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MODELLING DIAMETER DISTRIBUTION FOR A SEVEN YEAR *Eucalypus camaldulensis* STAND IN AFAKA FOREST RESERVE-NIGERIA USING WEIBULL FUNCTION

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ABSTRACT

Parameters of the Weibull function were estimated for a diameter distribution modelling of a seven year Eucalyptus (*Eucalyptus camaldulensis*) plantation in Afaka Forest Reserve. Two fit methods; Cumulative Density Function and Probability Density Function (Weibull-CDF and Weibull-PDF) were used. Twenty plots of 20 x 20 m were randomly selected for measurements. Stump diameter (Dst), diameter at breast height (Dbh), diameter at middle and top positions (Dm and Dt) of trees, and tree height for all the selected trees were measured. Pseudo coefficient of determination (Pseudo R^2), Residual Mean Square Error (RMSE), Anderson-Darling (A-D), and Kolmogorov-Smirnov (K-S) goodness-of-fit statistics were considered as model selection criteria. The average Dbh of the stand was 14.64cm, and the mean tree height was 6.02m.Weibull cumulative density function (Weibull-CDF) was the best fit method based on goodness-of-fit test (Pseudo R^2 = 0.98, RMSE = 0.0397, A-D = 1.1558, K-S = 0.0668). The diameter distribution models produced are considered only efficient for tree stands of seven years rotation age, while additional re-measurement data sets as well as other site factors are required for better models, hence the need for establishing permanent sample plots (PSP) in order to get additional information from re-measurements of the plots.

Keywords: Afaka forest reserve; Diameter distribution; *Eucalyptus camaldulensis*; Weibull function

INTRODUCTION

Growth and yield models are generally used to predict temporal and future growth of forest stands. In forestry, it is important to be able to make accurate future predictions of the mean values of growth variables based on repeated measurements through time made on units grouped hierarchically. In many forest management practices, decisions are based on yield projections that crucially depend on projections of plot level averages of tree height, basal area, and other morphometric variables (Hall and Bailey, 2001).

The structure of a forest consists of the distribution of trees and their respective size in relation to a unit area, being a result of species growth pattern, environmental conditions and forest management practices. In studying forest structure, diameter distribution is a basic, widely disseminated and applied tool that constitutes the simplest and most effective way to describe the characteristics of a given stand (Bartoszeck, 2000). Diameter distribution is an efficient indicator of forest growth and the most powerful way to describe the properties of a stand. The diameter variable is derived from direct measurement of trees and is well correlated to other important variables, including volume, production quality and exploration costs (Miguel *et al.,* 2010). According to Scolforo (1998), knowledge of diameter distribution in planted forests is a critical requirement to ensure that prediction or prognosis of yield is implemented. In analyzing diameter structure, there is an interest in describing diameter frequency distribution by using probability density functions. A probability density function defines the probability associated to each value of the variable in question, or else it can describe the relative and/or absolute frequency distribution of the various tree sizes (Campos and Leite, 2006). Diameter distribution models can estimate the number of trees per hectare per diameter class at present and future ages. The use of a volume, taper or volumetric ratio equation allows estimating yield per diameter class, which is an important tool in situations where multiple wood products are concerned (Miguel *et al.* 2010). Diameter-class distribution models have become a useful tool in forest management, growth and yield modeling and forests inventories. Various cumulative density function (CDF) and probability density functions (pdf) such as normal, log-normal, gamma, beta, Johnson's S_{B1} and Weibull have been utilized to characterize the diameter, cumulative and frequency distributions of forest stands.

Eucalyptus camaldulensis is a common and widespread species believe to have originated from the watercourses over much of mainland Australia. Eucalyptus has over 800 species (Brooker and Kleinig, 2001) established worldwide as a plantation grown species, with a dominant portion located in temperate Australian mainland. The species exhibits considerable morphological variation throughout its range and it ranges in size from single-stemmed, large-boled, medium-sized to tall trees. The versatile eucalyptus genus takes the largest share of plantations in the tropical world. It is grown equally for industrial and non-industrial purposes. FAO (1995) reported that the total eucalypt plantation area up to the end of 1990 was estimated at10.1million ha, out of which 52% is situated in tropical Asia, 40% in Tropical America and about 8% in Tropical Africa.

MATERIALS AND METHODS

Study Area

Afaka plantation is located at about 15km from Kaduna between latitudes $10^0 35'$ and $10^0 38'$N and longitudes $7^0 15'$ and $7^0 20'$E. It has an elevation of 610 m above sea level with an almost imperceptible slope towards the south or undulated layers in many sub soils. The plantation is situated in the Northern Guinea Savannah ecological zone (Keay, 1959).

The underlying rocks are mainly gneisses and schist of pre-Cambrian age buried under sandy frit materials. The soils are red or yellow loams, of weak structure with little textural variations, but with frequent occurrence of rounded ironstone concretions, sometimes associated with quartz gravel or occasionally forming hard crust. The pH of the soil varies between 4.5 and 5.5. The mean annual rainfall is 1300mm, but yearly totals show a wide range of variation particularly for the months of April and May. Relative humidity from January to March is very low (10%) during which temperatures may rise to over 37.7^0C (Adegbehin, 2002).

The introduction of exotic species started in 1958 and emphasis was given to the trial of eucalyptus and pines species within the reserve area. A more intensive management was later introduced, critical examination of fertilizer requirements, spacing and thinning regimes were also introduced to enhance growth performance of the species.

Sampling and Data Collection

A reconnaissance survey of the Eucalyptus plantation was carried out with an attempt to locate its boundaries and get acquainted with the general conditions of the reserves. The reconnaissance survey was also aimed to assist in planning for the full scale field work in order to complete the study in a timely manner. Sample plots were located by the use of some survey equipment such as prismatic compass, GPS, 50-meter tape, cutlass, and ranging poles. The Eucalyptus stand in Afaka Reserve is an even-aged plantation, therefore, simple random sampling was used to select 20 square plots of 20 x 20 m in dimension.

Within each randomly selected sample plot, preference was given to the enumeration of healthy trees with more typical growth forms; dead trees and trees with abnormal form were avoided. This is because in developing growth and yield models for effective management of forest plantations, only trees grade, soundness and size requirements for commercial logs or poles are considered relevant. While this sampling guideline appeared to introduce a bias in favor of better-formed trees, it is justified because only healthy trees with good form are of commercial value and therefore, require volume computation. The following measurements were carried out on all trees in the sample plot:

1. Stump diameter (35 cm above ground level) and diameter at breast height (at 1.3 m above ground) using the girth-diameter tape (in cm);
2. Stem diameter at the middle and upper positions of the tree using the Spiegel relaskop (in cm)
3. Merchantable height of selected trees using the Spiegel relaskop (in meter);
4. Total height of dominants and co-dominant trees using the Spiegel relaskop (in meter).

Data Analysis

Data collected were organized and screened for analysis. Descriptive statistical analysis was further carried out in order to summarize the data. All analysis carried out were conducted using SAS statistical package version 9.3, licensed to The University of British Columbia, Vancouver, Canada.

Fitting Diameter Distribution Models

Cumulative Density Function (CDF) and Probability Density Function (Pdf) of the Weibull were used to estimate the three parameter Weibull function. The parameters of the Weibull function are *a*, *b*, and *c* (location, scale, and shape) and the *a* parameter was considered to be lowest Dbh value in the data set, while *b* and *c* parameters were estimated and searched iteratively. The model forms are presented in Table 1.

Table 1: Description of the diameter distribution models

Model	Expression	References
Weibull CDF	$f(x) = 1 - exp\left[\left(\frac{x-a}{b}\right)^{c}\right]$	Bailey and Dell, 1973
Weibull PDF	$f(x) = \frac{c}{b}\left(\frac{x-a}{b}\right)^{c-1} exp\left[-\left(\frac{x-a}{b}\right)^{c}\right]$	Bailey and Dell, 1973

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Yield Computation and Goodness of Fit Test

Basal area of all trees in each diameter class was computed using the formula:

$$BA = \frac{\pi D^2}{4} \qquad (1)$$

Where BA = basal area; D = tree diameter at breast height

Newton's formula used for volume calculation is of the following form:

$$V = \frac{\pi h}{24}\left(Db^2 + 4Dm^2 + Dt^2\right) \qquad (2)$$

Where H = tree height; Db, Dm, and Dt are diameters at base, middle, and top positions of the tree, respectively.

The mean tree method (mean volume per plot was computed and extrapolated by hectre) was then used to estimate the volume of trees in each class. This facilitates the computation of volume of products (timber, poles, and fuelwood) that can be obtained from the plantations.

Each model was evaluated using some Goodness-of-Fit tests such as Pseudo R^2, RMSE, Anderson Darling test, and Kolmogorov Smirnoff test. Ordinary least squares (OLS) regression was also carried out in order to obtain corrected total sum of squares (SSY) for calculating RMSE and Pseudo R^2. Residual analysis was also computed in order to compare the estimated and actual values of the model parameters.

RESULTS AND DISCUSSION

Summary Statistics

For efficient and accurate growth and yield modelling, field data must be biologically and statistically valid. The data used were carefully obtained from the field and subjected to biological validation and the results indicated a normal distribution pattern as tree tapers from bottom to the top and the diameter decreases from bottom to the top as indicated in Table 2. This shows that the environmental factors are favorable for the normal growth and development of the species over time. Trees that are bigger in size also produce more volume compared with those with smaller diameters; this further confirms the biological validity of the results. The results of the descriptive statistics (Table 2) were found compatible with the works of Adegbehin (2002); Shamaki *et al.* (2011); Shamaki and Akindele (2013); and Shamaki and Ibrahim (2013).

One of the specific objectives of this study is to evaluate and project the number of multiple wood products to be obtained at the end of the scheduled age of silvicultural rotation (7years). From the results of diameter class distribution and the multiple products (Tables 3 and 4), it is evident that most trees fall within 10-20cm diameter class.

Specifically, diameter class 25-30 cm for this species has the minimum proportion of trees (1.7%). While the maximum proportion of the trees (64.1%) was in diameter class of 10-15 cm. In addition, the results revealed that the majority of the stands fall short of the required specification for sawn timber which is the primary objective for managing Afaka Plantation. Therefore the specified rotation age of seven years may not yield positive results in terms of timber quantity (height and size), unless if the stands are modified genetically and there is improvement in silvicultural activities. The higher number of trees in the size class that is suitable for timber production are very adequate for other important products such as fence post, poles and scaffolds within the specified rotation age. Miguel *et al.* (2010) observed a better yield (Average Dbh = 11.30 cm and Average height = 19.86 m) for a 7 year old, Eucalyptus in Brazil in their study. The diameter class distribution at the end of the rotation age favors timber size stands, and this could be as a result of environmental and other genetic variation across the regions of the world.

Table 2: Summary statistics of tree growth variables for Eucalyptus in the study area

Statistics	Dst(cm)	Dbh(cm)	Dm(cm)	Dt(cm)	Height(m)	BA(m^2)	Volume(m^3)
Mean	20.0	14.6	12.2	8.8	6.0	0.020	0.1253
Min	8.6	6.7	5.0	2.5	3.0	0.004	0.0075
Max	63.5	42.3	35.0	30.0	17.5	0.140	1.9240
SD	8.40	6.11	5.67	4.53	2.13	0.02	0.22
SE	0.39	0.30	0.26	0.21	0.10	0.00	0.01

Table 3: Diameter class distribution of the stand in the study area

Dbh class (cm)	Frequency	Proportion (%)
<10	54	11.4
10-15	304	64.1
15-20	65	13.7
20-25	11	2.3
25-30	8	1.7
>30	32	6.8

Table 4: Multiple wood products at the seven year projected rotation age

Species	Diameter (cm)	Length (m)	Product
Eucalyptus	≥30	3	Sawn Timber
	10-12	3	Fence Post
	20-25	10-15	Pole
	10-15	15	Scaffolds
	<5	-	Fuelwood

Source: Kaduna State Forest Management Board

Diameter Distribution Models

Weibull function is characterized by three parameters (location, scale and shape). The smallest diameter measured for each species was considered as the location parameter, while parameters *b* and *c* were iteratively searched in PROC NLIN module of SAS. The standard error recorded for the parameter estimates (Table 5) was relatively low for the

Weibull-CDF, but higher values of SE are recorded in Weibull-PDF. This indicates that Weibull-CDF is a better diameter distribution model for this species at this rotation age and location. Clutter *et al.* (1983) noted that the *b* and *c* parameters in diameter modelling must be not be negative, but the Weibull-PDF parameter estimates showed negative values of *c* parameter and this further disqualified the Weibull-PDF as diameter distribution model for this species.

In order to validate the models, the observed and predicted values were plotted in the residual analysis (Figure 1) and compared to see if they are closely the same. The observed values are represented by the scatter circles, while the line series represents the predicted values. Weibull-CDF smoothly fitted the predicted values up to about 85% and recorded a slight under and over prediction in the last 15% of the predicted values. This is an evident of wider distribution of the size classes. Four Goodness-of-Fit statistics (Pseudo R^2, RMSE, Anderson Darling, and Kolmogorov Smirnov) were used for comparison and validation of the models. From the results of the Goodness-of-Fit, Weibull-CDF showed a Pseudo R^2 value that is near the unity (Table 6) which is an indication of a good model. The RMSE, A-D, and K-S values are much lower in Weibull-CDF than the values recorded by Weibull-PDF. In general, the Weibull-CDF fitted the diameter distribution better than the Weibull-PDF. So, the Weibull PDF could be considered not appropriate for this Eucalyptus stands. Miguel *et al.* (2010) discovered that the fit of Weibull function was suitable to estimate the number of trees per diameter class by the Kolmogorov-Smirnov test while working with *Eucalyptus urophylla* stands in Brazil. To improve the volumetric estimators for the projected age of their study, the diameter distribution of the stand, as described by the Weibull function, was fitted according to site yield level. It is therefore, imperative to include site factors (e.g. stand age, spacing, density, and site index) in estimating the parameters of the diameter distribution. Unfortunately, such information were not available for this study. Therefore, the diameter distribution models developed in this study can only be applicable to the seven year rotation age of this species. Many studies (e.g. Liu *et al.* 2001; Cao, 2004; Nord-Larsen and Cao, 2006; and Palahi *et al.* 2006) used different methods to estimate parameters of the Weibull distribution using various kinds of transformations to linearize the function and subsequent estimation by linear regression, or by moment, or percentile estimation. Estimation of parameters by maximum likelihood have been found to produce consistently better goodness-of-fit statistics compared to other methods, but also put the largest demands on the computational resources (Cao, 2004).

Looking at the diameter distribution of this species (Figures 2), the distribution is left skewed which is an evidence of selective harvest of the bigger trees as they grow bigger, as the stand matures at the peak, it become less skewed and the variation increases. Towards the end of the distribution, the bigger trees were right skewed, probably because of the self-thinning of the smaller trees and partly due to the demand of the medium sized trees for poles and scaffolds. The finding of this study is supported with the studies on an even-aged stands of beech in Denmark (Nord-Larsen and Cao, 2006) and eucalyptus in Brazil (Miguel *et al.*, 2010).

Table 5: Parameter estimates for the diameter distribution models

Model	Parameters	Estimates	SE
Weibull-CDF	b	7.3966	0.0442
	c	2.1736	0.0454
Weibull-PDF	b	1.1378	0.7220
	c	-3.4182	2.2344

Table 6: Goodness of fit statistics for the diameter distribution models

Model	Pseudo R^2	RMSE	A-D	K-S
Weibull-CDF	0.984	0.0397	1.1558	0.0668
Weibull-PDF	0.012	0.3181	27.642	0.3246

*RMSE = Residual Mean Square Error, A-D = Anderson Darling, K-S = Kolmogorov Smirnov

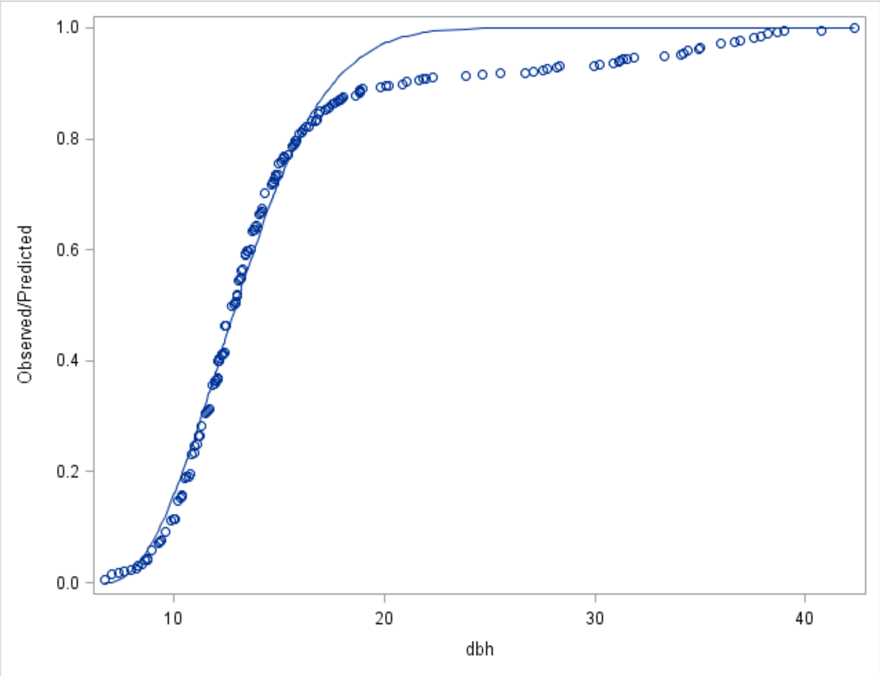

Figure 1: Observed and Predicted Weibull-CDF

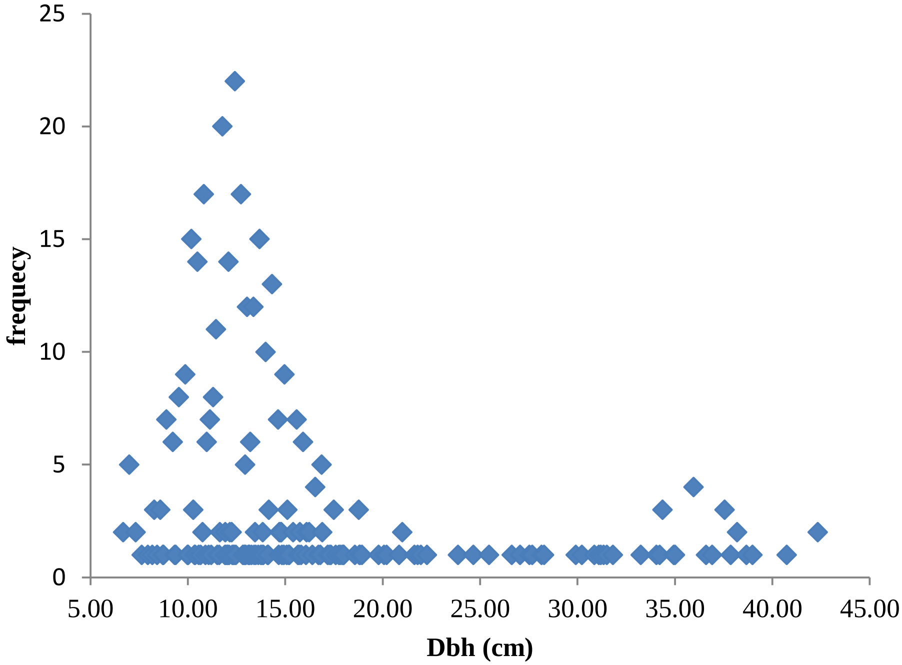

Figure 2: Shape of the diameter probability distribution for Eucalyptus stand

CONCLUSION

This study revealed that the growth performance of trees in terms of tree taper followed a normal distribution pattern, with diameter being reduced from bottom to the top of the trees. The frequency distribution of the size class is more concentrated in 10-20cm Dbh class. Five different products are considered obtainable from the stand viz: Sawn timber, fence post, pole, scaffolds, and fuelwood. On the average, diameter class of ≥30 cm is considered suitable for sawn timber, while other products fall within 10-20cm Dbh with the exception of fuelwood that smaller sizes of less than 5cm can be used.

Diameter distribution is skewed towards the left and slightly right skewed towards the end of distribution for the bigger sized trees. Weibull-PDF model was rejected on account of producing negative *c* parameter estimates and relatively higher sum of squares for error. Cumulative distribution function (CDF) of the Weibull function fitted better. The models developed are mostly applicable to the 7 years rotation age of this species in this location. Additional information is required to make the models of wider application.

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