also, the distribution of mothers and their switching after each period of childbirth shows that contraceptives are indeed effective in controlling conception. Lastly, the transition probabilities of the states are significantly greater than zero and the state are dependent on each other ($p < 0.05$). The study therefore concluded that increased birth gaps, elongated length of breastfeeding and the use of contraceptives by mothers in Lagos Metropolis when combined will ultimately lead to slowing childbearing and population growth rate thereby serving as strong instruments in population growth control.

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Markov chain had been applied in various areas by researchers to understand and optimize processes, phenomena, and operations as a predictive modelling tool for manpower management (Ezeugwu and Igbinosun, 2020; Ezeugwu and Ologun, 2017), Covid – 19 prediction (Arumugam and Rajathi, 2020), urban growth prediction (Ebrahimipour, *et al.*, 2016), Noodles brand switching among students (Karim, *et al*., 2020), etc. Besenczi, *et al*., (2021) also employed Markov chain theories and applications in analysing traffic flow in Kaggle, Porto. Huillet (2011) applied the Markov chain to model population growth for a rare catastrophic event. Nkemnole and Osunkeye (2016) employed a stochastic model Markov chain in their investigation of the endemic level and (knowledge of) persistent time of endemic diseases in a population going through a hospital record in Lagos State, Nigeria. Twumasi, *et al.* (2019) used the Markov chain to model infectious diseases transmissions in Ghana. Ghosh (2021) also applied the Markov chain model in modeling Moody's credit-rating migration in the international markets for the USA, European, and Emerging markets. Adekunle and Eboigbe (2021) also utilised the Markov chain approach to model the stock prices of the Nigerian oil and gas sector. The problem of population growth is becoming more obvious with the fear of hunger, malnutrition, afforestation, etc. are beginning to cause world leaders to think out of the box. Sub-Saharan Africa has the highest total fertility rate with Nigeria ranking 2nd only preceded by DR Congo which implies that its population will continue to rise (ICF International, 2018). Goujon (2019) observed that the world population will peak in the next century preceded by a probable addition of 2 – 3 billion people who ironically, will be born to presently poverty-stricken nations. Nigeria is currently ranked 7th in the world population though its peak period was not determined. This increase in

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population in the world, sub – Saharan Africa and in Nigeria, in particular, has posed serious challenges of malnourishment resulting from hunger, diseases, pollution, political and social unrest, depletion of earth resources without the possibility of sustainability due to population consumption pressure, expansion of agricultural land and reduction of natural forests and other woody vegetation and extinction of species, and all these if not checked will lead to greater catastrophe in the world (Tong and Qui, 2020; Goujon, 2019; Battin, 2018; Brandt, *et al*., 2017; Estrada, 2017).

MATERIALS AND METHODS

Population of the study: The study population comprises all women in the Lagos Metropolis that have had at least three conceptions/ children over their lifetime. The reason for this inclusion criteria was so that information on the number of months that elapsed between their first and second conception as well as between the second and third conception will be measured. Child spacing was usually measured as the period in months between childbirth and the next conception (Abe, 2014).

Data collection procedure and instrument: The data used in this study were collected using a self-designed questionnaire administered purposively to selected women who met the inclusion criteria. A total of 427 women were given the questionnaire to fill while those who could not read had it filled by the researcher or the assistants recruited for data collection after the contents were well explained to those women. The questionnaires used for data collection considered not only the demographic characteristics of the mothers in section A but section B included questions on the number of children ever born, their dates of birth, how long they were breastfed, the periods in months between child delivery and the next conception, mother's age at marriage and their age at birth of the first child, and section C on contraceptives awareness and use included questionnaires on if mothers ever used contraceptives, types of contraceptives used after birth of the first child and after the birth of the second child and the sources of their contraceptives.

Data analysis techniques: Modelling by Markov chain: The data collected was initially analysed using SPSS 26 to obtain the various frequencies and crosstabulations while further application of the Markov process was explored using the "Markovchain" package in R (Spedicato, *et al.*, 2021; Spedicato, 2021). This study measured four discrete birth gap states: <= 12 months (state 0), 13 – 18 months (state 1), 19 – 24 months (state 2) and > 24 months (state 3). Putting (X_i, i = 0, 1, 2, 3) to signify the birth gap periods at any state for any conception number and breastfeeding periods at any time t, X_i is a stochastic process with states 0, 1, 2, 3. For contraceptive use, five states to measure fives discrete contraceptives methods include Condoms (state 0), injectables (state 1), oral pills (state 2), implants (state 3), and other methods collectively (state 4) with (X_i, i =, 1, 2, 3, 4) to stand for types of contraceptives used. Hence, the first-order time homogenous discrete-time Markov dependency is modeled statistically as according to Eqn. (1) above as:

$$p(X_{n+1} = j | X_n = i, X_{n-1} = i_{n-1}, \dots, X_0 = i_0) = p(i,j) \quad (1)$$

$$= p(i,j) = p(X_{n+1} = j | X_n = i) \qquad (2)$$

And does not depend on the time n for temporary homogenous Markov chains.

Transition probabilities matrix: For a Markov chain, the current state (X_n) has a Markov property which implies that only X_n can predict the next state X_{n+1} while all the previous states, $X_{n-1}, \dots, X_0$ are irrelevant. Hence, given the current state, X_n, the transition probability which was stated in Eqn. (2) above.

The transition probability of moving from state i to state j in one step ($P_{i,j}$) for i, j = 0, 1, …, m,n is denoted in the matrix form as:

$$p(i,j) = \begin{pmatrix} p_{00} & p_{01} & p_{02} & p_{03} \\ p_{10} & p_{11} & p_{12} & p_{1n} \\ \vdots & \vdots & \vdots & \vdots \\ p_{m0} & p_{m1} & p_{m2} & p_{mn} \end{pmatrix} \quad (8)$$

Where

$$\sum_{j=0}^{3} p(i,j) = 1, \qquad i = 0, 1, 2, 3 \qquad (9)$$

Parameters of the Markov Chain: P_{ii}: the probability of remaining in a state I; P_{ij}: the probability of moving from state i to state j, i ≠ j.

NB: the time step to ensure the transition from one state to another is considered for a birth gap as between children born and breastfeeding periods (months).

Model Assumptions: Markov process assumes that the state of an individual depends solely on the state of the person at the previous step. Also, transition probabilities are constant over the period of study and therefore, independent of time. The states are recurrent in nature as any mother could move from one state to another and none of the states is absorbing in nature.

Transition Probabilities Estimation: The probabilities of transiting from one birth gap, breastfeeding gap, and contraceptive type to another and their standard errors in this study are estimated with the maximum likelihood estimation (MLE) technique. The number of women counted in each state for the three measured concepts of the birth gap, breastfeeding gap and contraceptive type are shown in tables 1 and 2 at each period. Note that while the birth gap and breastfeeding gap both have four states, contraceptive type have five states at each period.

Table 1: Number of women at any of the state of birth gap and breastfeeding gap for a period

Groups	**<= 12 months**	**13 – 18 months**	**19 – 24 months**	**> 24 months**
<= 12 months	X_{00}	X_{00}	X_{02}	X_{03}
13 – 18 months	X_{10}	X_{11}	X_{12}	X_{13}
19 – 24 months	X_{20}	X_{21}	X_{22}	X_{23}
> 24 months	X_{30}	X_{31}	X_{32}	X_{33}

For birth gaps and breastfeeding gaps

Table 2: Number of women at any of the states of contraceptive usage for a period

Groups	**Condoms**	**Injectables**	**Oral pills**	**Implants**	**Others**
Condoms	X_{00}	X_{00}	X_{02}	X_{03}	X_{04}
Injectables	X_{10}	X_{11}	X_{12}	X_{13}	X_{14}
Oral pills	X_{20}	X_{21}	X_{22}	X_{23}	X_{24}
Implants	X_{30}	X_{31}	X_{32}	X_{33}	X_{34}
Others	X_{40}	X_{41}	X_{42}	X_{43}	X_{44}

For the contraceptives used

The Markov principle stated that the transition events are independent of another, hence the likelihood of the probability of transition, P_{ij}, assumes a binomial distribution model according to (Twumasi, *et al.*, 2019).

$$\mathcal{L}(P_{ij}|N,x) = \binom{N_i}{x_{ij}} P_{ij}^{x_{ij}} \left(1 - P_{ij}\right)^{N_i - x_{ij}}, x_i = 0, 1 \ldots, \quad (10)$$

Where N_{ij} is the observed number of women transiting from state i to state j and from Eqn. (9)

$$\sum_j P(ij) = 1 \qquad (11)$$

The transition probability matrix translates to a multinomial distribution following from Eqn. (10) and the assumption that the transition probabilities over time are unchanging. Hence, the transition matrix probabilities are given as:

$$\hat{P}_{ij} = \frac{x_{ij}}{\sum_j x_{ij}} = \frac{x_{ij}}{N_i}, \qquad (12)$$

For i, j = 0, 1, 2, …, with standard errors from the sampling distributions of the ML estimates obtained as:

$$\widehat{s.e}(\hat{P}_{ij}) = \sqrt{\frac{\hat{P}_{ij}\,(1 - \hat{P}_{ij})}{N_i}} \qquad (13)$$

Metrics estimation for the concepts: The probability that a woman that is in state moves to the next state for the first time between w -1 and w steps for states i, j = 0, 1, 2, 3, 4 from the transition probabilities is given as:

$$f_{ij}^{(w)} = P(X_{n+w+1} = i, X_{n+w} = j, \ldots, X_{n+1} = 0 |\, X_n = 0)$$
$$= P_{ij}^{w-1} P_{ik} \qquad (14)$$

The expected time to move from state i to j has a closed-form solution computed as:

$$E\left(\tau'_{ij}\right) = \frac{\sum_{w=1}^{\infty} w f_{ij}^{w}}{P_r(i \to j)} = \frac{1}{1 - P_{ii}} \qquad (15)$$

For i, j = 0, 1, 2, …, i ≠ j, where the numerator, $\sum w f_{ij}^{w}$, is the expected value of the first passage from state i to state j and the denominator,

$$P_r(i \to j) = \frac{P_{ij}}{1 - P_{ii}} \qquad (16)$$

is the overall probability or lifetime probability of transitioning from state i to state j (Rotar, 2013; Capasso and Bakstein, 2015; Beichelt, 2016).

Estimation of the P^n Transition Matrix: To determine the nth-step transition probability matrices for each stance, the eigenvalues and the eigenvectors approach were used as proposed by Privault (2013). The P_{ij}^{n}, i, j = 0, 1, 2, … transition probability matrix is estimated for each concept using a decomposition procedure of the diagonalised form that utilises eigenvalues and eigenvectors corresponding to them as follows:

$$P^n = M\Lambda^n M^{-1} \qquad (17)$$

Where M is a 4 x 4 non-singular matrix (X_0, X_1, X_2, X_3) and X_j, (j = 0, 1, 2, 3) is the right eigenvectors which correspond respectively to the eigenvalues λ_j (j = 0, 1, 2, 3). Hence,

$$PX_j = \lambda_i X_j \qquad (18)$$

Where Λ^n is a diagonalised matrix with the diagonal elements given as:

$$\Lambda^n = \begin{pmatrix} \lambda_0^n & 0 & 0 & 0 \\ 0 & \lambda_1^n & 0 & 0 \\ 0 & 0 & \lambda_2^n & 0 \\ 0 & 0 & 0 & \lambda_3^n \end{pmatrix} \qquad (19)$$

Inferences on the Parameter of the Transition Matrix: Given a m state (1, 2, …, m) with the transition probability matrix P = (p_{ij}), i, j = 1, 2, …, m and n_{ij} is the number of observations from state i to state j and the total observation is $\sum_{i,j} n_{ij}$.

$$\sum_j n_{ij} = n_i, \quad \sum_i n_{ij} = n_j, \; i,j = 1, 2, \ldots, m \quad (20)$$

Assuming that p_{ij}, came from a Markov chain with a given transition probability P, testing that p_{ij} is of order zero (0), presupposes that the p_{ij}'s are independent. However, the study assumes a dependency in the categories or states of the Markov chain as that yields higher order models (Toivonen, *et al.*, 2020). Under the maximum likelihood estimation (MLE) procedure, the null hypothesis according to Medhi (2009) and Skuriat-Olechnowska (2005) is stated as:

$$H_0: p_{ij} = p_j, of\ order\ 0$$

The test criterion is:

$$\lambda = \prod_i \prod_j \left(\frac{\hat{p}_j}{\hat{p}_{ij}}\right)^{n_{ij}} \qquad (21)$$

$$where \quad \hat{p}_{ij} = n_j \Big/ \left(\sum_i \sum_j n_{ij}\right) \qquad (22)$$

The statistic under H_0 is given as:

$$-2log\lambda = 2\sum_i \sum_j n_{ij} \log\left(n_{ij} * \left(\sum_i \sum_j n_{ij}\right) \Big/ (n_{i.} n_{.j})\right) (23)$$

Eqn. 23 follows an asymptotic Chi-square distribution with $(m – 1)^2$ degrees of freedom, i.e.

$$-2log\lambda \sim \chi^2_{(m-1)^2} \qquad (24)$$

RESULTS AND DISCUSSION

We have stated clearly that the discrete states of the Markov chain model for birth gaps and length of breastfeeding are <=12 months (state 0), 13 – 18 months (state 1), 19 – 24 months (state 2), and >24 months (state 3) while for the contraceptives use are condoms (state 0), injectables (state 1), oral pills (state 2), implants (state 3) and other types (state 4). If we take X_i, i = 0, 1, 2, 3) for the birth gaps and breastfeeding periods and i = 0, 1, 2, 3, 4 for the contraceptive use represent the number of women at any state at any period t, which satisfies the first-order time-homogeneous Markov dependency from Eqn. (1). Hence, X_i fulfils the Markov chain properties with state spaces S = (0, 1, 2, …).

Table 3: Number of women at any state for birth gap at the end of the period

Groups	**<= 12 months**	**13 – 18 months**	**19 – 24 months**	**> 24 months**	**Total**
<= 12 months	12	8	4	6	30
13 – 18 months	1	25	15	14	55
19 – 24 months	5	14	52	42	113
> 24 months	2	11	44	212	269

Table 4: Number of women at any state for the length of breastfeeding at the end of the period

Groups	**<= 12 months**	**13 – 18 months**	**19 – 24 months**	**> 24 months**	**Total**
<= 12 months	160	38	3	0	201
13 – 18 months	58	139	11	5	213
19 – 24 months	4	12	21	8	45
> 24 months	1	2	12	18	33

Table 5: Number of women at any state for contraceptive usage at the end of the period

Groups	**Condoms**	**Injectables**	**Oral pills**	**Implants**	**Others**	**Total**
Condoms	81	4	6	3	3	97
Injectables	2	49	3	2	1	57
Oral pills	8	6	66	0	4	84
Implants	1	1	1	27	2	32
Others	4	3	3	2	63	75

The observed number of women in the sample that fall within the states delineated is given in tables 3 – 5 for the birth gap, length of breastfeeding, and the type of contraceptives. *Transition Probabilities Estimated:* With the MLE procedure, the transition probabilities with their corresponding standard errors are given in tables 6 – 7 while the transition probability matrices for each of the concepts is obtained and presented respectively. Table 6 presents the probabilities of transiting from state i to state j and their 95% confidence interval generated from the multinomial distribution for both birth gap and length of breastfeeding. Birth gap probabilities reveal that state 3 (> 24 months) has the highest probability of retaining its state with 0.788 implying that 78.8% of the childbearing women in Lagos metropolis who had given above 24 months before conception retained their birth gap while state 0 (<= 12 months) has the least probability of retaining its place with 0.400 showing that 40% of the women who observed 12 months or below before conception maintained that birth gap. Transiting from state 2 (13 – 18 months) to state 3 (19 – 24 months) has the highest probability gain of 0.372 indicating that 37.2% of childbearing mothers switched from waiting for 13 – 18 months to waiting for 19 – 24 months before their next conception while transiting from state 3 (19 – 24 months) to state 0 (<=12 months) has the least probability gain of 0.007 showing that only 0.75 of the women switched to 12 months or below from waiting for 19 – 24 months. Length of breastfeeding shows that state 0 (<=12 months) has the highest probability of returning at 0.796 while state 2 (19 – 24 months) has the least probability of 0.467. This shows that 80% of childbearing women in Lagos metropolis who previously breastfed their babies for 12 months or below stuck to that period, 65.3% of those who breastfed for 13 – 18 months retained the same period, 46.7% of those who breastfed for 19 – 24 months also stuck to the same period while 54.5% of those who breastfed for more than 24 months maintained same period. Transiting from state 3 to state 2 showed the highest switch as 36.4% of women who breastfed for above 24 months previously had switched to 19 – 24 months while none of the women switched from 12 months and below to breastfeeding for above 24 months.

Table 6: ML estimate of the transition probabilities for birth gaps and breastfeeding time of childbearing women in Lagos Metropolis

	Birth gap		Breastfeeding time	
Parameters	**Estimates (SE**	**95% Conf. Int**	**Estimates (SE**	**95% Conf. Int.**
P_{00}	**0.400 (0.089)**	0.225, 0.575	**0.796 (0.074)**	0.652, 0.940
P_{01}	0.267 (0.081)	0.108, 0.425	0.189 (0.071)	0.049, 0.329
P_{02}	0.133 (0.062)	0.012, 0.255	0.015 (0.022)	0, 0.058
P_{03}	0.200 (0.073)	0.057, 0.343	0	0
P_{10}	0.018 (0.018)	0, 0.053	0.272 (0.060)	0.155, 0.390
P_{11}	**0.455 (0.067)**	0.323, 0.586	**0.653 (0.064)**	0.527, 0.778
P_{12}	0.272 (0.060)	0.155, 0.39	0.052 (0.030)	0, 0.110
P_{13}	0.255 (0.059)	0.139, 0.37	0.023 (0.020)	0, 0.063
P_{20}	0.044 (0.019)	0.006, 0.082	0.089 (0.027)	0.036, 0.141
P_{21}	0.124 (0.031)	0.063, 0.185	0.267 (0.042)	0.185, 0.348
P_{22}	**0.460 (0.047)**	0.368, 0.552	**0.467 (0.047)**	0.375, 0.559
P_{23}	0.372 (0.045)	0.283, 0.461	0.177 (0.036)	0.107, 0.248
P_{30}	0.007 (0.005)	0, 0.018	0.030 (0.010)	0.010, 0.051
P_{31}	0.041 (0.012)	0.017, 0.065	0.061 (0.015)	0.032, 0.089
P_{32}	0.164 (0.023)	0.119, 0.208	0.364 (0.029)	0.306, 0.421
P_{33}	**0.788 (0.025)**	0.739, 0.837	**0.545 (0.030)**	0.486, 0.605

Table 7: ML estimate of the transition probabilities for the contraceptive type used by childbearing women in Lagos Metropolis

Parameters	**Estimates (SE**	**95% Conf. Int**	**Parameters**	**Estimates (SE**	**95% Conf. Int.**
P_{00}	**0.835 (0.068)**	0.702, 0.968	P_{23}	0	0
P_{01}	0.041 (0.036)	0, 0.112	P_{24}	0.048 (0.039)	0, 0.124
P_{02}	0.062 (0.044)	0, 0.148	P_{30}	0.031 (0.032)	0, 0.094
P_{03}	0.031 (0.032)	0, 0.093	P_{31}	0.031 (0.032)	0, 0.094
P_{04}	0.031 (0.032)	0, 0.093	P_{32}	0.031 (0.032)	0, 0.094
P_{10}	0.035 (0.034)	0, 0.101	P_{33}	**0.844 (0.066)**	0.714, 0.974
P_{11}	**0.860 (0.063)**	0.735, 0.984	P_{34}	0.063 (0.044)	0, 0.149
P_{12}	0.053 (0.041)	0, 0.133	P_{40}	0.053 (0.041)	0, 0.134
P_{13}	0.035 (0.034)	0, 0.101	P_{41}	0.040 (0.036)	0, 0.110
P_{14}	0.017 (0.024)	0, 0.065	P_{42}	0.040 (0.036)	0, 0.11
P_{20}	0.095 (0.054)	0, 0.200	P_{43}	0.027 (0.029)	0, 0.084
P_{21}	0.071 (0.047)	0, 0.164	P_{44}	**0.840 (0.067)**	0.709, 0.971
P_{22}	**0.788 (0.075)**	0.639, 0.933			

The type of contraceptives used shows that state 1 (injectables) has the highest probability of returning at 0.860 while state 3 (oral pills) has the least probability of 0.788. This shows that 83.5% of women who used condoms previously retained the type, 86% of childbearing women in Lagos metropolis who previously injectables stuck to that type, 78.8% of those who used oral pills maintained the same type, 84.4% of those who used implants retained the type while 84% of those who other types also stuck to those types. Transiting from state 3 (oral pills) to state 2 (injectables) showed the highest switch as 9.5% of women who used oral pills previously had switched to injectables while none of the women switched from injectables to oral pills.

Transition Probability Matrices: The transition probability matrices for each of birth gaps (BG), length of breastfeeding (BFL), and types of contraceptives used (Contr.) are presented in Eqns. 25 – 27.

$$\underline{P}_{BG} = \begin{pmatrix} 0.400 & 0.267 & 0.133 & 0.200 \\ 0.018 & 0.455 & 0.272 & 0.255 \\ 0.044 & 0.124 & 0.460 & 0.372 \\ 0.007 & 0.041 & 0.164 & 0.788 \end{pmatrix} \qquad (25)$$

$$\underline{P}_{BFL} = \begin{pmatrix} 0.796 & 0.189 & 0.015 & 0 \\ 0.272 & 0.653 & 0.052 & 0.023 \\ 0.089 & 0.267 & 0.467 & 0.177 \\ 0.030 & 0.061 & 0.364 & 0.545 \end{pmatrix} \qquad (26)$$

$$\underline{P}_{Contr.} = \begin{pmatrix} 0.835 & 0.041 & 0.062 & 0.031 & 0.031 \\ 0.035 & 0.860 & 0.053 & 0.035 & 0.017 \\ 0.095 & 0.071 & 0.786 & 0 & 0.048 \\ 0.031 & 0.031 & 0.031 & 0.844 & 0.063 \\ 0.053 & 0.040 & 0.040 & 0.027 & 0.840 \end{pmatrix} \quad (27)$$

The transition matrices above reveal that 40% of the women remained with a birth gap of 12 months or below, 26.7% switched to 13 – 18 months birth gap from 12 months or below, 13.3% switched to 19 – 24 months and 20% switched to above 24 months of gap between births. Similarly, while only about 2% of women who previously gave an interval of 13 – 18 months switched to 12 months or below, 45.5% retained the birth gap, 27.2% switched to 19 – 24 months and 25.5% switched from that state to above 24 months. Also, amongst women who gapped 19 – 24 months, only 4.4% switched to 12 months or below, 12.4% switched to 13 – 18 months, 46.0% retained the period and 37.2% switched to above 24 months. Lastly, amongst women who had a birth gap of above 24 months, below 1% switched to 12 months and below, 4.1% switched to 13 – 18 months, 16.4% switched to 19 – 24 months and 78.8% remained with above 24 months of birth gap. The matrices for the length of breastfeeding above reveal that 79.6% of the women remained with 12 months or below, 18.9% switched to 13 – 18 months breastfeeding length from 12 months or below, 1.5% switched to 19 – 24 months and none switched to above 24 months of breastfeeding length. Similarly, while only about 27.2% of women who previously breastfed for 13 – 18 months switched to 12 months or below, 65.3% retained the length of breastfeeding, 5.2% switched to 19 – 24 months and 2.3% switched from that state to above 24 months. Also, amongst women who breastfed for 19 – 24 months, only 8.9% switched to 12 months or below, 26.7% switched to 13 – 18 months, 46.7% retained the period and 17.7% switched to above 24 months. Lastly, amongst women who had a birth gap of above 24 months, below 3% switched to 12 months and below, 6.1% switched to 13 – 18 months, 36.4% switched to 19 – 24 months and 54.5% remained with above 24 months of breastfeeding their babies. The transition matrix for the type of contraceptive used above reveal that 83.5% of the women remained with condom, 4.1% switched to injectables, 6.2% switched to oral pills, 3.1% switched to implants and other types respectively. Similarly, 86% of those who used injectables remained with the type, 3.5% switched to condoms and implants respectively, 5.3% switched to oral pills, and 1.7% to other contraceptives. Also, while 78.6% of those who used oral pills retained it, 9.5%, 7.1%, 0%, and 4.8% switched to condoms, injectables, implants, and other types respectively. In addition, while 84.4% remained with implants, 3.1% switched to condoms, injectables, and oral pills respectively and 6.3% of them switched to others types of contraceptives. Lastly, while 84% of the Lagos Metropolis women continued with other types of contraceptives, 5.3% switched to condoms, 4% switched to injectables and oral pills respectively and 2.7% switched to implants.

Transition Probability Diagrams: The transition probability diagrams of the birth gap, length of breastfeeding, and type of contraceptives used by childbearing women of Lagos Metropolis is shown in figure 1 – 3.

*Estimating the nth – step and the Steady States Probabilities***:** Using the initial probability of p_0 as [1, 0, 0, 0) for both birth gap and length of breastfeeding and [1, 0, 0, 0, 0] for the type of contraceptives used at t = 0 by childbearing women in Lagos Metropolis, the nth=step and steady-state probabilities are given in tables 8 – 10. The estimates of the nth-step state probabilities in table 8 show the various probabilities of remaining in the various states of the birth gaps. It reveals that those that have a birth gap of 12 months or below went from 40% of the women at step 1 to only 2.9% at step 10 and retained that proportion for the foreseeable future. Also, women who gave a period of 13 – 18 months interval between births went from 26.7% at the first step to 11.7% at the 10th step and reached an equilibrium proportion of 11.6%. Furthermore, while women who took 19 – 24 months break in between births increased from 13.3% at step 1 to 25% at the 10 steps and retained the ¼ mark at equilibrium. Lastly, giving a birth gap of above 24 months grew from 20% of the women at step 1 to 60.5% at the steady step.

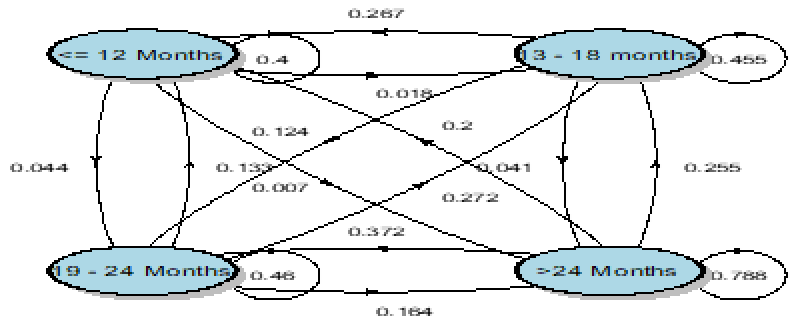

Fig. 1: Transition graph of the probability of the birth gap of childbearing women in Lagos Metropolis, Nigeria drawn with R

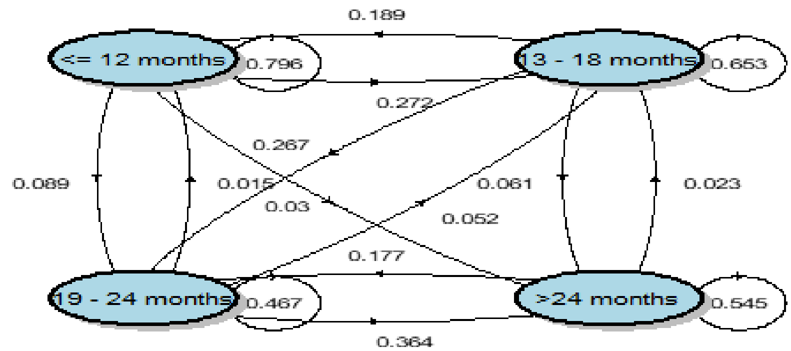

Fig. 2: Transition graph of the probability of the breastfeeding time of childbearing women in Lagos Metropolis, Nigeria drawn with R

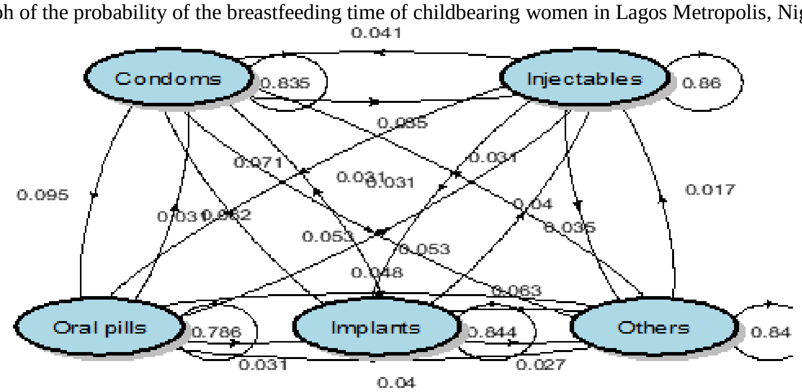

Fig. 3: Transition graph of the probability of the types of contraceptives used by childbearing women in Lagos Metropolis, Nigeria drawn with R

The implication is that with an increase in the number of months taken in between births, more women will give less birth which translates to a reduction in the number of children born and a reduction in the population growth in the metropolis. The estimates of the nth-step state probabilities in table 9 show the various probabilities of remaining in the various states of the length of the period of breastfeeding babies. It reveals that those women that breastfed for 12 months or below dropped from 79.6% at step 1 to only 62.5% at step 10 and to 51.4% for the foreseeable future. Also, women who breastfeed for 13 – 18 months rose from 18.9% at the first step to 35.3% at the 10th step and retained the same equilibrium proportion. Furthermore, breastfed for 19 – 24 months increased from only 1.5% at step 1 to 7.7% at the 10 steps and attained 8.3% at equilibrium. Lastly, women that breastfed for above 24 months grew from 0% at step 1 to 5.0% at the steady step. The implication is that with the increase in the period of breastfeeding, fewer pregnancies will result which translates to a reduction in the number of children born and also a reduction in the population growth in the metropolis. The estimates of the nth-step state probabilities in table 10 show the various probabilities of remaining in the various states of the types of contraceptives used by Lagos metropolis women. It reveals that those who used condoms dropped from 83.5% at the first step to 31.5% at the 10th step and further to slightly below a quarter at the steady-state.

Table 8: The nth-step and steady-state of the probabilities of birth gap

nth – step	<= 12 months	13 – 18 months	19 – 24 months	> 24 months
1	0.400	0.267	0.133	0.200
2	0.172	0.253	0.220	0.355
5	0.039	0.143	0.256	0.562
10	0.029	0.117	0.250	0.604
20	0.029	0.116	0.250	0.605
Steady state (π_j)	0.029	0.116	0.250	0.605

Table 9: The nth-step and steady-state of the probabilities of the length of breastfeeding

nth – step	<= 12 months	13 – 18 months	19 – 24 months	> 24 months
1	0.796	0.189	0.015	0
2	0.686	0.278	0.029	0.007
5	0.565	0.347	0.059	0.029
10	0.525	0.353	0.077	0.045
20	0.510	0.353	0.085	0.052
Steady state (π_j)	0.514	0.353	0.083	0.050

Also, women who used injectables grew from 4.1% at step 1 to 21.4% at the 10th step and peaked at about ¼ at the equilibrium period. In addition, women who used oral pills increased from 6.2% to 18.9% at the 10th step and 18.7% at the steady-state. Lastly, women who used implants also increased from 3.1% at the first step to 13.6% at the steady-state while those that used other contraceptive types grew from 3.1% to 18.3% at the equilibrium step.

Metrics estimates: The expected time and the overall or lifetime probabilities of the concepts – birth gaps, length of breastfeeding, and contraceptives types used were obtained using Eqns. (15) and (16) and presented in tables 11 – 13. Table 11 shows that birth gaps of 12 months or below has an expected time of 1.67 and a recurrence time of 34.7, 13 – 18 months have an expected time of 1.83, and recurrence time of 8.6, 19 – 24 months have an expected time of 1.85 and recurrence time of 4.0 while above 24 moths birth gap have an expected time of 4.72 with a recurrence time of 1.6. The proportions of Lagos metropolis women that will remain or transit from one birth gap to another for a lifetime are also shown. Table 12 shows that length of breastfeeding of 12 months or below has an expected time of 4.90 with a recurrence time of 1.95, 13 – 18 months have an expected time of 2.88 with a recurrence time of 2.8, 19 – 24 months have an expected time of 1.88 and recurrence time of 12.03 while above 24 months birth gap have an expected time of 2.20 with a recurrence time of 19.93. The proportions of Lagos metropolis women that will remain or transit from one length of breastfeeding to another for a lifetime are also shown. Table 13 shows the proportions of Lagos metropolis women that will remain or transit from one contraceptive type to another for a lifetime.

Table 10: The nth-step and steady state of the probabilities of birth gap

nth – step	Condom	Injectables	Oral pills	Implants	Others
1	0.835	0.041	0.062	0.031	0.031
2	0.707	0.076	0.105	0.054	0.058
5	0.470	0.152	0.168	0.096	0.144
10	0.315	0.214	0.189	0.122	0.160
20	0.252	0.245	0.188	0.134	0.181
Steady state (π_j)	0.245	0.249	0.187	0.136	0.183

Table 11: Estimated expected time and overall or lifetime probability of birth gaps

Parameter	<= 12 months	13 – 18 months	19 – 24 months	> 24 months
E(τ)	1.67	1.83	1.85	4.72
Recurrence time	34.66	8.59	4.01	1.65
<= 12 months	0.240	0.160	0.080	0.120
13 – 18 months	0.010	0.248	0.148	0.139
19 – 24 months	0.024	0.067	0.248	0.201
> 24 months	0.001	0.009	0.035	0.167

Table 12: Estimated expected time and overall or lifetime probability of length of breastfeeding

Parameter	<= 12 months	13 – 18 months	19 – 24 months	> 24 months
E(τ)	4.90	2.88	1.88	2.20
Recurrence time	1.95	2.83	12.03	19.93
<= 12 months	0.162	0.038	0.003	0
13 – 18 months	0.094	0.227	0.018	0.008
19 – 24 months	0.047	0.142	0.249	0.094
> 24 months	0.014	0.028	0.166	0.248

Table 13: Estimated expected time and overall or lifetime probability of contraceptives types used

Parameter	Condom	Injectables	Oral pills	Implants	Others
E(τ)	6.06	7.14	4.67	6.41	6.26
Recurrence time	4.08	4.02	5.36	7.34	5.45
Condom	0.137	0.007	0.010	0.005	0.005
Injectables	0.005	0.120	0.007	0.005	0.002
Oral pills	0.020	0.015	0.168	0	0.010
Implants	0.005	0.005	0.005	0.131	0.010
Others	0.008	0.006	0.006	0.004	0.134

It also reveals that condoms use have an expected time of 6.06 periods with a recurrence time of 4.08, injectables have an expected time of 7.14 with a recurrence time of 4.02, oral pills have an expected time of 4.67 with a recurrence time of 5.36, implants have expected time of 6.41 with a recurrence time of 7.34 while other contraceptives use to have an expected time of 6.26 with a recurrence time of 5.45.

Hypothesis Testing: The hypothesis of the insignificance of the recurrence of the states (p_{ii}) was tested using the t statistic and the zero-order probability of transiting from state i to state j tested with chi-square distribution is presented in Tables 14, 15 and 16. Table 14 shows that the proportions of Lagos metropolis women who fall in the states of birth gap and length of breastfeeding are all significantly greater than zero ($p < 0.05$) with above 24 months (78.8%, se = 2.5%) giving the highest proportions while 12 months or below (40%, se = 8.9%) has the least proportion of birth gaps. Also, 12 months or below (79.6%, se = 7.4%)) has the highest proportion of women while 19 – 24 months (46.7%, se = 4.7%) has the least proportion of women with the length of breastfeeding. Table 15 shows that the proportions of Lagos metropolis women who fall in the states of type of contraceptive used are all also significantly greater than zero ($p < 0.05$) with injectables (86.0%, se = 6.3%) giving the highest proportions while oral pills (78.8%, se = 7.5%) has the least proportion of contraceptive types used.

Table 14: The probability of recurrence of the states (p_{ii}) of the birth gap and length of breastfeeding and their statistical significance

	Birth gap		Length of breastfeeding	
Parameter	**P_{ii} (se(p))**	**t (p)**	**P_{ii} (se(p))**	**t (p)**
<= 12 months	0.400 (0.089)	4.472 (<0.001)	0.796 (0.074)	10.820 (<0.001)
13 – 18 months	0.455 (0.067)	6.770 (<0.001)	0.653 (0.064)	10.164 (<0.001)
19 – 24 months	0.460 (0.047)	9.815 (<0.001)	0.467 (0.047)	9.943 (<0.001)
> 24 months	0.788 (0.025)	31.630 (<0.001)	0.545 (0.030)	17.967 (<0.001)

Table 15: The probability of recurrence of the states (p_{ii}) of the type of contraceptive used and their statistical significance

Parameter	**P_{ii}**	**t (p)**
Condom	0.835 (0.068)	12.279 (<0.001)
Injectables	0.860 (0.063)	13.651 (<0.001)
Oral pills	0.788 (0.075)	10.507 (<0.001)
Implants	0.844 (0.066)	12.788 (<0.001)
Others	0.840 (0.067)	12.537 (<0.001)

Table 16: Chi-square test of order zero of the Markov chain

Variables	**Chi-Square (χ^2)**	**df.**	**p(>χ^2)**
Birth gap	76.205	9	<0.001
Length of breastfeeding	145.222	9	<0.001
Types of Contraceptives used	208.077	16	<0.001

Table 16 shows that the probability of transiting from state i to state j (for birth gaps, length of breastfeeding, and types of contraceptives used is not significantly independent (zero-order) ($p < 0.05$). This implies that the states are dependent on one another and the probability of switching is significantly greater than zero. The study had considered how the number of months in between births (birth gap), numbers of months women breastfeed their babies (length of breastfeeding) and the types of contraceptives used by women are used to control the population growth of Lagos metropolis. Markov chain which estimated the transition probabilities of switching from one state i to another state j was the tool employed for such estimation. The study found that the transition probability matrix is not zero order as the probabilities of switching from one state to another depending on the state under consideration. This was in line with the submission of many studies that majority of Markov chains probability transition matrix follow the first order because of the dependence of the states on each other (Azizah, *et al.*, 2019).

It also found that the proportion of women who in the long run gapped their births rose from 2.9% of 12 months and below to 60.5% of above 24 months. This shows that over time, over half of the child-bearing women in Lagos metropolis will take above 24 months interval between births which will result actually in the reduction in the growth of the population. This finding aligned with the demographic transition theory as expounded by Goujon (2019) that population growth will slow down and population will eventually go down. This can be established by the fact that most Nigerians no longer strive for large number of children leading to slowing or declining population growth rate (Battin, 2018). Also, with long birth gaps, mothers' risk during pregnancy and the survival of the babies is greatly enhanced in line with the assertions of Levine, *et al*., (2006) who reported that women who gapped in state 4 months are more likely to survive during pregnancy and childbirth than those who in the first state and that children born with state 4 gap are less likely to suffer malnutrition and more likely to survive beyond age 5. Also, it found that the length of breastfeeding will drop from over half within 12 months and below state to about 5% for above 24 months. This implies that while the number of months taken to breastfeed babies dropped coupled with the increase in the period in between births, more women will have more time for more productive engagements and better care of the households. This double transitioning will no doubt help improve the economic well-being of families which agreed with Joe, *et al.* (2018) that demographic transitions supports economic growth. It further found that while the proportion of women who use condoms dropped drastically, those engaging in the other contraceptives (injectables, oral pills, implants, etc.) increased. This goes to show that there is better awareness of contraceptives and their usefulness and the more women indulge in the use, the less likely are they to take in (conceive) – which will eventually lead to a drop in the population growth in the metropolis. This finding also aligned with several studies which surmised that contraception when effectively and consistently implemented will lead to slowing population growth rate, healthier women, reduction in maternal mortality because unwanted pregnancies, improved children's health through long child spacing (birth gaps) and elongated breastfeeding, and better economic conditions for households (Schenk, 2021; Ikegwu, *et al.*, 2020; Verma, *et al.*, 2019; Battin, 2018). In other words, the use of contraceptives in family planning will prevent a significant proportion of infant deaths through prevention of "mistimed and underspaced" births and Levine, *et al*. (2006) noted that birth gaps (child spacing) of over 24 months (state 4) could reduce infant mortality by close to half. The campaign on the consistent and effective use of contraceptives is to reduce conception and with the aid of barrier methods like condom protect users against sexually transmitted infections (STIs). The inference results reveal that the transition probability matrix for each of the concepts is not zero order as the states are dependent on each other and the probability of transiting from one state (i) to another state (j) is significantly greater than zero ($p < 0.05$). This agree with the submissions of other researchers show observed that the transition matrix of a Markov chains is dependent and that their test where also significant (Jale, *et al*., 2019). Hence, it attested to the fact that the number of state spaces signified for the each of birth gap, length of breastfeeding and type of contraceptives used were adequate. Finally, with increased birth gaps, lengths of breastfeeding and contraception among childbearing women in the metropolis, more families will settle for fewer number of children coupled with the high cost of maintaining and training large families. This sits well with Bongaarts (2017) that fertility will drop to two births per woman, reduced family size and better lifestyle and welfare also leading to reduced mortality. The decline in population growth rate will also affect living conditions as high population was found to be a major cause of increases in the prices of commodities, poverty and depletion of natural resources (Romano and Fletcher, 2018; Dong, *et al*., 2018; Nguyen and Kinnucan, 2017; Estrada, 2017).

Conclusions: The study concludes that with an increase in the proportion of women who take longer periods in between births, the growth of the population in the Lagos metropolis will be slowed down. Secondly, freeing the time spent on breastfeeding coupled with the increased interval between pregnancies, more women will have a better time to take care of the families and engage in better productive activities which will boost their family welfare. Also, the increased awareness about contraceptives which resulted in the increased use of other contraceptives other than condoms will further help families plan their childbearing resulting in not only reduction in the growth of families but better welfare and care of households through the productivity of mothers.

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