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Euclidean Null Controllability of Nonlinear Infinite Delay Systems with Time Varying Multiple Delays in Control and Implicit Derivative

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ABSTRACT: Sufficient conditions for the Euclidean null controllability of non-linear delay systems with time varying multiple delays in the control and implicit derivative are derived. If the uncontrolled system is uniformly asymptotically stable and if the control system is controllable, then the non-linear infinite delay system is Euclidean null controllable. @JASEM

The control processes for many dynamic systems are often severely limited, for example, there may be delays in the control actuators. Models of systems with delays in the control occur in population studies. Most specifically models of systems with distributed delays in the control occur in the study of agricultural economics and population dynamics, Arstein (1982), Arstein and Tadmor (1982). In most biological populations the accumulation of metabolic products may inconvenience a population and this result in the fall of birth rate and increase in death rate. If it is assumed that total toxic action in the birth and death rates is expressed by an integral term in the logistic equation then an appropriate model is the integro-differential equation with infinite delays. Several authors have studied these systems and established sufficient conditions for the controllability and null controllability of these systems, Chukwu (1992), Gopalsany (1992).

Chukwu (1980) showed that if the linear delay system

$$\dot{x}(t) = L(t, x_t)$$

is uniformly asymptotically stable and

$$\dot{x}(t) = L(t, x_t) + B(t)u(t)$$

is proper, then

$$\dot{x}(t) = L(t, x_t) + B(t)u(t) + f(t, x_t, u(t))$$

is Euclidean null controllable, provided f satisfies certain growth and continuity condition.

Sinha (1985) studied the non-linear infinite delay system

$$\dot{x}(t) = L(t, x_t) + B(t)u(t) + \int_{-\infty}^{0} A(\theta)x(t+\theta)d\theta + f(t, x_t, u(t)) \qquad (1)$$

and showed that (1) is Euclidean null controllable if the linear base system

$$\dot{x}(t) = L(t, x_t) + B(t)u(t) \qquad (2)$$

is proper and the free system

$$\dot{x}(t) = L(t, x_t) + \int_{-\infty}^{0} A(\theta)x(t+\infty)d\theta \qquad (3)$$

is uniformly asymptotically stable, provided that f satisfies some growth conditions.

Balachandran and Dauer (1996) studied the null controllability of the non-linear system

$$\dot{x}(t) = L(t, x_t) + \sum_{i=0}^{n} B_i(t)u(h_i(t)) + \int_{-\infty}^{0} A(s)x(t+s)ds + f(t, x(t), u(t)) \qquad (4)$$

$$x(t) = \phi(t) \qquad t \in [-\infty, t_0]$$

Hale (1974) provided sufficient conditions for the stability of systems of the form

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$$\dot{x}(t) = L(t,x_t) + \sum_{i=0}^{n} B_i(t-h_i) + \int_{-\infty}^{0} B(\theta)x(t+\theta)d\theta \quad (5)$$

The aim of this paper is to study the null controllability of systems of the form

$$\dot{x}(t) = L(t,x_t) + \sum_{i=0}^{n} B_i(t)u(h_i(t)) + \int_{-\infty}^{0} d_\theta H(t,\theta,x(t))x(t+\theta) + f(t,x(t),\dot{x}(t),u(t)) \quad (6)$$

$$\dot{x}(t) = \phi(t) \qquad t \in [-\infty, t_0]$$

where $L(t,\phi)$ is continuous in t, linear in ϕ, and is given by

$$L(t,\phi) = \sum_{k=0}^{n} A_k(t)\phi(-h_k) \quad (7)$$

BASIC ASSUMPTIONS AND PRELIMINARIES.

Let E^n be an n-dimensional linear space with norm $|.|$. In equation (6) A_i is a continuous $n \times n$ matrix function for $0 \le h_i \le h$ and $H(t,\theta,x(t))$ is $n \times n$ matrix valued function which is measurable in $(t,\theta,x(t))$, and $H(t,\theta,x(t))$ is of bounded variation in $(\theta,x(t))$ in $(-\infty,\infty)$. The matrix function $B_i(t)$ $i = 0,1,2,\ldots\ldots\ldots n$ are $n \times p$, continuous in t *and* $h = t_0 - \min_i h_i(t_0)$ *where* $h_i(t)$ are defined below. Here $x \in E^n$ *and* $u \in E^p$. Let $0 \le h \le \gamma$ be given real numbers (γ *may be* $+\infty$). The function $\eta : [-\gamma, 0] \to (0,\infty)$ is lebesque integrable on $[-\gamma,0]$, positive and non-decreasing. Let $B([-\gamma,0], E^n)$ be the Banach space of functions which are continuous. Let $B([-\gamma,0], E^n)$ be the Banach space of functions which are continuous and bounded on $[-\gamma,0]$ such that

$$|\phi| = \sup_{s\in[-h,0]}|\phi(s)| + \int_{-\gamma}^{0} \eta(s)|\phi(s)|ds \qquad < \infty$$

for any $t \in R$ and any function $x : [t-\gamma, t] \to E^n$, *let* $x_t : [-\gamma,0] \to E^n$ be defined by

$$x_t(t) = x(t+s) \qquad s \in [-\gamma, 0]$$

Assume that the function $h_i[t_o,t_1] \to R$ $\quad i = 0,1,2,\ldots\ldots\ldots\ldots n$ are twice continuously differentiable and strictly increasing in $[t_0,t_1]$, further

$$h_i(t) \le t \qquad for \;\; t \in [t_o,t_1] \;\; i = 0,1,\ldots\ldots\ldots\ldots m.$$

Let us introduce as in Kantorovich (1992) the following time lead function r_i with

$$r_i(t) : [h_i(t_o), h_i(t_1)] \to [t_0,t_1]$$

such that $r_i(h_i(t)) = t$ $\quad for \;\; i = 0,1,\ldots\ldots\ldots\ldots\ldots\ldots\ldots, m \quad t \in [t_0,t_1]$ ..Further assume that $h_i(t) = t$ and for $t = t_1$ the function $h_i(t)$ satisfies the inequalities

$$h_m(t_1) \le h_{m-1}(t_1) \le \ldots\ldots\ldots \le h_{m+1}(t_1) \le t_0 = h_m(t_1) < h_{m-1}(t_1) = \ldots\ldots\ldots = h_1(t_1) = h_0(t_1) = t_1 \quad (8)$$

Let the fundamental matrix X satisfy the equation

$$\frac{\partial X(t,s)}{\partial t} = L(t, X_t(.,s)) \qquad t \ge s$$

$$0, \qquad t-h \le t \le s$$

$$X(t,s) = I \qquad t = s$$

where $X_t(.,s)(\theta) = X(t+\theta,s), \quad -h \le \theta \le 0$ .Then the solution of (6) is given by

$$x(t) = X(t,s)\phi(t_0) + \int_0 X(t,s)\sum_{i=0}^{n} B_i(s)u(h_i(s))ds + \int_0 X(t,s)\left(\int_{-\gamma}^{0} d_\theta H(s,s-\theta,x(\theta))\right)x(s+\theta)ds$$

$$+ \int_0 X(t,s)f(s,x(s),\dot{x}(s),u(s))ds \quad for \quad t_0 \le t \le t_1$$

$$x(t) = \phi(t) \quad for \quad t \in (-\infty, t_0] \tag{9}$$

with initial state $Y(t_0) = (x(t_o);\phi,\eta)$ *where* $u(s) = \eta(s)$ *for* $s \in [t_0 - h, t_0]$ *and* $x(t,t_0,\phi)$ is the solution of $\dot{x}(t) = L(t,x_t)$. Using the time lead function and the inequalities (8) we have,

$$x(t_1) = x(t_1;t_0,\phi) + \sum_{i=0}^{m} \int_{h_i(t_0)}^{t_0} X(t,r_i(s))B_i(r_i(s))\dot{r}_i(s)\eta(s)ds + \sum_{i=m+1}^{n} \int_{h_i(t_0)}^{h_i(t_1)} X(t_1,r_i(s))B_i(r_i(s))\dot{r}_i(s)\eta(s)ds$$

$$+\sum_{i=0}^{m} \int_0^{t_1} X(t_1,r_i(s))B_i(r_i(s))\dot{r}_i(s)u(s)ds + \int_0^{t_1} X(t_1,s)\left(\int_{-\gamma}^{0} d_\theta H(s,\theta,x(s))x(s+\theta)\right) \tag{10}$$

$$+ \int_0^{t_1} X(t_1,s)f(s,x(s),\dot{x}(s),u(s))ds$$

For brevity introduce the following notations

$$H(t,\eta) = \sum_{i=0}^{m} \int_{h_i(t_0)}^{t_0} X(t,r_i(s))B_i(r_i(s))\dot{r}(s)\eta(s)ds + \sum_{i=m+1}^{n} \int_{h_i(t_0)}^{h_i(t)} X(t,r_i(s))B_i(r_i(s))\dot{r}(s)\eta(s)ds \tag{11}$$

$$q(t_1,\eta) = x(t_1;t_0,\phi) + H(t_1,\eta) + \int_0^{t_1} X(t_1,s)f(s,x(s),\dot{x}(s),u(s)) \tag{12}$$

$$+ \int_0^{t_1} X(t_1,s)\left(\int_{-\gamma}^{0} d_\theta H(s,\theta,x(s))x(s+\theta)\right)$$

$$G(t,s) = \sum_{j}^{i} X(t,r_j(s))B_j(r_j(s))\dot{r}_j(s) \tag{13}$$

Consider the homogeneous systems

$$\dot{x}(t) = L(t,x_t) + \sum_{i=0}^{n} B_i(t)u(h_i(t)) \tag{14}$$

$$\dot{x}(t) = L(t,x_t) + \int_{-\infty}^{0} d_\theta H(t,\theta,x(t))x(t+\theta) \tag{15}$$

The controllability matrix of system (14) at time t is given by

$$W(t,s) = \int_0 G_m(t,s)G_m^{\ T}(t,s)ds \tag{16}$$

where T denotes matrix transpose.
Chukwu (1992), established null controllability of systems of the form (14)

DEFINITION: The system (6) is said t o be null controllable if for each $\phi \in B([-\gamma,0], E^n)$ there is a $t_1 \geq t_0$, $u \in L_2([t_0,t_1])$ u is a compact convex subset of E^p such that the solution $x(t;t_0,\phi,u)$ of (6) satisfies $x_{t_0}(t_1;\phi,u)=\theta$ *and* $x(t_1;t_0,\phi,u)=0$

MAIN RESULT

THEOREM: Suppose that the constraint set u is an arbitrary compact subset of E^p and that

i. Assume that system (15) is uniformly asymptotically stable, so that the solution $x_t(t_0,\phi)$ *satisfies* $\|x_t(t_0,\phi)\| \leq me^{-\alpha(t-t_0)}\|\phi\|$ for some $\alpha > 0$, $m > 0$
ii. The linear control system (14) is controllable
iii. The continuous function f satisfies
$\|f(t,x(t),\dot{x}(t),u(t))\| \leq \exp(-\beta t)\pi(x(t),\dot{x}(t),u(t))$ for all
$(t,x(t),\dot{x}(t),u(t)) \in [t_0,\infty] \times B([-\gamma,0], E^n) \times L_2([t_0,t_1], u)$

Where

$$\int_0^\infty \pi(x(.),\dot{x}(.),u(.))ds \leq k < \infty \quad and \quad \beta - \alpha \geq 0$$

then (6) is Euclidean null controllable.

Proof: Since (15) is controllable, hence it is proper in E^n *and* $W^{-1}(t_0,t_1)$ exists for each $t_1 > t_0$. Suppose the pair of functions x, u form a solution pair to the following equations

$$u(t) = -G_m(t_1,t_0)W^{-1}(t_0,t_1)q(t_1,\eta) \tag{17}$$

for some suitable chosen $t_0 \leq t \leq t_1$, $u(t)=\eta(t)$, $t \times [t_0-h,t_0]$ and

$$x(t) = x(t;t_0,\phi) + H(t,\eta) + \int_0^t G_m(t,s)u(s)ds + \int_0^t \int_{-\gamma}^0 d_\theta(s,\theta,x(t))x(s+\theta)ds$$
$$+ \int_0^t X(t,s)f(s,x(),\dot{x}(),u())ds$$

$$x(t) = \phi(t) \quad t \in [t_0-h,t_0] \tag{18}$$

then u is square integrable on $[t_0-h,t_0]$ *and* x is a solution of (6) corresponding to the control u with initial state $Y(t_0) = x(t_0;\phi,\eta) = 0$. Now it is shown that $u:[t_0,t_1] \rightarrow u$ is a compact constraint subset of E^n, that is $|u| \leq a$ for some constant $a > 0$. Since (15) is uniformly asymptotically stable and B_i are continuous in t, it follows that

$$\left|G_m^T(t_1,t)W^{-1}\right| \leq c_1 \; for \; some \; c_1 > 0$$

$$|x_t(t_0,\phi)| \leq c_2 \exp[-\alpha(t_1-t_0)] \; for \; some \; c_2 > 0$$

$$|H(t,\eta)| \leq c_3 \exp[-\alpha(t_1-t_0)] \; for \; some \; c_3 > 0 \; hence$$

$$|u(t)| \leq c_1\left[c_2 \exp[-\alpha(t_1-t_0)] + c_3 + \int_0^{t_1} m \exp[-\alpha(t_1-s)]\exp(-\beta s)\pi(x(),u())ds\right]$$

and therefore

$$|u(t)| \leq c_1[c_2+c_3]\exp[-\alpha(t_1-t_0) + km\exp(-\alpha t_1)] \tag{19}$$

since $\beta - \alpha \geq 0$ *and* $s \geq t_0 \geq 0$. From (18) t_1 can be chosen so large that $|u(t) \leq a, t \in [t_0, t_1]$ which proves that u is an admissible control for this choice of t_1.

It remains to prove the existence of a solution pair of the integral equation (16) and (17). Let B be the Banach space of all functions $(x,u):[t_0 - h, t_1] \times [t_0 - h, t_1] \to E^n \times E^p$ where $x \in B([t_0 - h, t_1], E^n)$ *and* $u \in L_2([t_0 - h, t_1], E^n)$ with the norm defined by

$$\|(x,u)\| \leq \|x\|_2 + \|u\|_2 \text{ Where}$$

$$\|x\|_2 = \left[\int_0^{t_1} |x(s)|^2 ds\right]^{1/2}, \quad \|u\|_2 = \left[\int_0^{t_1} |u(s)|^2 ds\right]^{1/2}.$$

Define the operation $T: B \to B$ *by* $T(x,u) = (y,v)$ where

$$v(t) = -G_m(t_1, t) W^{-1}(t_0, t_1) q(t_1, \eta) \quad for \ t \in [t_0, t_1] \equiv J \tag{20}$$

$$v(t) = \eta(t) \qquad for \ t \in [t_0 - \gamma, t_0]$$

$$y(t) = x(t; t_0, \phi) + H(t, \eta) + \int_0^t G_m(t,s) v(s) ds + \int_0^t \int_{-\gamma}^0 X(t,s) d_\theta H(s, \theta, x(s), x(s+\theta)) ds$$
$$+ \int_0^t X(t,s) f(s, x(), \dot{x}(), u()) ds \qquad for \ t \in J \tag{21}$$

and $y(t) = \phi(t)$ *for* $t \in [t_0 - \gamma, t_0]$.

Because of the various assumptions on our system and the estimate from (17) to (19) it is clear that $|v(t)| \leq a, \ t \in J$ *and also* $v: [t_0 - h, t_0] \to u$, *so* $|v(t)| \leq a$ hence

$\|v\|_2 \leq a(t_1 + h - t_0)^{1/2} = \beta_0$ *next*

$$|y(t)| \leq c_2 + c_3 \exp[-\alpha(t - t_0)] + c_4 \int_0^t |v(s)| ds + km \exp(-\alpha t_1) \quad where$$

$$c_4 = \sup |G_m(t,s)| \quad since \ \ \alpha > 0, t \geq t_0 \geq 0, \ it \ follows \ that$$

$$|y(t)| \leq c_2 + c_3 + c_4 a(t_1 - t_0) + km \equiv \beta \qquad t \in J$$

$$|y(t)| \leq \sup |\phi(t)| = \sigma, \quad t \in [t_0 - h, t_0],$$

hence, if $\lambda = \max\{\beta, \sigma\}$, *then* $\|y(t)\|_2 \leq \lambda (t_1 + h - t_0)^{1/2} = \beta_1$.

Let $r = \max\{\beta_0, \beta_1\}$, *then letting* $Q(r) = \{(x,u) \in B: \|x\|_2 \leq r, \|u\|_2 \leq r\}$.

It follows that $T: Q(r \to Q(r))$. since $Q(r)$ is closed, bounded and convex, by Rieze's theorem (Kantorovich and akilov, 1992) it is relatively compact under T. The Schauders theorem implies that T has a fixed point $(x,u) \in Q(r)$., this fixed point (x,u) of T is a solution pair of the set of integral equations (20) and (21). Hence the system (6) is Euclidean null controllable.

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