PRINT ISSN 1119-8362
Electronic ISSN 1119-8362

Full-text Available Online at
https://www.ajol.info/index.php/jasem
https://www.bioline.org.br/ja

J. Appl. Sci. Environ. Manage.
Vol. 26 (4) 565-570 April 2022

Combined Effect of Radiant Heat, Local Nusselt Number and Skin Friction Coefficient of a Third Grade Fluid through a Channel Flow in a Saturated Porous Medium

***[1]PETER, BA; [2]IDOWU, SA; [3]OGUNSOLA, AW; [4]SANGOTAYO, EO**

*[*1]Federal University of Petroleum Resources, Effurun, Department of Mathematics, Delta State, Nigeria*
[2]Olabisi Onabanjo University, Department of Mathematical Sciences, Ago-Iwoye, Nigeria
[3]Ladoke Akintola University of Technology, Department of Pure and Applied Mathematics, Ogbomoso, Nigeria
[4]Kampala International University, Department of Mechanical Engineering, Kampala, Uganda
**Corresponding Author Email: peter.benjamin@fupre.edu.ng*

ABSTRACT: This paper investigates the combined effects of radiant heat, local Nusselt number and Skin friction coefficient on a channel flow of third grade fluid in the presence of saturated porous medium. It is assumed that the fluid has temperature-dependent variable viscosity and thermal conductivity. The non-linear governing differential equations are obtained and tackled numerically using Spectral element technique and C++ programming language. Increase in the values of radiation parameter and third grade material parameter consequently increase the local Nusselt number and skin friction coefficient. Graphical results showing the effects of various physical parameters are presented and discussed quantitatively. It is concluded that Williamson's fluid parameter served as a cooling factor in diverse improved technological processes in order to avoid different machines from overheating and enhance their efficiencies.

DOI: https://dx.doi.org/10.4314/jasem.v26i4.2

Impact factor: http://sjifactor.com/passport.php?id=21082

Google Analytics: https://www.ajol.info/stats/bdf07303d34706088ffffbc8a92c9c1491b12470

Dates: Received: 17 January 2022; Revised: 13 March 2022; Accepted: 07 April 2022

Keywords: Porous media, Non-Newtonian fluids, Viscosity, Thermal conductivity, Radiant energy

Non-Newtonian fluids behaviour is encountered in human's day to day activities as well as in industrial processes. In the recent times, the study of Newtonian and non-Newtonian fluids through porous media has attracted the attention of many researchers following its large area of utilizations in industrial and engineering processes, particularly in applied geophysics, geology, geothermal reservoir, petroleum design, automobile exhaust system and material processing to mention but just a few. Due to the recent increase in the demand for the production of crude oils, and the materials whose relationships are non-linear; it has become imperative to have a better knowledge of non-Newtonian fluids flow. The fluid thermal conductivity varies exponentially with temperature is fundamental in the developments of energy transfer (Ogunsola and Peter, 2014; Chinyoka and Makinde, 2015; Chinyoka and Makinde, 2012; Das *et al.*, 2016). Several researchers have considered flow of third grade fluid under different physical situations (Fosdick and Rajagopal,s 1980; Hayat *et al.*, 2009 and Massoudi and Christe, 1995). Haroon *et al.*, *(*2011); Hayat *et al.*,(2009) examined the flow of a receptive power-law poiseuille fluid. The results revealed the effect of power-law index on the flow. Hayat and Ahmed (2014) studied the influence of Joule heating in third grade fluid flow. Jayeoba and Okoya (2012) considered the steady flow of a third-grade fluid in a channel. Makinde 2009 examined the influence of inherent irrevocable in a pipe flow. Furthermore, (Rivlin and Erickson, 1995; Peter *et al.*, 2019) analysed stress contortion relation for isotropic substances. Szeri and Rajagopal (1985) investigated on the flow of a non-Newtonian fluid in equidistant plates. Yurusoy *et al.*, (2008) examined entropy perusal of a third-grade fluid flow viscosity in annular conduit. The results revealed that increase in variable viscosity parameter decreases the velocity flow. However, this study improved on Massoudi and Christe (1993) by incorporating radiant heat and temperature-dependent variable thermal conductivity. This paper investigates the combined effects of radiant

**Corresponding Author Email: peter.benjamin@fupre.edu.ng*

heat, local Nusselt number and Skin friction coefficient on a channel flow of third grade fluid in the presence of saturated porous medium.

MATERIALS AND METHODS

The study of reactive fluid flow through a porous medium with radiant energy is considered with variable properties.

$$-\frac{1}{\rho}\frac{\partial \bar{P}}{\partial \bar{x}} + \frac{1}{\rho}\frac{d}{d\bar{y}}\left(\bar{\mu}(T)\frac{dw}{dy}\right) + \frac{\sqrt{2}\mu(T)\Gamma}{\rho}\left(\frac{dw}{d\bar{y}}\right)\frac{d^2w}{d\bar{y}^2} - \frac{\bar{\mu}(T)w}{\rho K} = 0 \qquad (1)$$

$$\frac{1}{\rho C_P}\frac{d}{d\bar{y}}\left(k(T)\frac{dT}{d\bar{y}}\right) - \frac{dq_r}{d\bar{y}} + \mu\frac{w^2}{\rho C_P K} + \frac{2\beta_3}{\rho C_P}\mu\left(\frac{dw}{d\bar{y}}\right)^4 + \frac{Qc_0A}{\rho c_p}e^{-E/RT} = 0 \qquad (2)$$

Subject to the following boundary conditions $w(\bar{y}) = 0, T(\bar{y}) = T_0$ initial condition $w = -\tan\phi$, $T(0) = T_0$ at $y = 0$, $w = V\sin\phi$, $T(\bar{y}) = T_0$ at $y = h$ (3)

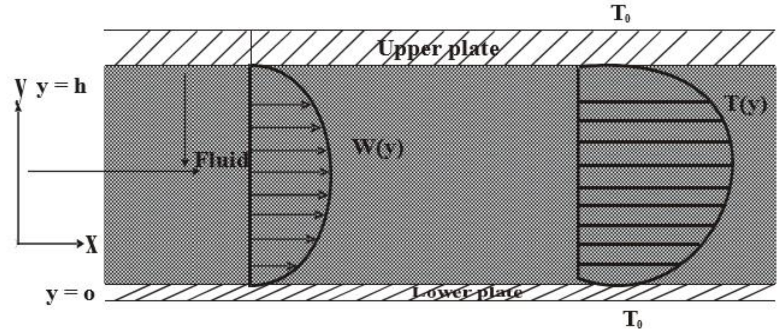

Fig 1: Flow geometry

Where

$\sqrt{2}\mu\Gamma$ is the Williamson's fluid material, Γ is the time constant, $\rho r = -\frac{dq_r}{dy}$, q_r is the radiant heat flux, E is the activation energy, Q is the heat of reaction, R is the universal gas constant, T is the absolute temperature, the heat capacitance is ρc_p, material derivatives, the porous medium permeability is K, A is the rate constant, $k(T) = k_0 e^{m(T-T_0)}$, T_0 is the initial temperature, ρ is the density, m is a numerical exponent, n is the thermal conductivity variation and b is the variable viscosity.

The dimensionless variables are as follows:

$$\varepsilon = \frac{RT_0}{E}, \theta = \frac{E(T-T_0)}{RT_0^{\ 2}}, u = \frac{w\rho h}{\mu_0}, y = \frac{\bar{y}}{h}, P = \frac{\bar{P}\rho h^2}{\mu_0^2}, x = \frac{\bar{x}}{h}, C = -\frac{\partial P}{\partial x}\frac{\mu_0^2}{\rho h^3}, \beta = \frac{\sqrt{2}\mu_0\Gamma}{\rho^2 h^3},$$

$$\Pr = \frac{\mu_0 c_p}{k_0}, \psi = \frac{QEAh^2C_0}{RT_0^{\ 2}c_p}e^{-E/RT}, \alpha_1 = \frac{bRT_0^2}{E}, n_1 = \frac{nRT_0^2}{E}, \qquad (4)$$

$$Da = \frac{K}{h^2}, R_d = \frac{4\sigma^* T_\infty^3}{k^* k_0}, \tan\phi = \frac{\sin 180^0}{\cos 180^0} = \frac{0}{-1} = 0$$

From equations (2.1)-(2.4) the dimensionless differential equations are given blow:

$$C + e^{-\alpha_1\theta}\frac{d^2u}{dy^2} - \alpha_1 e^{-\alpha_1\theta}\frac{d\theta}{dy}\frac{du}{dy} + e^{-\alpha_1\theta}\beta\frac{du}{dy}\left(\frac{du}{dy}\right)^2 - uS = 0 \qquad (5)$$

$$\frac{1}{\Pr}\frac{d^2\theta}{dy^2}\left(e^{\Lambda_1\theta}+\frac{4R_d}{3}\right)+\psi e^{\theta/1+\varepsilon\theta}+\left(\frac{du}{dy}\right)^2\left\{e^{-\alpha_1\theta}+2\beta\left(\frac{du}{dy}\right)^2\right\}+e^{-\alpha_1\theta}u^2S=0 \qquad (6)$$

$$u(0)=0, u(1)=0$$

$$\theta(0)=0, \theta(1)=0 \quad (7)$$

RESULTS AND DISCUSSION

The numerical results for various values of physical variables are described in Figures 2-12. Figures 2- 4 shows that Williamson's fluid parameter β, R_d radiation parameter and ε activation parameter increases (being transfer of heat energy away from the source to the surrounding fluids) which leads to an increase in the temperature of the fluid. The result is in agreement with the literature. Consequently, increase in the thermal boundary layer which in turn provides an additional means to diffuse energy. This result is applicable to electrical power generation, polymer melts, cooling of electronic components, and recovery of oil from oil spillage (i.e., thermal radiation parameter is the parameter responsible for a decrease in the viscosity of the fluid), bakery industry and thermal insulation engineering. Increasing the values of Nu leads to a corresponding decrease in the heat transfer rate for laminar and convective flows. Nu Increases for both R_d and ε parameters.

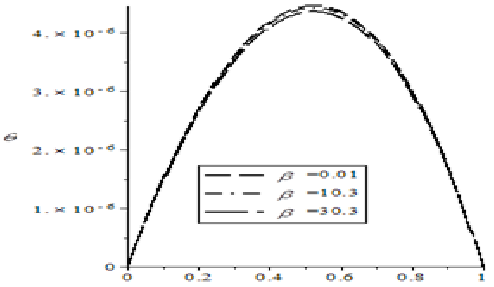

Fig 2: Graph of the temperature function θ when $\Lambda_1=1.5,\ \varepsilon=0,0.1,0.25,\delta=\alpha_1=1.0.$

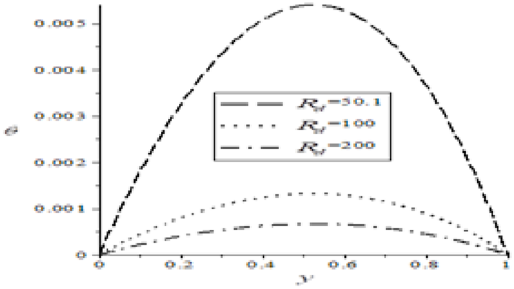

Fig 3: Graph of the temperature function θ when $\Lambda_1=1.5,\ \varepsilon=0,0.1,0.25,\delta=\alpha_1=1.0.$

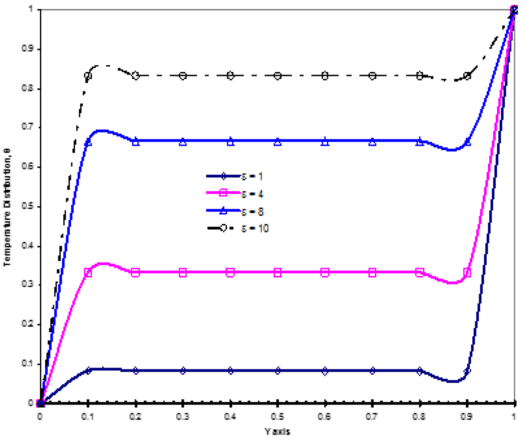

Fig 4: Graph of the temperature function θ against ε when $Nu=5,\ R_d=0.2,\ C_f=0.1,\ S=0.2,$ $\beta=0.1,\alpha_1=0.1,\ C=0.1$

It is seen from figure 3 that increase in R_d corresponds to a decrease on the fluid temperature consequently, leading to a high value of the rate of heat transfer within the fluid channel.From figure 2 Williamson's fluid parameter lead to increase in the rate of heat transfer of the fluid medium which is in agreement with previous studies. Skin-friction coefficient decreases when $R_d,\ \varepsilon,\ Nu,\ S,\ \beta$ and α_1 are increased. Skin-friction being the parameter that measures the rate of shear stress within fluid channel. From figures 5-12 the increased in the parameters correspond to a decrease in the rate of mass transfer of the fluid. Thermal stability of the system is achieved for minimum and maximum value of R_d, Nu, ε while maximum value is reached for $R_d,\ \varepsilon,\ Nu,\ Da,\ \beta,\alpha_1$ and β is the non-Newtonian Williamson fluid parameter. Higher values of activation parameter are significant on the rate of mass transfer. It is noted that ε increases the fluid temperature due to exothermic chemical reaction. Increased in the values of R_d slower the rate of diffusion, i.e. it implies that R_d decreases the rate of heat transfer which consequently leads to the accumulation of heat energy in the fluid channel due to frictional heating which in turns leads to a decrease in the rate of heat transfer of the flow system.

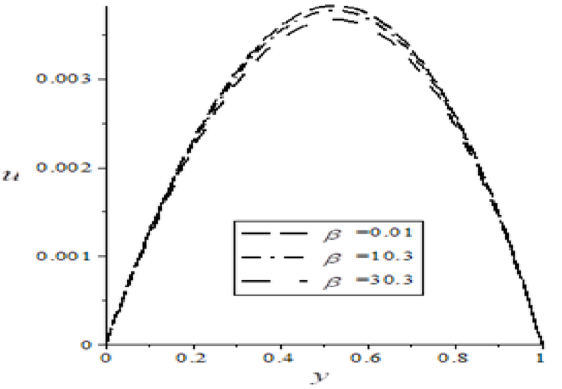

Fig 5: Graph of the velocity function u when $\Lambda_1 = 1.5,\ \varepsilon = 0, 0.1, 0.25, \delta = \alpha_1 = 1.0.$

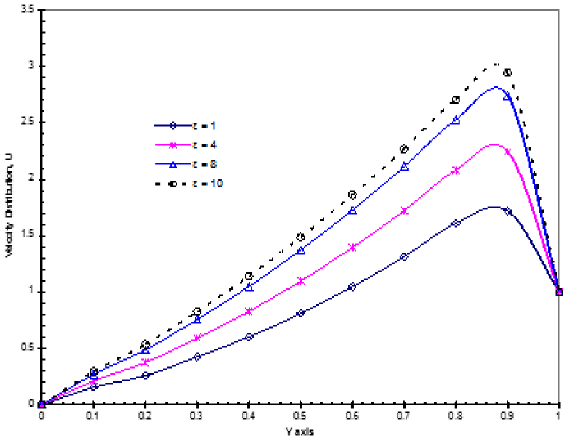

Fig 6: Graph of the velocity function u against ε when $Nu = 5,\ R_d = 0.2,\ C_f = 0.1,\ S = 0.2,\ \beta = 0.1, \alpha_1 =$

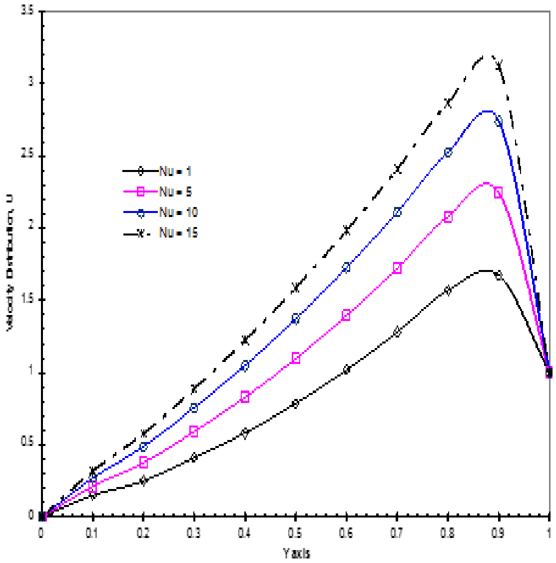

Fig 7: Graph of the velocity function u against Nu when $R_d = 0.2,\ \varepsilon = 4,\ C_f = 0.1,\ S = 0.2,$ $\beta = 0.1, \alpha_1 = 0.1,\ C = 0.1$

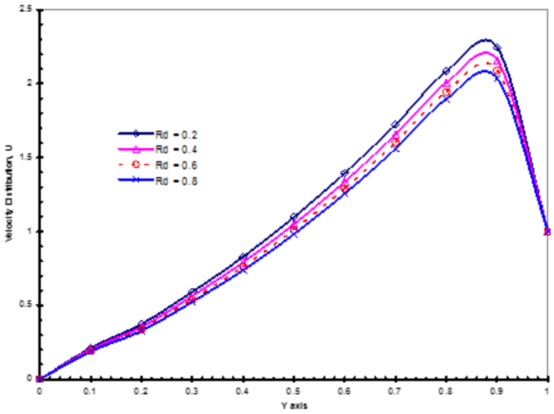

Fig 8: Graph of the velocity function u against R_d when $Nu = 5,\ \varepsilon = 4,\ C_f = 0.1,\ Da = 0.2,$ $\beta = 0.1, \alpha_1 = 0.1,\ C = 0.1$

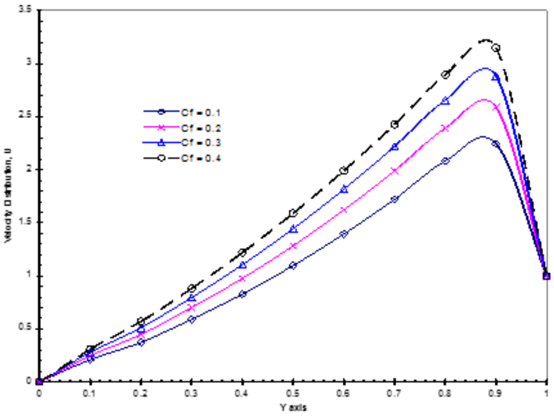

Fig 9: Graph of the velocity function u against C_f when $Nu = 5,\ R_d = 0.2,\ \varepsilon = 4,\ S = 0.2,$ $\beta = 0.1, \alpha_1 = 0.1,\ C = 0.1$

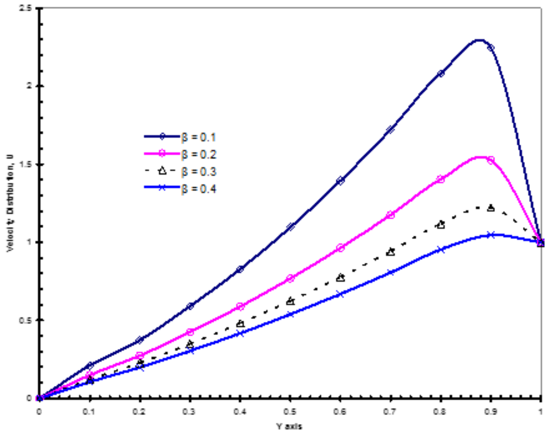

Fig 10: Graph of the velocity function u against β when $Nu = 5,\ R_d = 0.2,\ \varepsilon = 4,\ C_f = 0.1,$ $S = 0.2, \alpha_1 = 0.1,\ C = 0.1$

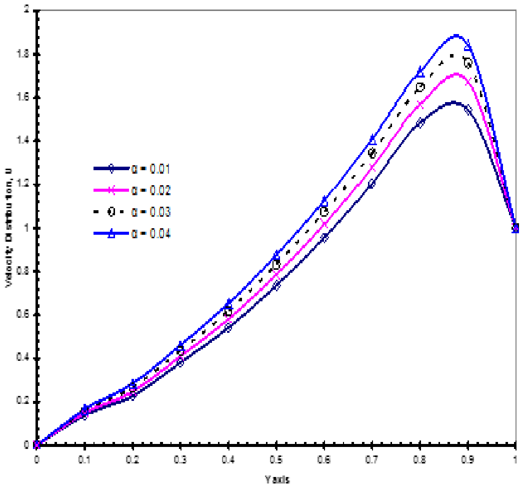

Fig 11: Graph of the velocity function u against α_1 when
$Nu = 5,\ R_d = 0.2,\ \varepsilon = 4,\ C_f = 0.1,$
$S = 0.2,\ \beta = 0.1,\ C = 0.1$

Declarations

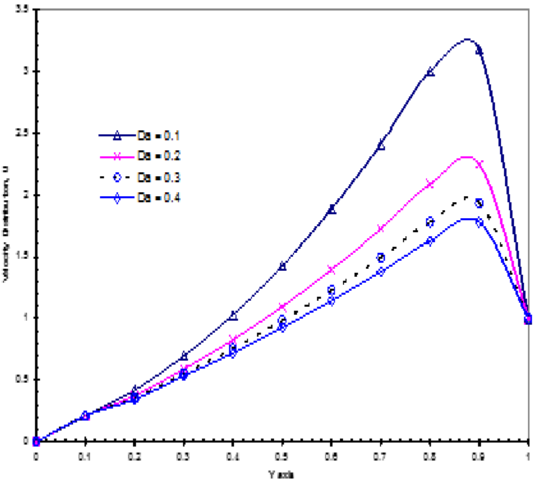

Fig 12: Graph of the velocity function u against Da when
$Nu = 5,\ R_d = 0.2,\ \varepsilon = 4,\ C_f = 0.1,$
$\beta = 0.1, \alpha_1 = 0.1,\ C = 0.1$

Increase in Da corresponds to increase in the diffusion rate. This study combined the effects of radiant heat, the local Nusselt number and skin friction coefficient, temperature dependent viscosity and variable thermal conductivity on reactive flow of third grade fluids through a porous medium computationally. The result of this study is of significant interest in the area of electrical power generation, petroleum exploration and refinery. Petroleum is very reactive and non-Newtonian in nature. For engineering purpose, the flow model of our problem represents the oils well and as R_d radiation parameter is increasing there is quick recovery of oil from the oils well. Also, the results of this study are of great interest in production processing, for the safety of life and proper handling of the materials during processing.

Conclusion: The findings are as follows: It is noticed that thermal stability of the fluid system is achieved and maximum value is reached. It is seen that increased R_d, Nu and ε parameters correspond to an increase in the rate of heat transfer of the fluid channel. It is noted that ε increases the fluid temperature due to exothermic chemical reaction. It is observed that there is a steady increase in both fluid velocity and temperature with an increase in the reaction rate and fluid viscosity.

Acknowledgements: The authors appreciate the effort of the independent readers and the constructive comments given by the reviewers.

REFERENCES

Chinyoka, T; Makinde, OD (2015). Unsteady and porous media flow of reactive non-Newtonian fluids subjected to buoyancy and suction/ injection. *Inter. J. Num. Meth. Heat and Fluid Flow.* 7 (1):1682-1704.

Chinyoka, T; Makinde, OD (2012).Unsteady hydrodynamic flow of reactive variable viscosity third-grade fluid in a channel with convective cooling. *Inter. J. Numer. Methods in Fluids.* 69:353-365.

Das, T; Jana, RN; Makinde, OD (2016). Transient natural convection in a vertical channel filled with nanofluids in the presence of radiation. *Alex. Engr. J.* 55:253-262.

Fosdick, RL; Rajagopal, KR (1980).Thermodynamics and stability of fluids of third grade. Proc. R. Soc. Lond. 369: 351-377.

Haroon, T; Ansari,AR; Sidiqui, AM; Jan, SU (2011). Analysis of poiseuille flow of a reactive power-law fluid between parallel plates. *J. Appl. Mathem. Sci.* 5(55):2721-2746.

Hayat, T; Faroog, MA; Javed, T; Sajid, M (2009). Partial slip effects on the flow and heat transfer characteristics in third grade.*J. Non-linear Analysis. Real World Appl.*10:725-755.

Hayat, T; Anum, S; Ahmed, A (2014). Effect of Joule heating and thermal radiation in flow of third grade fluid over radiative surface. *J. Non-linear Anal. Appl. Mathem*. Faculty of Science, Saudi Arabia. 9(1):1-12

Jayeoba, OJ; Okoya, SS (2012). Approximate analytical solutions for pipe flow of a third-grade fluid with models of viscosities and heat generation/absorption. *J. Nig. Mathem. Society*. 31:207-228.

Makinde, OD (2015). Thermal decomposition of unsteady non-Newtonian MHD Couette flow with variable properties. *Int. J. Numer. Methods Heat and Fluid Flow*. 25(2):252-264.

Massoudi, M; Christe, I (1993). Heat transfer and flow of third grade fluid in a pipe. *Mathem. Modelling Sci. Comp.* 2:1273-1275.

Massoudi, M; Christe, I (1995). Effects of variable viscosity and viscous dissipation on the flow of third grade fluid in a pipe. Inter. *J. Non-Linear Mech.* 30(5):687-699.

Ogunsola, AW; Peter, BA (2014). Non-Newtonian fluid flow with Arrhenius reaction between heated parallel plates through a porous medium. *Inter. J. Innov. Tech. Exploring. Eng.* 3(11), 137-142

Peter, BA; Ogunsola, AW; Itodo, AE; Idowu, SA; Mundu, MM (2019). Reacting Flow of Temperature-Dependent Variable Permeability through a Porous Medium in the Presence of Arrhenius Reaction. *Amer. J. Mathem. Comp. Sci.*4 (1):11-18.

Rosseland, (1936). Theoretical Astrophysics. Oxford University, New York, NY, USA.

Szeri, AZ; Raagopal, KR (1985). Flow of a Non-Newtonian fluid between heated parallel plates. *Inter. J. Non-Linear Mech.* 20(2):91-101.

Yurusoy, M; ayrakoekem, H; Kapucu, M; Aksoy, F (2008). Entropy analysis for third grade fluid flow with Vogel's models of viscosity in annular pipe. *Inter. J. Non-Linear Mech.* 43:588-599.