ESTIMATION OF ORTHOMETRIC HEIGHT USING EGM2008 AND GPS OVER NAIROBI COUNTY AND ITS ENVIRONS

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Abstract
Global Navigation Satellite Systems (GNSS) have been used widely in 3-dimensional positioning globally, regionally and locally. Global Positioning System (GPS) is one of the most extensively used GNSS in Earth sciences. GPS which employs World Geodetic System adopted in 1984 (WGS84) as the reference system has extensively been used for height determination and has brought a revolution on how relative and absolute heights on earth's surface are determined. GPS measures ellipsoidal heights above a reference ellipsoid (WGS84). Although these heights can be useful in deformation surveys, machine monitoring and guidance, they are not applicable in engineering projects (e.g. sewer lines, pipelines and road construction among others) where heights referenced to an equipotential surface (geoid) are required. The separation between the geoid and a reference ellipsoid (geoid undulation) is necessary in converting ellipsoidal height into orthometric height. In this study we determine geoid undulation from Earth Gravitational Model of 2008 (EGM2008) using freely available Alltrans EGM2008 calculator software version 3.002 at 18 GPS/levelling points. The determined geoid undulations are used to determine estimated orthometric heights from ellipsoidal heights. We then model the differences between spirit-levelled orthometric and estimated orthometric heights by a four parameter model (first order polynomial) at 11 GPS/levelling points using least squares technique for improvement on the estimated orthometric heights. 7 GPS/levelling points are used for testing the performance of the four parameter model over Nairobi County and its environs. The standard deviations of the differences between observed and estimated orthometric heights (obtained from EGM2008 and GPS) at all GPS/levelling points (18) and 7 test points are ±0.52 and ±0.35 m respectively. When the four parameter model is applied, the standard deviations of the differences between spirit-levelled and improved estimated orthometric heights at the 7 test points reduces to ±0.10 m, representing an improvement of 71%. The accuracy of ±0.10 m obtained at the test points may be sufficient for some engineering projects that do not require very high orthometric height accuracy.

Key words: EGM2008, Geoid, GPS, Orthometric height, Ellipsoidal height, Alltrans software version 3.002

1.0 Introduction

Accurate estimation of orthometric heights from ellipsoidal heights is one of the current research areas for geodesists. This is because there is a need to obtain orthometric heights from ellipsoidal heights, determined by Global Navigation Satellite Systems (GNSS). Recovery of orthometric heights from ellipsoidal heights has been studied by several authors (e.g., Amos and Featherstone, 2009; Miyahara et al., 2014; Odera and Fukuda, 2015). Earth Gravitational Model of 2008 (EGM2008) is a spherical harmonic model of the earth's gravitation potential and is an example of a gravity field model developed from combination solutions i.e. both terrestrial and satellite data covering the entire earth surface, although at varying scales. It was developed by least squares combination of ITG-GRACE03S gravitational model (Mayer-Gürr, 2007). This model was computed at the Institute of Theoretical Geodesy of the University of Bonn in Germany and is based on 57 months of GRACE (Gravity Recovery and Climate Experiment) satellite-to-satellite tracking data and its associated error covariance matrix with the gravitational information obtained from a global set of area-mean free-air gravity anomalies defined in a 5 arc-minute grid (Palvis et al, 2012).

Just like its predecessors (Earth Gravity Model of 1996 and Earth Gravity Model of 1984), it was developed by the United States National Geospatial Intelligence Agency (NGA) formerly National Imagery and Mapping Agency (NIMA). The primary product of EGM2008 model is the set of estimated spherical harmonic coefficients to degree 2190 and order 2159 (Pavlis et al., 2012), from which gravity anomaly, height anomaly, geoid undulation and deflections of the vertical can be determined. The accuracy of EGM2008 can be established from comparisons with independent data that were not used in its development. The data includes preliminary gravitational models, orbit fits tests, GPS/levelling, and Astro-geodetic deflections of the vertical among other data sets. Several authors have evaluated the performance of EGM2008 in different parts of the world (e.g., Abd-Elmotaal, 2009; Claessens et al., 2009; Huang and Véronneau, 2009; Hirt et al., 2010; Pavlis et al., 2012; Featherstone and Olliver, 2013; Odera and Fukuda, 2013; Abeho et al., 2014). Accurate computation of geoid undulation from EGM2008 is generally labour intensive and more time is required for computation of geoid undulation at many points. In this study we used Alltrans software version 3.002, retrieved from http://www.alltrans.soft112.com/, for estimation of geoid undulation. This is because it is freely available, faster and can be used for computation of geoid undulations at a number of points. We first determine geoid undulations from EGM2008 using Alltrans software version 3.002 at 18 GPS/levelling points and derive estimated orthometric heights from ellipsoidal heights. The differences between spirit-levelled and estimated orthometric heights are then modelled by a four parameter model (first order polynomial) at 11 GPS/levelling points using least squares technique to obtain improved estimated orthometric heights. 7 GPS/levelling points are used for testing the performance of the four parameter model over Nairobi County and its environs. A geoid model for the study area is also determined and presented in this paper.

2.0 Materials and Methods

2.1 Levelling Data

Levelling is the operation required in the determination, or more strictly, the comparison of heights of points on the surface of the earth (Bannister et al., 1992). Levelling is useful in designing highways, railways and canals, setting out projects according to planned elevations, calculating volumes of stacks, earthworks and embankments, investigating and laying out of drainage systems among other uses. The levelling height datum used in Kenya is the mean sea level. Ideally, the mean sea level should be determined for a period of approximately 19 years. There are various methods of determining difference in elevation of points. They include; taping methods, differential levelling, barometric levelling, trigonometric levelling and the modern methods such as GPS levelling.

Spirit levelling is a surveying technique that employs spirit levels to orient the line of sight to coincide with the horizontal line in order to determine change in elevations between two points. The levelling procedure is performed by taking a backsight reading to a levelling staff placed vertically at a benchmark, then reading a foresight on a staff placed on a point whose height is to be determined as illustrated in the Figure 1.

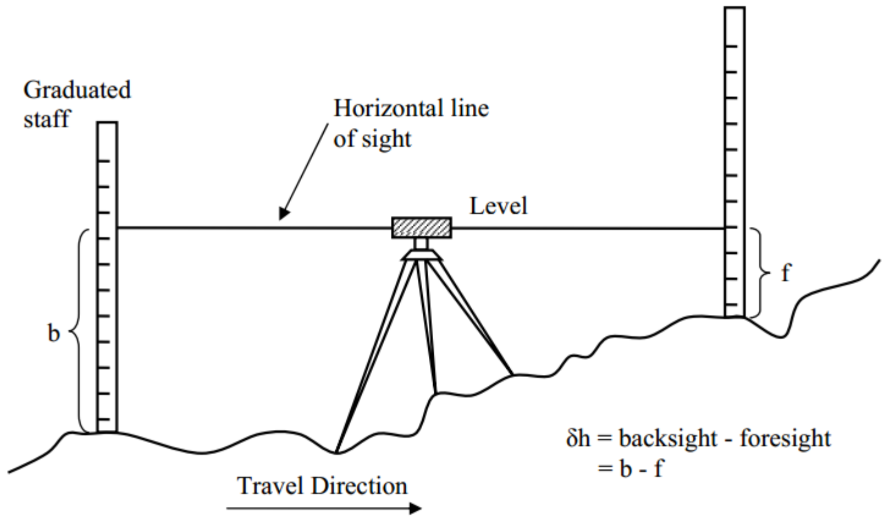

Figure 1: Spirit levelling procedure

In practice, the distance between benchmark and point whose height is to be determined is vast and in this case, levelling is done along the path connecting the benchmark and point of interest at predetermined distances that minimize the effect of curvature of the earth (i.e. adopting short distance where both the horizontal and level surfaces are coincident). For precise levelling, gravity measurements are taken at the stations occupied by the level and later used to adjust the observed height differences. Such heights are generally referred to as orthometric heights with the geoid as the references surface. Normally mean sea level is used as the best directly measurable approximation of the geoid.

GPS has introduced a revolution on how absolute reduced levels and relative levels of points on earth's surface can be determined and has been welcomed by users hence used in lieu of classical levelling techniques. Classical techniques (spirit levelling and trigonometric levelling) are not only tedious but also time consuming. During the development of GPS the focus was typically on horizontal control with the ability of GPS to measure height being seen as an added extra (Higgins, 1999) but today, GPS levelling technique is widely employed. GPS heights are generally referred to as ellipsoidal heights with the reference ellipsoid (WGS84 in this case) as the reference surface. In this study we have used 18 precisely levelled points (using spirit level and GPS) within Nairobi County and its environs. It should be noted that each of the 18 points has both ellipsoidal height and orthometric height. Figure 2 shows distribution of GPS/levelling points in the study area.

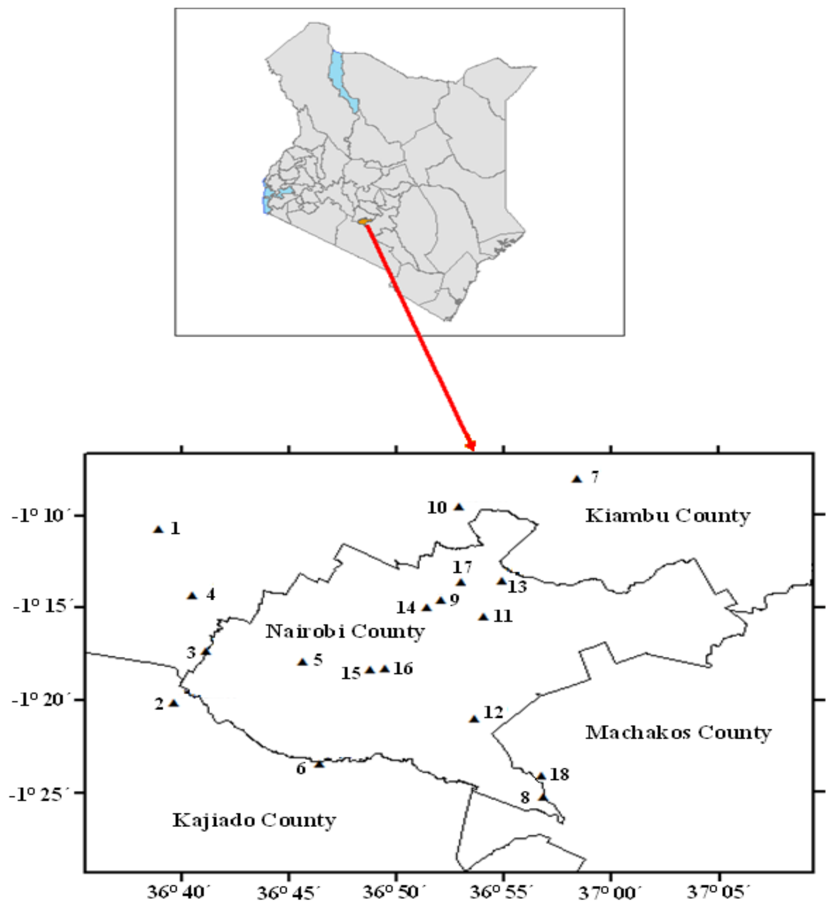

Figure 2: Distribution of GPS/levelling points within Nairobi County and its environs

2.2 Relationship between Orthometric, Ellipsoidal and Geoid Heights

The separation between the geoid and reference ellipsoid is referred to as geoid height or commonly geoid undulation. Knowledge of this parameter (geoid undulation) is necessary to enable conversion of GPS derived ellipsoidal height (h) to physically meaningful orthometric height (H), commonly used in many practical applications. Figure 3 gives a general relationship between the geoid height, ellipsoidal height and orthometric height.

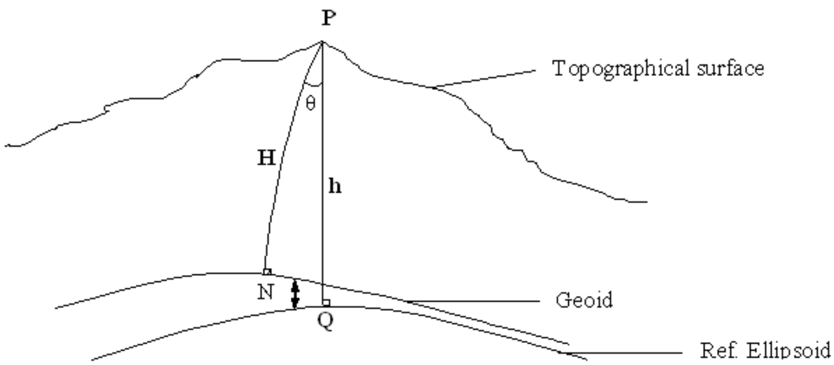

*Figure 3: Relationship between geoid, ellipsoidal and orthometric heights (*Odera et al., 2014). The deflection of the vertical (θ) is normally ignored in most practical applications, hence the orthometric height (H), may be obtained from ellipsoidal height as,

$$H = h - N \quad (1)$$

where, h is the ellipsoidal height, H is the orthometric height and N is the geoid undulation.

The ellipsoidal height (h) is normally determined from GNSS observations (GPS in this study) while the orthometric height is obtained through spirit levelling (precise levelling in this study). The geoid undulation is the link between ellipsoidal and orthometric heights. This explains one of the most important applications of geoid undulations or a geoid model. We explain determination of geoid undulations in the next section (2.3).

2.3 Determination of Geoid Undulation

Geoid undulation can be computed by a number of methods, e.g. gravimetric, satellite geometric, astro-geodetic and combined case among others. The choice of the method to be used depends on the availability of data sets, although it is expected that the combined case (where all available data sets are used) would produce more accurate geoid undulations. In this study, we have used EGM2008 which is a gravimetric technique. We therefore describe determination of geoid undulation from EGM2008.

The EGM2008 model represents geoid undulation as a function of spherical harmonics. It is complete to spherical harmonic degree and order 2159, and contains additional coefficients extending to degree 2190 and order 2159. Details about computation of geoid undulations from geopotential models can be found in Heiskanen and Moritz, 1967; Rapp, 1971; Smith, 1998; Torge, 2001, among other authors. The geoid undulation implied by EGM2008 ($N_{EGM\,2008}$) can be obtained as,

$$N_{EGM\,2008} = \frac{GM}{r\gamma}\sum_{n=2}^{n_{\max}}\left(\frac{a_{ref}}{r}\right)^{n}\sum_{m=0}^{n}(\overline{C}^{*}_{nm}\cos m\lambda + \overline{S}_{nm}\sin m\lambda)\overline{P}_{nm}(\cos\theta) \quad ,...(2)$$

where, GM is the product of the universal gravitational constant and mass of the earth, r is the geometric distance between the centre of the earth and the computation point, a_{ref} is a scaling parameter associated with a particular GGM (EGM2008), $\overline{P}_{nm}(\cos\theta)$ are the fully normalized associated Legendre functions for degree n and order m, $\overline{C}^{*}_{nm}$ and $\overline{S}_{nm}$ are fully normalized spherical harmonic coefficients after reduction by the even zonal harmonics of the reference ellipsoid and $n_{\max}$ is the finite maximum degree of a GGM (EGM2008).

For precise determination of geoid undulations, two additional quantities are added to equation (2). These are zero degree term and a conversion term used to convert height anomaly into geoid undulations. It is not clear whether Alltrans software version 3.002 applies the two additional terms although the zero degree term is a small constant value, hence does not affect the standard deviation. The conversion of height anomaly into geoid undulation is necessary for orthometric height system. However, the relative difference between height anomaly and geoid undulation in a small area is also small, hence Alltrans software version 3.002 is feasible for rapid estimation of geoid undulations. In this study we have taken care of the two quantities by applying a bias parameter in the determination of orthometric height from ellipsoidal height.

2.4 Numerical Tests

In this study, we compute geoid undulation using Earth Gravitational Model of 2008 (EGM2008). It is achieved in this study by evaluating equation (2) using Alltrans software version 3.002. This technique is faster and can be used for determining geoid undulation at any point on the earth surface provided the position is known in terms of latitude and longitude. Its drawback is the fact that the determined geoid undulation is not as accurate as the one obtained from geometric method. On the other hand, geoid undulation obtained by the geometric method (Odera et al., 2014) is very accurate. However, it requires direct spirit and GPS/levelling data at each point, which is indeed an uphill task especially in a large area. Again, this method can only be used at discrete points. It is therefore difficult to have a good coverage in an area, hence limiting in terms of application.

We therefore determine geoid undulations at 18 GPS/levelling points using gravimetric method (equation 2) over Nairobi County and its environs (Figure 2). The computed geoid undulations are then used to determine estimated orthometric heights ($H_{estimated}$) at 18 GPS/levelling points as,

$$H_{estimated} = h - N_{EGM\,2008} \quad \text{……………………………………………………………………(3)}$$

where, $N_{EGM\,2008}$ is the gravimetric geoid undulation (obtained from equation (2) using Alltrans EGM2008 calculator software version 3.002). The difference between the estimated orthometric height and the actual (spirit levelled) orthometric height is given as,

$$\Delta H = H - H_{estimated} \quad \text{……………………………………………………………………….(4)}$$

The orthometric height differences in equation (4) are modelled using first order polynomial to determine corrections to the estimated othometric heights so as to obtain improved orthometric heights. To achieve this, the data points (18 GPS/levelling points) are divided into two: 11 points are used for the determination of the polynomial coefficients while 7 points are used for cross-validation (test points). The distribution of data points and validation points is shown in Figure 4. The first order polynomial adopted in this study is of the form,

$$\Delta H = K_O + K_1\lambda + K_2\phi + K_3\lambda\phi + bias = Corr. \quad \text{…………………………………… (5)}$$

where K_O, K_1, K_2 and K_3 are coefficients of the first order polynomial, ϕ and λ are the geodetic latitude and longitude (in radians) of the point respectively, $bias$ is the mean of the differences between the actual orthometric heights and estimated orthometric heights. The middle part of equation (5) represents a correction to the estimated orthometric height ($Corr.$). The improved orthometric height of a point is then obtained from the ellipsoidal height as,

$$H_{improved} = h - N_{EGM\,2008} + K_O + K_1\lambda + K_2\phi + K_3\lambda\phi + bias \qquad \text{.....................(6)}$$

It should be noted that the first two terms on the right hand side of equation (6) give estimated orthometric height from EGM2008 using Alltrans software version 3.002 (equation 3). Equation (6) enables fairly accurate recovery of orthometric heights from ellipsoidal heights using EGM2008 after determination of the four parameters or coefficients and a $bias$ parameter. Finally we compare improved orthometric height and actual levelled orthometric height (H) as,

$$\delta H = H - H_{improved} \qquad \text{...(7)}$$

The geoid undulations of Nairobi County and its environs on a 1×1 arc-minute is also computed and presented in the results section. The geoid undulation map (geoid model) of Nairobi County and its environs gives accurate information on variation and trend of geoid undulations over the area of study.

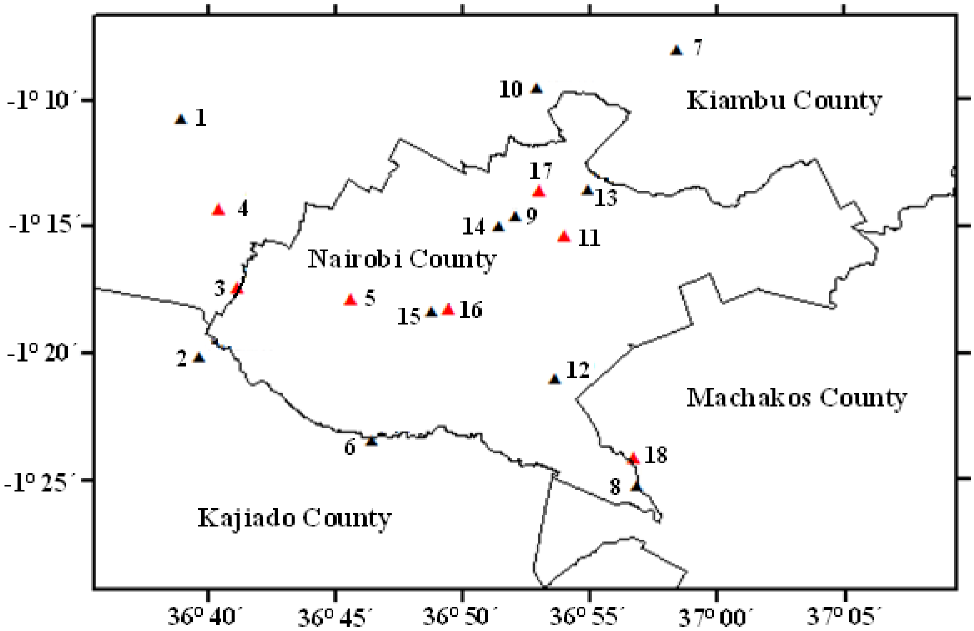

Figure 4: Distribution of polynomial evaluation points (small black triangle) and test points (small red triangle) over Nairobi County and its environs

3.0 Results and Discussion

The statistics of the differences between estimated orthometric and actual levelled heights are given in Table 1. The improvement on the estimated orthometric height is obtained by modelling ΔH at 11 points (Figure 4) using a first order polynomial. As indicated earlier, the $bias$ is taken as the mean (0.76 m in this case). The coefficients of the first order polynomial are obtained as, K_O = 38.27±0.10 m, K_1 = -47.49±0.16 m, K_2 = 4461.99±4.76 m and K_3 = -6400.84 ±7.40 m.

These coefficients or parameters are used in the computation of corrections to the estimated orthometric heights to obtain improved orthometric heights at 7 test points. The 7 test points are excluded in the determination of the first order polynomial coefficients to facilitate a cross-validation test. The statistics of the differences between actual orthometric heights and improved orthometric heights at the seven test points are given in Table 2. The geoid model over Nairobi County and its environs on a 1×1 arc-minute is presented in Figure 5. The geoid undulations (geoid model) within the study area vary from -16.99 m to -13.55 m with a mean of -15.20 m and standard deviation of ±0.82 m. We note the NW-SE trend in the geoid model of the study area (Figure 5).

The mean and standard deviation (SD) of the differences between actual and estimated orthometric heights at 18 GPS/levelling points in the study area are 0.76 m and ±0.52 m respectively. This indicates that the geoid undulations obtained from EGM2008 using Alltrans software version 3.002 are close to the geoid undulations obtained from GPS/levelling data. The estimated orthometric heights from ellipsoidal heights are sufficient for accurate feasibility studies for a number of engineering projects. They can also be used in developing maintenance dredging drawings and construction of earth/rock fill structures e.g. dams and floodwalls.

Table 1: Statistics of the differences between estimated and levelled orthometric heights (units in m)

Point	$N_{EGM\,2008}$	h	$H_{estimated}$	H	ΔH
1	-15.39	2127.72	2143.11	2144.19	1.08
2	-16.37	1918.13	1934.50	1934.59	0.09
3	-16.06	1878.00	1894.06	1894.69	0.63
4	-15.74	1979.61	1995.35	1996.13	0.78
5	-16.13	1777.99	1794.12	1794.63	0.51
6	-16.69	1699.55	1716.24	1716.20	-0.04
7	-15.11	1517.45	1532.56	1534.39	1.83
8	-16.84	1532.58	1549.42	1549.54	0.12
9	-15.77	1603.79	1619.56	1620.72	1.16
10	-15.27	1573.76	1589.03	1590.49	1.46
11	-15.86	1580.15	1596.01	1596.98	0.97

Continued…					
Point	$N_{EGM\,2008}$	h	$H_{estimated}$	H	ΔH
12	-16.46	1619.87	1636.33	1636.67	0.34
13	-15.66	1571.65	1587.31	1588.51	1.20
14	-15.81	1628.45	1644.26	1645.27	1.01
15	-16.18	1663.36	1679.54	1680.10	0.56
16	-16.16	1645.01	1661.17	1661.84	0.67
17	-15.67	1594.50	1610.17	1611.34	1.17
18	-16.74	1573.38	1590.12	1590.20	0.08
Minimum					-0.04
maximum					1.83
Mean					0.76
SD					0.52

Table 2: Statistics of the differences between improved and levelled orthometric heights (units in m)

Point	$N_{EGM\,200}$	h	$H_{estimated}$	ΔH	$Corr.$	$H_{improved}$	H	δH
3	-16.06	1878.00	1894.06	0.63	0.44	1894.50	1894.69	0.19
4	-15.74	1979.61	1995.35	0.78	0.75	1996.10	1996.13	0.03
5	-16.13	1777.99	1794.12	0.51	0.51	1794.63	1794.63	0.00
11	-15.86	1580.15	1596.01	0.97	0.99	1597.00	1596.98	-0.02
16	-16.16	1645.01	1661.17	0.67	0.59	1661.76	1661.84	0.08
17	-15.67	1594.50	1610.17	1.17	1.15	1611.32	1611.34	0.02
18	-16.74	1573.38	1590.12	0.08	0.22	1590.34	1590.20	-0.14
Minimum				0.08				-0.14
maximum				1.17				0.19
Mean				0.69				0.02
SD				0.35				0.10

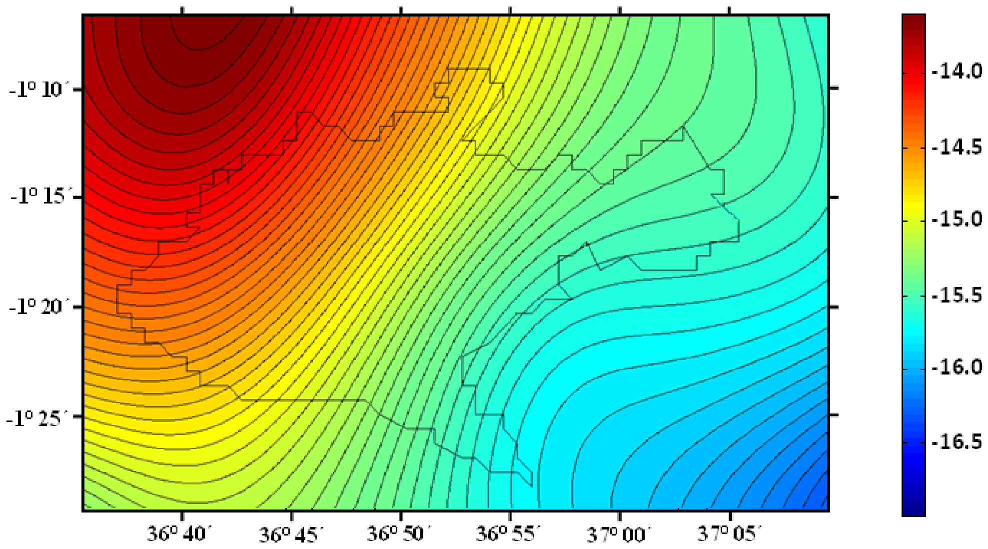

Figure 5: Geoid model over Nairobi County and its environs (units in m)

The mean and standard deviation (SD) of the differences between actual and directly estimated orthometric heights at 7 GPS/levelling test points in the study area are 0.69 m and ±0.35 m respectively while the mean and standard deviation of the differences between actual and improved orthometric heights at 7 GPS/levelling test points in the study area are 0.02 m and ±0.10 m respectively (Table 2). This shows that the use of first order polynomial improves the accuracy of determined orthometric heights obtained directly from EGM2008 using Alltrans software version 3.002 by 71% (i.e. from ±0.35 m to ±0.10 m).

4.0 Conclusions

This study has shown that geoid undulations from EGM2008 (obtained using Alltrans software version 3.002) are close to the ones obtained from GPS/levelling data. When such geoid undulations are used to recover orthometric heights from ellipsoidal heights, accuracy values of ±0.52 m and ±0.35 m are obtained at 18 GPS/levelling points and 7 GPS/levelling test points respectively. A first order polynomial is used to improve the accuracy of orthometric heights at 7 test points. The standard deviation of the differences between actual and improved orthometric heights at 7 GPS/levelling test points in the study is ±0.10 m, indicating that the use of first order polynomial improves the accuracy of determined orthometric heights obtained directly from EGM2008 using Alltrans software version 3.002 by 71%. The final heights obtained using the procedure described may be sufficient for the implementation of engineering projects that do not require very high vertical accuracy. Geospatial scientists and engineers can easily determine fairly accurate orthometric heights for most construction design and implementation using the procedure developed in this study. It should be noted that the parameters determined in the current study can only be used within the study area. The implementation of the procedure described in this study is fairly simple to facilitate practical application.

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