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Solving Minimum Cost Multi-Commodity Network Flow Problem Using Lexicographic Goal Programming Approach

***OLUSINA, JO; NSE, OU**

Department of Surveying & Geoinformatics, Faculty of Engineering, University of Lagos, Lagos State, Nigeria
Email: joolusina1@yahoo.com; kingso4heaven@yahoo.com

ABSTRACT: In an urban transport system, a dysfunction often occurred as demand for transportation infrastructure exceeds available supply. The result includes traffic congestion, higher travel time and cost, higher emission of harmful gases and general reduction in quality of life. In this research, an attempt was made to minimize travel time on three urban road segments using Lexicographic Goal Programming. The positive and negative deviations from the goals were minimized. A minimum cost multi-commodity network flow problem with multiple objectives was successfully modelled using LINDO 6.1. The modelling technique provided a solution that effectively minimized travel time by 50%.

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Transportation networks are complex systems, and come in a variety of forms, such as road, rail, air, and waterway networks. The aim of transportation within the context of utility services is to aid travel of persons and goods from place to place within the most reasonable time frame. Transportation networks provides for the functioning of societies and economies through the movement of people, goods, and services from one place to another. From an economic perspective, demand for transportation is represented by the users of the transportation system while the supply is represented by the underlying network topology and the ability to meet the demand given the cost characteristics (Sheffi, 1985). An equilibrium occurs when the number of trips between an origin (e.g. residence/place of employment) and destination (place of employment/residence) equals the travel demand given by the market price, typically, represented by the travel time for the trips (Nagurney and Dong, 2004).

In urban conurbation of developing nations, such as Lagos, several transportation related problems exists, including inadequate road capacity to meet swelling demand and faulty network topology (intersections/junctions). They in no small way affect travel time. Many road users, desire minimized travel time within urban centers. Travel time minimization is the attempt to reduce the time it takes to travel from one point in space to another (Björklund and Carlén, 2012).

Some Network Flow Models (Optimization Techniques) for City Traffic Congestion include works by: Zawack and Thompson (1983) in their work discussed the use of a space-time network to model traffic flows over time for a capacitated road transportation system having one-way and two-way streets; Jisheng, et al (2015) presented a space-time network-based modeling framework for integrated fixed and mobile sensor networks, in order to provide a rapid and systematic road traffic monitoring mechanism; Fisk (1983) in his paper drew correspondences between two game theory models (Nash noncooperative and Stackelberg games) and solved the problem of carriers competing for intercity passenger travel and signal optimization in transportation systems; Easley and Kleinberg (2010), developed models for network traffic using the game-theoretic ideas, i.e. the Braess's Paradox, which shows that adding capacity to a network can sometimes actually slow down the traffic. Also, Sifaleras (2013) presented a wide range of problems concerning minimum cost network flows, and give an overview of the classic linear single-commodity Minimum Cost Network Flow Problem (MCNFP) and some other closely related problems, either tractable or intractable; Geranis, et al (2009) presented for the first time an exterior point dual simplex-type algorithm for the Minimum Cost Network Flow Problem (MCNFP) named Dual Network Exterior Point Simplex Algorithm

**Corresponding Author Email: joolusina1@yahoo.com*

(DNEPSA) for the MCNFP; while on Goal Programming, Tamiz and Jones, 2010 and Schniederjans, 1984 described Goal Programming as a Multi-Objective Decision Making (MODM) technique that allocates scarce resources (road capacity) to competing needs (vehicles) in an optimal manner when the decision maker has multiple objectives (minimize travel time, maximize speed and maximize the number of vehicles that a route can carry), so long as the problem can be expressed by an objective function and linear inequality constraints. Using the Goal Programming model, a goal is expressed as a mathematical equation and a deviational variable(s) assigned to it. Deviational variables can be positive or negative. The model involves two types of constraints: system (i.e. on existing capabilities or available resources) and goal constraints (decision maker's desired goals). In most cases, objectives are in conflict with each other and may or may not be equally important. In Lexicographic Goal Programming, objective functions are ordered according to their importance. This implies that goals of higher priority must be met before those of lower priority are considered. Preference weight of positive infinity is assigned to a goal of higher priority compared to that of the goal of next-lower priority. The general Lexicographic Goal Programming model (also known as the pre-emptive Goal Programming) can be written as follows (Ignizio, 1976 and Tamiz, et al, 2010):

$$Lexi \min Z = \left[\sum_{i-1} x_j(d_i^- + d_i^+), \ldots, \sum_{i-1+m} x_j(d_i^- + d_i^+), \ldots, \sum_{i-1+n} x_j(d_i^- + d_i^+)\right] \quad (1)$$

Subject to: $\left(\sum_{j=1}^{n} a_{ij}\, x_j\right) + d_i^- \mid d_i^+ = b$, i = 1,2,..........,n, $\quad i \in j, \quad j \in (1, \ldots \ldots, P)$ (2)

where: d_i^- and d_i^- - under- and over- achievements of the *ith* goal respectively;

a_{ij} - constant for representing the preemptive priority of deviational variables d_i^- and d_i^+; j - the index set of goals placed in the *1+ith* priority level; *n* - total number of attributes separated in *P* priority levels; $a_{ij}\, x_j$ - the linear function of the *ith* goal; b_i - an aspiration level of the *ith* goal; *X* **-** the vector of decision variables; *F* **-** the feasible set of constraints.

Lexicographic Goal Programming was used in this work being an approach that solves Goal Programming problems with priorities on objectives in order achieve travel time minimization, speed maximization and volume of traffic maximization) during peak periods on urban highways in Lagos.

MATERIALS AND METHODS

*The Study Area***:** Although Lagos State is the smallest state in Nigeria, with an area of 356,861 hectares of which 75,755 hectares are wetlands, the State has an estimated population of 17 million out of a national estimate of 150 million. The rate of population growth is about 600,000 per annum with a population density of about 4,193 persons per sq. km. (Lagos Official Website, 2009). With its high population always witness heavy demand for transportation causing heavy congestion on a daily basis. Unlike in advanced economies, there is no integrated or multimodal transportation system. Actually, there is poor inter-face between Rail and Road, thus making it difficult to enjoy the ease and benefits of Inter-modal passenger services (Nwanze, 2002 and Olusina, 2008). Till now water transportation maximum efficiency has not been achieved (Adejare, et al, 2011). The study area selected for this work are three highways within the Lagos Metropolis (i.e. Ikorodu Road/Murtala Mohammad Highway [from Mile 12 to Oyingbo]), Herbert Macaulay Way [from Jibowu to Adekunle Interchange] and Lagos Badagry Expressway [from Eric Moore to Apapa/Oshodi Expressway Interchange]). Figure 1 shows a graphical representation of the study area, showing the specific roads rendered in red, blue and purple lines.

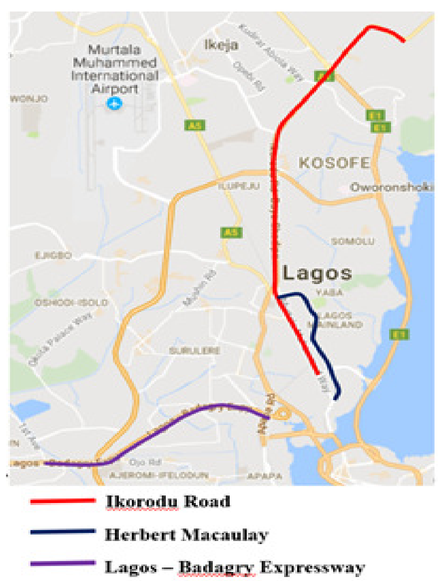

Fig 1: Selected Highways in the Study Area (Google Map, 2016).

Data Types and Sources- Data used for this research were collected from both primary and secondary sources.

Primary data sources include field surveys (e.g. vehicle traffic, travel time data, etc.), while Secondary data include traffic and travel time data collected from regulatory bodies (e.g. Lagos Metropolitan Transport Agency [LAMATA], The Federal Road Safety Commission [FRSC], scholarly materials and journals, academic works and textbooks).

Data Collection: One directional travel time minimization using the Lexicographic Goal Programming approach of heterogeneous traffic flows, on three straight and level stretches of a multi-lane divided roads, in the megacity of Lagos (Ikorodu Road [Mile 12 - Oyingbo], Herbert Macaulay Way [Jibowu junction - Adekunle Interchange] and Lagos - Badagry Expressway [Eric Moore - Apapa/Oshodi Expressway Interchange]) were studied. Roads width is 3.65m and the Roads Design capacity is 2000pcu/hr. per lane. The attributes of data collected are shown in Tables 1 and 2, while the Route Characteristics are shown in Table 2.

Table 1: Data Collected.

Activity	Duration	Type of Data Collected
Hourly (morning and evening) peak periods traffic counts at selected points along the routes	3-months	Classified count vehicles: (i) Long vehicles- Trailers/Large buses/Trucks, (ii) Personal vehicles- Cars/SUVs/Small vans, (iii) Commercial Vehicles- Taxi cabs, Small 4-wheeled goods vehicles and Danfo minibuses.
Travel time data (Test Vehicle Technique [FHWA-PL-98-035 Report] and LAMATA from 2009 – 2011).		Travel time and speed, and delay for each segment along the survey routes
Global Positioning System (GPS) observation.		Test vehicle position, speed, day of survey, time of' survey (7am - 10am and 4pm - 7pm) and route.

Table 2: Routes Characteristics and Number of GPS Observation runs

S/No.	Route	Route Length (Km)	Ave. Observed Travel Time (Mins)	No. of Runs
1	Ikorodu Road	14.5	70	6
2	Herbert Macaulay Road	5.13	38	6
3	Lagos – Badagry Exp.	6.03	43	6

For this research, BRT Buses were not considered in the modelling process because they travel on dedicated lanes exclusively provided for their use along the roads under review. Since all the routes are not having the same number of lanes, therefore, compartmentalization of each route based on segment route characteristics was adopted. For example, between Ojota and Maryland, along Ikorodu road, has a distance of 2.6km and total width of 7.3m (2 lanes) with three major vehicle classification types- (i) Trailers/Large buses, (ii) Cars/SUVs, (iii) Minibuses. The vehicles Passenger Car Unit (PCU) values and the average vehicular speed are shown in Table 3.

Table 3: Observed Traffic Characteristics on Ojota - Maryland Corridor.

Variable	Vehicle	PCU	Average Speed (Km/h)	Passenger Capacity	Average Hourly Volume	Total Speed
X1	Large Buses	3	23	56	71	1,633
X2	Minibuses	1.6	26	15	809	21,034
X3	Car/Jeep	1	30	2	2,685	80,550
TOTAL					**3,565**	**103,217**

Data Processing- Relevant technologies and applications used in the development of the work include: LINDO 6.1 software - to solve the minimum cost multi-commodity network flow problem using Lexicographic Goal Programming approach was used in this research; Microsoft Excel 2007 - used for sorting, arranging, summing and analyzing observed traffic counts and travel times along the routes; GPS Utilities Software- was used to downloaded GPS data onto the computer e.g. GPS signal/position error, errors in accurate recording of departure and arrival times, gaps in recording of delay times, etc.

Mathematical Model Formulation: From the Highway LOS Ratings (Victoria Transport Policy Institute, 2013), if traffic volume is reduced from 2,000 to 1,800 veh./hr. (a 10% reduction), a roadway will be shifted from LOS E to LOS D, which increases traffic speeds by about 15 mph (a 30% increase). This indicates that on a congested roadway, small reductions in traffic volume can relatively minimize travel times. For this research, the goal was to achieve a LOS D transportation system on the selected routes. Observed traffic values at peak hours in the selected routes revealed a pattern that is consistent and close to Class E Level of Service where there is a high traffic volume usually higher than the road's capacity, flow becomes irregular, speed is minimized and rarely reaches the posted limit. Using the data in Table 3, the overall Objective is to minimize total "cost" of travelling along the link (i.e. *minimize Travel Time*) by adopting a notional speed of 60km/h.

Desired Goals: The following desired goals have been arranged in order of importance: (1) To realize a Class D LOS as much as possible; (2) To utilize route capacity up to 90 percent (i.e. 3,600 PCU); (3) To carry an average of 15,000 persons per hour; and (4) To allow up to 1,000 PCU for minibuses per hour. These are all pre-emptive goal targets. Since all goals have been established, the mathematical formulation of the travel time minimization using the Lexicographic Goal Programming approach i.e. solving the objective/goal with the highest priority first, follows (Ignizio, 1976):

Functional/System/Hard Constraints: The following were the identified hard constraints: (i) *Constraint 1 (Speed Constraint [Goal 1])*- maximum observed average speed at peak periods was 26 km/h per day [i.e. nearly E (LOS)]. The goal is to change the E (LOS) to D (LOS) at a notional speed of 60km/h. (ii) *Constraint 2 (Route Capacity Constraint [Goal 2])*- Design Route capacity ≤ 1800 PCU/hr/lane, and for a 2-lane road, the total design capacity is 3600 PCU/hr. Summing all the individual PCU values of the different vehicles must yield the capacity requirement of the route. (iii) *Constraint 3 (Passenger Constraint i.e.* Goal 3 [P3]*)*- To carry an average of 15,000 persons per hour i.e. Passengers ≥ 15,000. (iv) *Constraint 4 (Flow Constraint [Goal 4])*- Vehicles are all simultaneously in motion in platoons with minimal headways, longer travel times and lower speeds [i.e. Cars/SUVs (X_3) ≥ Minibuses (X_2) ≥ Large Buses (X_1)]. Allow up to 1,000 minibuses per hour i.e. Minibuses (X_2) ≥ 1000/hr.

Where the Goals are the Priorities (i.e. Goal 1 = P1, Goal 2 = P2, Goal 3 = P3 and Goal 4 = P4, and P1 ≥ P2 ≥ P3 ≥ P4).

The *Decision Variables* chosen were used to formulate all goals. All of these goals are determined by the number of vehicles that can pass through a section of road per hour. Therefore: X_1 - Heavy Vehicles (Molue Buses); X_2 - Minibuses; and X_3 - Cars / SUVs.

*Writing the Lexicographic Goal Programming Model***:** If P_i represents the Pre-emptive priority goals, P_1, P_2,..............., P_i (from the highest priority to the lowest), and since Goal Programming Model entailed combining the constraints and the objective function, therefore, the Lexicographic Goal Programming Model is given in Equations 3- 10.

Solving the Model: This Minimum Cost Multi-commodity Network Flow with goals problem was solved as a linear program, using Lindo 6.1. The Linear nature of the model enabled the solution by Linear Lexicographic Goal Programming Model.

$$\textit{Lexi} \min Z = P_1 d_{p1}^- + P_2(d_{p2}^- + d_{p2}^-) + P_3 d_{p3}^- + P_4 d_{p4}^- \quad (3)$$

Subject to (All the Constraints)

Goal 1: $45X_1 + 55X_2 + 60X_3 + d_{p1}^- - d_{p1}^+ = 150000$ (Average Speed Goal) (4)

Goal 2: $3X_1 + 1.6X_2 + X_3 + d_{p2}^- - d_{p2}^+ = 3500$ (Capacity Goal) (5)

Goal 3: $56X_1 + 15X_2 + 2X_3 + d_{p3} - d_{p3}^+ = 15000$ (Passenger Goal) (6)

Goal 4: $X_2 + d_{p4}^- - d_{p4}^+ = 1000$ (Minibus Goal) (7)

$3X_1 + 1.6X_2 + X_3 \le 3600$ (Capacity: Hard Constraint) (8)

$X_3 \ge 2000$ (Cars/SUVs: Hard Constraint) (9)

$3X_1 + 1.6X_2 \le 1600$ (Buses: Hard Constraint) (10)

RESULTS AND DISCUSSION

The prevalent/existing vehicular distribution is given in Figure 2. The figure shows that there are more single occupant vehicles along all the roads per time than other vehicle types. The implication of this is that, more space is used up by a few commuters, which does not in any way measure up to the demand for road use by other road users.

Implementation of Model, with All the Goals
Implementation of Model with Goal 1- The goal here was to minimize the underachievement of the desired Class D Level of Service where the average speed is given at 60km/h. This is implicitly achieved by multiplying the total volume of vehicles by the average speed. The output summary of results for Goal 1 from the implementation of the model in LINDO 6.0 is shown in Table 4.

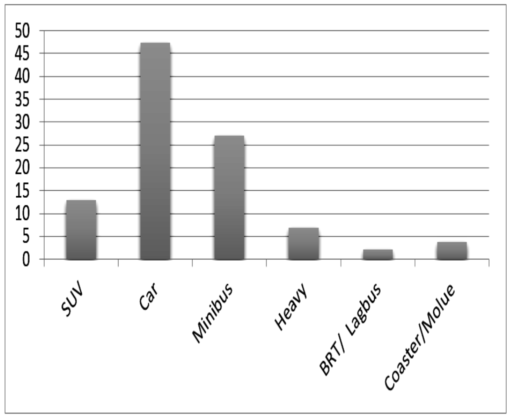

Fig 2: Observed distribution of prevalent Vehicle Types in all studied Roads (in Percentages).

Table 4: Summary of Results for Goal 1

Resource	Target Level	Underachievement	Overachievement	Achieved
Total Speed	160,000	0	0	160,000
Capacity	3000	0	400	3,400
Passengers/Hour	15000	0	7133	22,133
Minibuses	900	0	100	1,000

From Table 4 we can therefore derive the average speed, traffic flow, travel time as well as density of the traffic stream. From the capacity value of 3,400 PCU and the individual PCU values of different vehicle types, a flow of 2,901 vehicles/hr was obtained.

Total Vehicles = $\frac{Flow}{hr}$ = 2,901 veh/hr.; Average Speed = $\frac{160,000}{2,901}$ = 57.82km/hr.;

Density = $\frac{Flow}{Speed} = \frac{2,901}{57.82}$ = 50veh/km;

Travel Time = $\frac{Distance}{Speed} = \frac{2.6}{57.82}$ = 0.045hrs.

Consequent upon the overachievement of the route capacity goal, number of passengers goal and number of Minibuses goal, the Model implementation was further achieved with Goals 1 and 2; Goals 1, 2 and 3; and with all Goals:

Implementation of Model with Goals 1 and 2- Implementing the model for goal 2 using the achieved value of goal 1, results showed that the second pre-emptive goal is not satisfied with an overachievement value of the route capacity target by 7%, an overachievement of the number of passengers per hour by 5146 and an overachievement of the number of minibuses at 225 was observed.

Implementation of Model with Goals 1, 2 and 3- Furthermore, the third pre-emptive goal was sequentially solved, assuming satisfaction of the upper- priority goals at the levels already attained. From the output, the third pre-emptive goal was satisfied with an overachievement of 5148 Number of Passengers.

*Implementation of Model with all Goals:*_With the attainment of higher priority goals at their respective levels of achievement, the model with all pre-emptive goals was implemented. The output showed that Goal 4 is achieved with an overachievement of 225 Minibuses. The results show stability of decision variable values between goals 2, 3 and 4. This can be attributable to the fact that with the given resources and constraints, the decision maker's goals are met to a certain degree of achievement.

Post Optimal Analysis: Sensitivity Post-optimal analysis was carried out in order to investigate the robustness of the solutions of the model.

Sensitivity Analysis It re-orders the pre-emptive goals in a priority level different from the baseline model. The reordering of pre-emptive priorities may reveal restrictions a higher pre-emptive goal places on a lower pre-emptive goal. The solutions from each pre-emptive goal scheme will depict how the flow of the different "commodities" attains the goals differently. This will bring insight to the decision maker, particularly in situations where the results of the prioritizations have resulted in unintended consequences. Four variations were made to the baseline model as depicted in Table 5.

Implementation of Variations 1 - 4 of the Model: Variation 1 interchanges the second, fourth and third pre-emptive goals. P2 which is to maintain route capacity utilization at 80.5% becomes the main priority. The model was implemented to minimize overachievement of the capacity goal. Similarly, the implementation of Variations 2, 3 and 4 of the Model was carried out and the various results are shown in Table 6.

From Table 6, minimizing the negative deviation of the total speed of vehicles yielded an optimal value of 67 Large Buses, 1000 Minibuses and 1700 Passenger Cars. Route capacity however is placed at 3400 PCU/hour which is 85% of route capacity while 22133 persons are commuted in the hour. Minimizing the positive and negative deviations from the target route capacity while holding the achieved value goal 1 constant, produced a departure from the initial achieved route capacity values. The results obtained eliminated large vehicles and gave preference for minibuses and passenger cars. The target route capacity value was not achieved because an overachievement of 323 PCU/hr. was obtained. This route capacity value was better than the initial value of 400 obtained when the total speed values were minimized. The results seem to suggest that in trying to solve the problem of traffic congestion, consideration must be given to high occupancy vehicles with low impedance values than vehicles with a higher impedance value.

	Table 5. Re-Ordering of Priorities				**Table 6. Summary of Results of the Different Variations.**					
					Total Speed	**Route Capacity**	**Persons/Hr.**	**Large Buses**	**Minibuses**	**Passenger Cars**
Base Model	P1	P2	P3	P4	**160,000**	**3,323**	**20,146**	**0**	**1,125**	**1,635**
Variation 1	P2	P4	P1	P3	**132,300**	**3,000**	**22,800**	**120**	**900**	**1,290**
Variation 2	P4	P1	P2	P3	**160,000**	**3,400**	**22,133**	**67**	**1,000**	**1,700**
Variation 3	P3	P2	P4	P1	**156,900**	**3,410**	**23,620**	**120**	**900**	**1,700**
Variation 4	P1	P4	P3	P2	**160,000**	**3,400**	**22,133**	**67**	**1,000**	**1,700**

Comparing the travel times derived with travel times observed from the test vehicle process, Table 7 shows the variations in travel times. The derived travel times depends on a controlled volume of traffic rather than the laissez faire system of traffic, which is the prevailing capacity allocation method on the roads.

Table 7: Comparison between Observed Travel Time values and the Derived Travel Time

S/ No	**Route**	**Observed Travel Time (Min.)**	**Derived Travel Time (Min.)**	**Differenc e (Min.)**	**Remarks**
1	Ikorodu Road	48.3	21.8	26.5	55% Reduction
2	Herbert Macaulay	14.6	7.3	7.3	50% Reduction
3	Lagos - Badagry Expressway	15.7	8.0	7.7	49% Reduction

And it can be observed that if the volume is controlled with respect to the road capacity, the average vehicle speed increases significantly, and the travel time reduces conversely until the appropriate value is obtained.

Conclusion: Lexicographic Goal Programming Approach has been applied to allocate resources, mostly, when multiple conflicting goals of shortest possible travel times, maximum number of vehicles and increase in passenger carriage as much as possible on the road. It was used to derive a vehicle mix in terms of Passenger Car Units, Vehicle Occupancy and Vehicle Speed on the three selected roads at a given time in order to reduce time of commuting from one point to another. Having implemented the model, decision maker's initial goal priorities were changed and by examining the solutions, an estimate of the trade-offs between goals were obtained.

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