Derivation of Stochastic Equations for Computational Uncertainties in Petro-Physical Properties Using a Simplified Algorithm

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ABSTRACT: This paper presents a simple mathematical algorithm or procedure for computing the uncertainties at the various percent of data input, using the stochastic approach of simulating the input variables to compute the output variables. A simple algorithm was used to derive stochastic equations for some selected petro-physical parameters using the relative standard deviations techniques (σ). These equations also known as reference work equations were found to produce reasonably accepted magnitude of uncertainties in the different parameters associated with cores. Equations were derived for the percent uncertainties in the values of the pore volume of the core - V_p, the fluid saturation – S_w, S_{or}, the porosity of the core – Φ, formation factor - F, bulk density – ρ_B, the derived porosity - Φ_L and the derived permeability – K_{BU}, Kro, Krw. The uncertainty equations can also be used to define the maximum level of uncertainty that can be tolerated in any independent variable if the maximum uncertainty to be tolerated in the dependent variable is known. © JASEM

Keywords: Stochastic equations, petrophysical parameters, percent uncertainties, Mathematical algorithm, standard deviations, partial derivatives, degree of accuracy, empirical models**.**

LIST OF ABBREVIATION

a=	Intercept values of F at $S_w = 1.0$	P_a=	Atmospheric pressure
A=	Cross sectional area	P_1=	Initial reference volume pressure for Boyle's law grain volume (psig)
B_o=	Oil formation volume factor (res BBL/STB)	P_2=	Final reference volume pressure for Boyle's law grain volume (psig)
CEC=	Cation exchange capacity (milliequivalent/100grams)	P_u=	Porosity units (%)
ΔP_b=	Pressure drop (Base permeability)	P_w=	Flowing wellbore pressure (psig)
ΔP=	Pressure drop	P_{1h}=	Ideal wellbore pressure after 1hour shut in (psig)
F=	Formation factor (R_o/R_w)	Q_v=	Cation exchange capacity/unit pore volume (milliequivalent/cc)
h=	Formation thickness (feet)	q_g=	Gas flow rate at atmospheric pressure (cm^3/s)
K_L=	Liquid permeability (Darcies)	q_L=	Liquid flow rate (cm^3/s)
K_g=	Gas permeability (Darcies)	q_o=	Oil flow rate (cm^3/s)
K_{ro}=	Oil relative permeability (fraction)	q_t=	Total flow rate [$q_o + q_w$] (cm^3/s)
K_{rw}=	Water relative permeability (fraction)	q_w=	water flow rate (cm^3/s)
K_{BU}=	Build up permeability (md)	R=	Radius (cm)
K_o=	Oil permeability	r_e=	Drainage radius (ft)
L=	length (cm)	r_w=	Wellbore radius (ft)
m=	Cementation exponent (formation factor equation)	RI=	Resistivity index (R_t/R_o)
n=	saturation exponent	R_o=	Resistivity of 100% water saturation rock (ohm-m)
R_w=	Resistivity of saturating brine (ohm-m)	R_t=	Resistivity of partially water saturated rock (ohm-m)
RSD=	Relative Standard Deviation ($\sigma X/X$), (fraction)	S_{or}=	Residual oil saturation (fractional pore volume)
S_w=	Water saturation (fractional pore volume)	t_L=	Acoustic travel time of formation (micro s/ft)
t_{ma}=	Acoustic travel time of matrix rock (micro s/ft)	t_f=	Acoustic travel time of fluid (micro s/ft)
V_p=	Pore volume (cc)	V_B=	Bulk volume (cc)
V_G=	Grain volume (cc)	V_R=	Reference volume (cc)
V_{mc}=	Matrix cup volume (cc)	V_{oi}=	Initial Oil volume (cc)
V_{op}=	Produced Oil volume (cc)	W_D=	Dry weight (g)
W_I=	Immersed weight (g)	W_S=	Saturated weight (g)
Φ=	Porosity (fraction)	ρ_b=	Bulk density (g/cm^3)
ρ_s=	Saturating fluid density (g/cm^3)	ρ_f=	Fluid density (g/cm^3)
ρ_{ma}=	Matrix density (g/cm^3)	ρ_{wc}=	Volume correction of distilled water to brine volume in sample, fraction > 1.0
ρ_{HG}=	Mercury density	μ_g=	Viscosity of gas (cp)
μ_L=	Viscosity of Liquid (cp)	μ_o=	Viscosity of Oil (cp)
μ_w=	Viscosity of water (cp)	σX=	Standard Deviation in value of X (unit same as X)

The derivation or computation of petrophysical parameters values is an important step in processing of geological or geophysical data as well as in the interpretation of the subsurface data. Many papers have been presented in respect of the computation or derivation of petrophysical parameters either analytically or by stimulation of models. Determination of the degree of uncertainties associated with the method of computation or derivation of the petrophysical parameters is not a recent development, but determining stochastically the degree of uncertainties in the computed values of the parameters has improved the manners and method of computation. Use of Monte Carlo technique to compute the degree of uncertainties dates back to 1964, when it was used for business decisions (Hertz, 1964). Different authors have used different mathematical models (Stochastically) extensively in analysis, such as in ground water hydrology (Freeze, 1970); (Smith *et al.,* 1979) and for petroleum reservoir simulation (Cekirge et al, 1981); (Smith *et al.*, 1982). A model is said to be stochastic if it contains random variables which have probability distributions, such that the uncertainties in input

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variables are simulated to predict uncertainties in output variables (stochastic approach). Most stochastic models are solved by the Monte Carlo technique, which has been used extensively in quantifying uncertainties in petrophysical deliverables (Bishnu *et al.*, 2005), investigation of errors in relative permeability estimates from the JBN technique (Tao *et al.,* 1984) and to evaluate uncertainties in engineering calculations (Walstrom *et al.*, 1986). Monte Carlo modeling is a flexible technique that allows different interpretation models to be incorporated and the degree of uncertainties computed easily. Given the random variables as a function of normal distribution, then the degree of uncertainty in the expected output variable can be computed or determined empirically from the uncertainties in the random input variables by using the simple algorithm (root mean square equation).

METHODOLOGY

*Stochastical Method***:** To illustrate the importance of the technique used for computing the uncertainties, a simple mathematical model is derived for some basic petrophysical parameters. Given a petrophysical parameter by a mathematically variable Y which is proven to be dependent on some independent variables X_i: (X_1, X_2…X_n) and the random error in Y as ΔY. The random error ΔY can be computed from the random errors existing in the independent variables X_i, if these errors are given as ΔX_1, ΔX_2…..ΔX_n.

Then, the root mean square error equation can be applies only if the independent variables are not related as:

$$Y = f(X_1, X_2, X_3, \ldots\ldots\ldots\ldots\ldots\ldots X_n) \quad 1$$

and

$$\Delta Y = \pm \left[\left(\frac{\delta Y}{\delta X_1} \Delta X_1 \right)^2 + \left(\frac{\delta Y}{\delta X_2} \Delta X_2 \right)^2 + \left(\frac{\delta Y}{\delta X_3} \Delta X_3 \right)^2 \right]^{1/2} 2$$

The technique or method for determining the level of uncertainty was previously applied to selected core measurements by Hook (1983) and also applied to water saturations calculated from the Waxman-Smits equation by Freedman et al (1985) and in the analysis of the Archie equation components by Chen et al (1986).

The random errors in many different physical processes can be approximated by a normal distribution, thus the errors are probabilistic in nature, hence stochastic in approach. For the above errors values, errors ΔX_1, ΔX_2,…… ΔX_n and ΔY can be represented by the standard deviations of the values as σX_1, σX_2, ….. σX_n and σY respectively.

Thus using a simple stochastic technique for the above equations, the following simple algorithms (steps) are derived and used:

1. Deriving an equation relating the dependent variable Y to its independent variables X, i.e

$$F = \frac{R_o}{R_w} \quad 3$$

2. Next compute the partial derivatives of the parameters:

$$\frac{\delta Y}{\delta X} \, i.e.., \quad \frac{\delta F}{\delta R_o} \, and \quad \frac{\delta F}{\delta R_w} \quad 4$$

Substituting the partial derivatives (Equation 3 and 4) into the derived error equation given in equation 2 above, and manipulating the resulting equation to usable form, we derive

$$\sigma F = \pm F \left[\left(\frac{\sigma R_o}{R_o} \right)^2 + \left(\frac{\sigma R_w}{R_w} \right)^2 \right]^{1/2} \quad 5$$

or

$$\frac{\sigma F}{F} = \pm \left[\left(\frac{\sigma R_o}{R_o} \right)^2 + \left(\frac{\sigma R_w}{R_w} \right)^2 \right]^{1/2} 6$$

In equation 5 and 6, the ratio of the standard deviation (σF) to the value of the variable (F) is referred to as the relative standard deviation ($\sigma F/F$) and this is commonly computed in fraction or percent.

Finally, derive the degree or level of the parameter uncertainty ($\sigma R_0/R_0$) and $\sigma R_w/R_w$ in the parameter and calculate the resultant relative standard deviation of the dependent variable ($\sigma F/F$). If the uncertainty in a given case is expressed as the relative standard deviation with units of fraction or percent (i.e., $\sigma R_0/R_0$), no actual values of R_0 is necessary required to solve the equations. But in some cases the numerical values must be assigned (This technique or approach is used in deriving the uncertainty level of the parameters), such as in the calculation of pore volume by difference of bulk volume (V_B) and grain volume (V_G), where numeric values of V_B and V_G is assigned. In deriving the equation for uncertainty level of some petrophysical parameters, the independent variables and the equations commonly used to determine core data basic properties such as pore volume, bulk volume, grain volume and porosity are shown in Table 1 (Amaefule, 1989). Similar data relating to core residual oil saturation, electrical properties and specific and relative permeability are also shown in the table as well as variables for well test data and log derived properties. Partial derivatives for the parameters were determined for each independent variable, and the resultant uncertainty equations derived as shown for some selected variable in Table 2, these equations are also known as

Reference Work Equations: These equations allows for insertion of any standard deviation or relative standard deviation desired into the uncertainty equation for subsequent calculation of uncertainty of the computed variable. For example, insertion of the percentage uncertainty in R_0, which is represented by ($\sigma R_0/R_0$), and the percentage uncertainty in R_w, which

is represented by ($\sigma R_w/R_w$) allows for the calculation of the percentage uncertainty in F, represented by ($\sigma F/F$) as shown in Table 2 (Amaefule, 1989).

Table 1: Core Data Computed Variables, Independent Variables and Models Equations (Amaefule, 1989)

Computed Variables	Independent Variables	Model Equations
Basic Properties		
V_P (Saturation)	W_S, W_D, ρ_s	$V_P = \dfrac{(W_S - W_D)}{\rho_s}$
V_p (Boyle's Law) Direct Injection	P_1, P_2, V_R	$V_P = \dfrac{V_R(P_1 - P_2)}{P_2}$
V_p (Difference)	V_B, V_G	$V_P = V_B - V_G$
V_B (Caliper)	R, L	$V_B = \pi R^2 L$
V_B (Hg Immersion)	W_I, ρ_{HG}	$V_B = \dfrac{W_1}{\rho_{Hg}}$
V_G (Saturation)	W_S, W_I, ρ_s	$V_G = \dfrac{W_S - W_I}{\rho_s}$
V_G (Boyle's Law) Matrix Cup	P_1, P_2, V_R, V_{mc}	$V_G = \dfrac{V_R(P_2 - P_1) + P_2 V_{mc}}{P_2}$
Φ_1	V_B, V_G	$\phi_1 = \dfrac{(V_B - V_G)}{V_B}$
Φ_2	V_P, V_B	$\phi_2 = \dfrac{V_P}{V_B}$
Φ_2	V_P, V_G	$\phi_3 = \dfrac{V_P}{(V_P - V_G)}$
ρ_{ma}	W_D, V_G	$\rho_{ma} = \dfrac{W_D}{V_G}$

Computed Variables	Independent Variables	Model Equations
Saturations		
S_{or} (Volumetric balance)	V_{oi}, V_{op}, V_p	$S_{oi} = \dfrac{(V_{oi} - V_{op})}{V_p}$
S_w (Gravimetric)	W_S, W_D, ρ_w, ρ_H	$S_W = \dfrac{(W_S - W_D) - \rho_H}{V_P(\rho_W - \rho_H)}$
Electrical Properties		
F	R_o, R_w	$F = \dfrac{R_o}{R_w}$
RI	R_t, R_o	$RI = \dfrac{R_t}{R_o}$
m	a, F, Φ	$m = \dfrac{\text{Log } a - \text{Log } F}{\text{Log } \phi}$
n	R_t, R_o, S_w	$n = \dfrac{\text{Log } R_o - \text{Log } R_t}{\text{Log } S_w}$
Specific Permeability		
K_L	q_L, A, ΔP, μ_L, L	$K_L = \dfrac{q_L \mu_L L}{A \, \Delta P}$
K_g	q_a, A, P_1, P_2, ΔP, μ_g, L, P_a	$K_g = \dfrac{2q_a \mu_g L P_a}{A(P_1^2 - P_2^2)}$
Relative Permeability		
K_{ro}	q_L, q_o, ΔP_b, ΔP	$K_{ro} = \dfrac{q_o \Delta P_b}{q_t \, \Delta P}$
K_{rw}	q_t, q_w, ΔP_b, ΔP, μ_w, μ_o	$K_{rw} = \dfrac{q_w \Delta P_b \mu_W}{q_t \, \Delta P \mu_o}$
Well Test Data		
K_{BU}	q_o, μ_o, B_o, m, h	$K_{BU} = \dfrac{162.6 q_o \mu_o B_o}{mh}$
Log Derived Properties		
Φ (Acoustic)	t_L, t_{ma}, t_f	$\phi_2 = \dfrac{t_L - t_{ma}}{t_f - t_{ma}}$
Φ (Density)	ρ_{ma}, ρ_f, ρ_b	$\phi = \dfrac{\rho_{ma} - \rho_b}{\rho_{ma} - \rho_f}$
S_w	a, Φ, m, R_w, R_t, n	$S_W = \left[\dfrac{a R_w}{\phi^m R_t}\right]^{1/n}$

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Table 2: Derived Uncertainties Equations for Some Selected Petrophysical Properties (Core Data) (Amaefule, 1989)**:**

Computed Variables	Derived Uncertainties Equation
1. Basics Properties	

- Pore Volume (V_p)

V_P (saturation)

$$\frac{\sigma V_P}{V_P} = \pm\left[\left(\frac{\sigma W_S}{W_S - W_D}\right)^2 + \left(\frac{\sigma W_D}{W_S - W_D}\right)^2 + \left(\frac{\sigma \rho_S}{\rho_S}\right)^2\right]^{1/2}$$

V_P (Boyle's Law)

$$\frac{\sigma V_P}{V_P} = \pm\left[\left(\frac{\sigma V_R}{V_R}\right)^2 + \left\{\left(\frac{\sigma P_2}{P_2}\right)\left(\frac{P_1}{P_1 - P_2}\right)\right\}^2 + \left(\frac{\sigma P_1}{P_1 - P_2}\right)^2\right]^{1/2}$$

V_P (Difference)

$$\frac{\sigma V_P}{V_P} = \pm\left[\left(\frac{\sigma V_B}{V_B - V_G}\right)^2 + \left(\frac{\sigma V_G}{V_B - V_G}\right)^2\right]^{1/2}$$

- Bulk Volume (V_B)

V_B (Caliper)

$$\frac{\sigma V_B}{V_B} = \pm\left[\left(\frac{2\sigma R}{R}\right)^2 + \left(\frac{\sigma L}{L}\right)^2\right]^{1/2}$$

V_B(Hg Immersion)

$$\frac{\sigma V_B}{V_B} = \pm\left[\left(\frac{\sigma W_1}{W_1}\right)^2 + \left(\frac{\sigma \rho_{Hg}}{\rho_{Hg}}\right)^2\right]^{1/2}$$

Grain Volume (V_G)
V_G (Saturation)

$$\frac{\sigma V_G}{V_G} = \pm\left[\left(\frac{\sigma W_S}{W_S - W_1}\right)^2 + \left(\frac{\sigma W_1}{W_S - W_1}\right)^2 + \left(\frac{\sigma \rho_S}{\rho_S}\right)^2\right]^{1/2}$$

Porosity (Φ)
Φ_1

$$\frac{\sigma \phi_1}{\phi_1} = \pm\frac{V_G}{V_B - V_G}\left[\left(\frac{\sigma V_G}{V_G}\right)^2 + \left(\frac{\sigma V_B}{V_B}\right)^2\right]^{1/2}$$

Φ_2

$$\frac{\sigma \phi_2}{\phi_2} = \pm\left[\left(\frac{\sigma V_P}{V_P}\right)^2 + \left(\frac{\sigma V_B}{V_B}\right)^2\right]^{1/2}$$

Matrix Density (ρ)
ρ_{ma}

$$\frac{\sigma \rho_{ma}}{\rho_{ma}} = \pm\left[\left(\frac{\sigma W_D}{W_D}\right)^2 + \left(\frac{\sigma V_G}{V_G}\right)^2\right]^{1/2}$$

2. Saturations
S_{or} (Volumetric Balance)

$$\frac{\sigma S_{or}}{S_{or}} = \pm\left[\left(\frac{\sigma V_P}{V_P}\right)^2 + \left\{\left(\frac{\sigma V_{oi}}{V_{oi}}\right)\left(\frac{V_{oi}}{V_{oi} - V_{op}}\right)\right\}^2 + \ldots\ldots + \left\{\left(\frac{\sigma V_{op}}{V_{op}}\right)\left(\frac{V_{op}}{V_{oi} - V_{op}}\right)\right\}^2\right.$$

3. Electrical Properties
 - F (formation factor)

$$\frac{\sigma F}{F} = \pm\left[\left(\frac{\sigma R_o}{R_o}\right)^2 + \left(\frac{\sigma R_w}{R_w}\right)^2\right]^{1/2}$$

RI (Resistivity Index) –
(R_t/R_o)

$$\frac{\sigma RI}{RI} = \pm\left[\left(\frac{\sigma R_t}{R_t}\right)^2 + \left(\frac{\sigma R_o}{R_o}\right)^2\right]^{1/2}$$

- m (Cementation exponent)

- n (Saturation exponent)

$$\frac{\sigma m}{m} = \pm\frac{1}{\ln\left(\frac{a}{f}\right)}\left[\left(\frac{\sigma a}{a}\right)^2 + \left(\frac{\sigma F}{F}\right)^2 + \ldots\ldots \ldots\ldots + \left(\frac{\ln\left(\frac{a}{f}\right)}{\ln \phi}\bullet\frac{\sigma \phi}{\phi}\right)^2\right.$$

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$$\frac{\sigma n}{n} = \pm\left[\left(\frac{\sigma S_W}{S_W}\bullet\frac{1}{\ln S_W}\right)^2 + \left(\frac{\sigma R_t}{R_t}\bullet\frac{1}{\ln(R_t/R_o)}\right)^2 + ...\ ...... + \left(\frac{\sigma R_o}{R_o}\bullet\frac{1}{\ln(R_t/R_o)}\right)^2\right]^{\frac{1}{2}}$$

4.Specific Permeability

K_L(Liquid Permeability)

$$\sigma \ln K_L \cong \pm\left[\left(\frac{\sigma q_L}{q_L}\right)^2 + \left(\frac{\sigma A_L}{A}\right)^2 + \left(\frac{\sigma \Delta P}{dP}\right)^2 + \left(\frac{\sigma \mu_L}{\mu_L}\right)^2 + ... + \left(\frac{\sigma L}{L}\right)^2\right]$$

Note:

$$\frac{\sigma K_L}{K_L} \cong \left[(\ell^{\pm\sigma} \ln K_L) - 1\right]$$

5.Relative Permeability
K_{ro} (Oil relative
    Permeability

$$\frac{\sigma K_{ro}}{K_{ro}} = \pm\left[\left(\frac{\sigma q_t}{q_t}\right)^2 + \left(\frac{\sigma q_o}{q_o}\right)^2 + \left(\frac{\sigma \Delta P_b}{\Delta P_b}\right)^2 + \left(\frac{\sigma \Delta P}{\Delta P}\right)^2\right]$$

6. Well Test Data

K_{BU}(Build-up
    Permeability)
        Note:

$$\sigma \ln K_{BU} \cong \pm\left[\left(\frac{\sigma q_o}{q_o}\right)^2 + \left(\frac{\sigma \mu_o}{\mu_o}\right)^2 + \left(\frac{\sigma B_o}{B_o}\right)^2 + ...\ .. + \left(\frac{\sigma m}{m}\right) + \left(\frac{\sigma h}{h}\right)^2\right]$$

$$\frac{\sigma K_{BU}}{K_{BU}} \cong \left[(\ell^{\pm\sigma} \ln K_{BU}) - 1\right]$$

7. Log Derived Properties

Φ (Acoustic)

$$\frac{\sigma \phi}{\phi} = \pm\left[\begin{array}{l}\left\{\left(\frac{\sigma t_L}{t_L}\right)\left(\frac{t_L}{t_L - t_{ma}}\right)\right\}^2 + \left\{\left(\frac{\sigma t_f}{t_f}\right)\left(\frac{t_f}{t_f - t_{ma}}\right)\right\}^2\left(\frac{\sigma t_L}{t_L}\right)^2 + ...\ ....\ ....\\ ....\ .... + \left\{\left(\frac{\sigma t_{ma}}{t_{ma}}\right)\left(\frac{t_{ma}}{t_f - t_{ma}}\right)\left(\frac{t_L - t_f}{t_f - t_{ma}}\right)\right\}^2\end{array}\right]$$

S_w (Water Saturation –
fractional pore volume)

$$\frac{\sigma S_W}{S_W} = \pm\frac{1}{n}\left[\begin{array}{l}\left(\frac{\sigma a}{a}\right)^2 + \left(\frac{\sigma R_W}{R_W}\right)^2 + \left(\frac{\sigma R_t}{R_t}\right)^2 + ...\ ....\\ .... + m^2\left(\frac{\sigma \phi}{\phi}\right)^2 + (m - \ln \phi)^2\left(\frac{\sigma m}{m}\right)^2 + ... + (n*\ln S_W)^2\left(\frac{\sigma n}{n}\right)^2\end{array}\right]$$

RESULTS AND DISCUSSION

Derivation of Uncertainty Equation Of Selected Petro physical Parameters: Stochastic technique is used to derive equations for some selected petrophysical parameters using the relative standard deviation equations obtained and the root mean square technique. Some of the derived equations shown in Table 2, are used in various ways, the uncertainty assigned can be of equal or unequal value depending on each of the independent variables. Experimentally, some of the variables are less rigidly controlled with large standard deviations, from the mean value or technique limitations derived values that vary within larger percentages than others. The differences in values are accounted for in the uncertainty equations, which will give an appropriate uncertainty values in the computed variables. Examples of the derivation of stochastic equations using the simple algorithm and the parameters in Table 2 are:

Pore Volume: The equation for calculating the percent uncertainty in pore volume Vp resulting for uncertainty in core dry weight, saturated weight and density of saturated fluid is given as

$$V_P = \frac{(W_S - W_D)}{\rho_S} \qquad\qquad 7$$

$$\frac{\sigma V_P}{V_P} = \pm \left[\left(\frac{\sigma W_S}{W_S - W_D} \right)^2 + \left(\frac{\sigma W_D}{W_S - W_D} \right)^2 + \left(\frac{\sigma \rho_S}{\rho_S} \right)^2 \right]^{1/2} \qquad\qquad 8$$

$$\frac{\sigma V_P}{V_P} = \pm \left[\left(\frac{\sigma W_S}{W_S} \bullet \frac{W_S}{W_S - W_D} \right)^2 + \left(\frac{\sigma W_D}{W_D} \bullet \frac{W_D}{W_S - W_D} \right)^2 + \left(\frac{\sigma \rho_S}{\rho_S} \right)^2 \right]^{1/2} \qquad 9$$

Porosity: The equation for calculating the percent uncertainty ($\frac{\sigma \phi}{\phi}$) in porosity resulting from uncertainty in core bulk and grain volumes is given as

$$\phi = \frac{V_B - V_G}{V_B} \qquad\qquad 10$$

$$\frac{\sigma \phi}{\phi} = \pm \frac{V_G}{V_B - V_G} \left[\left(\frac{\sigma V_G}{V_G} \right)^2 + \left(\frac{\sigma V_B}{V_B} \right)^2 \right]^{1/2} \qquad\qquad 11$$

Formation Factor: The equation for calculating the percent uncertainty ($\frac{\sigma F}{F}$) in formation factor (F) for uncertainty in resistivity of 100% saturated sample resistivity R_o and water resistivity R_w is given as

$$F = \frac{R_o}{R_w} \qquad\qquad 12$$

$$\frac{\sigma F}{F} = \pm \left[\left(\frac{\sigma R_o}{R_o} \right)^2 + \left(\frac{\sigma R_w}{R_w} \right)^2 \right]^{1/2}. \qquad\qquad 13$$

Permeability: The equation for calculating the percent uncertainty $\frac{\sigma K_{BU}}{K_{BU}}$ in well build-up permeability for uncertainty in flow rate, viscosity, formation volume factor, slope and thickness is given as

$$\ln K_{BU} = \ln \left(\frac{162.6 \, q_o \mu_o B_o}{mh} \right) \qquad\qquad 14$$

$$\sigma \ln K_{BU} = \ln \left(\frac{\sigma q_o}{q_o} \right)^2 + \left(\frac{\sigma \mu_o}{\mu_o} \right)^2 + \left(\frac{\sigma B_o}{B_o} \right)^2 + \left(\frac{\sigma m}{m} \right)^2 + \left(\frac{\sigma h}{h} \right)^2 \qquad 15$$

The derived stochastic equations makes its easy and simple to determine the degree of uncertainties of any petrophysical parameters, these equations can easily be verified with derived or experimentally petrophysical parameters values .

effects. These uncertainty equations are generic solutions, this allows for uncertainty opportunity or environs that allows for dependent variables for any assigned value of uncertainty in the independent variables.

Conclusions: A generalized uncertainty equation utilizing the root mean square equation has been developed for some selected laboratory core measurements, well and log data parameters. These equations can be used to compute the degree or percent uncertainty of each independent variable to the total uncertainty of the dependent variable, this procedure assist in identifying the independent variables contributing most to the uncertainty level and the parameters to target for reduction in the degree of uncertainty of the parameters or uncertainty

High correlation between the observed and calculated uncertainty for laboratory derived porosity and formation factor support the equations used to theoretically assess uncertainty in a dependent variable when reasonable values of uncertainty in the independent variables are assigned. The uncertainty equations can also be used to define the maximum level of uncertainty that can be tolerated in any independent variable if the maximum uncertainty to be tolerated in the dependent variable is known.

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