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EXPERIMENTAL DETERMINATION OF THE POISSON'S RATIO (μ) AND GAUGE FACTOR (k) VALUES OF A PRACTICAL STRAIN GAUGE (120-SRA, FOIL TYPE)

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Abstract

A technique for determining the gauge factor (k) and the Poisson's ratio (μ) of a typical 120-SRA foil type bounded strain gauge has been experimentally and analytically investigated. The results show that its gauge factor (k) is 1.42, its Poisson ratio (μ) is 0.21, and that the strain gauge is of the zinc material and highly sensitive for measuring loads as low as 1g, based on the methodology as well as the dimensions of the beam employed in this (research) work.

Keywords: Experimental Determination, Foil type strain gauge, Sensitivity, Poisson's Ratio and Gauge Factor.

1. Introduction

Industrial and scientific establishments have placed the measurements of physical parameters such as position, temperature force, pressure, sound, light and the rate of fluid flow as a paramount process (Jones, 1994). In each of these measurements, a transducer is employed. Transducers have become convenient, economical and highly efficient in operation because they convey various physical and chemical quantities into related electrical signals, which are applicable in measurements, amplification, transmission, and control. (Jones, 1972) A strain gauge is a wire whose resistance changes by a small amount when it is lengthened or shortened by small amount. It is bonded to the structure whose strain is to be measured. The strain suffered by the strain gauge then equals that of the structure. These strain gauges are mostly arranged using the Wheatstone bridge format and the bridge circuit usually consist of the working and temperature compensation gauges produces which produces an electrical output voltage when the structure to which they are attached is strained by external load. (Devor, 1990)

A tensile stress tends to elongate the wire and thereby increase its length and decrease it cross-sectional area. The combined effect leads to an increase in resistance R, given by:

$$R = \frac{\rho l}{A} \tag{1}$$

where ρ = resistivity, l = the length, A = the cross sectional, and R = the resistance of the conductor.

One parameter of the wire, which is important to strain gauge measurement, is the gauge factor k, defined as:

$$k = \frac{\Delta R / R}{\Delta l / l} \tag{2}$$

where ΔR. = change in resistance when the structure is under stress,
Δl = change in length when the structure is under stress.

But:

$$R = \frac{\rho l}{A} = \frac{\rho l}{\pi d^2 / 4} = \frac{4 \rho l}{\pi d^2}$$

Under stress,

$$R + \Delta R = \frac{4\rho}{\pi} \frac{l + \Delta l}{(d - \Delta d)^2}$$

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$$= \frac{4\rho l}{\pi d^2}\frac{(l+\Delta l/l)}{(1-\Delta d/d)^2} \qquad = R\left(1+\frac{\Delta l}{l}\right)\left(1+\frac{2\Delta d}{d}\right)$$

$$\therefore \frac{\Delta R}{R} = \frac{\Delta l}{l} + \frac{2\Delta d}{d} = \frac{\Delta l}{l}\left[1+\left(\frac{2\Delta d}{d}\Big/\frac{\Delta l}{l}\right)\right]$$

But the Poisson's ratio, μ, is given by:

$$\mu = \frac{\text{strain in the lateral direction}}{\text{strain in the axial direction}} = \frac{\Delta d}{d}\Big/\frac{\Delta l}{l} = \frac{l\Delta d}{d\Delta l} \qquad (3)$$

Then *k* becomes,

$$k = \frac{\Delta R}{R}\Big/\frac{\Delta l}{l} = \frac{l\Delta R}{R\Delta l} = 1+2\mu \qquad (4)$$

Thus Strain,

$$G = \frac{\Delta l}{l} \qquad (5)$$

Therefore, Eq. 2 can be written as:

$$k = \frac{\Delta R/R}{G} = \frac{\Delta R}{RG} \qquad (6)$$

For strain gauge application, a high degree of sensitivity is desirable. Sensitivity is also a function of the ability of the beam carrying the strain gauge to be flexible (bend) while retaining it elastic limit (Jones & Childers, 1999). For a strain gauge to be sensitive, k must be large which implies large resistance with a relative low value of μ (typically < 0.5). Table 1 shows gauge materials with their respective ¼ and k for some materials. (Larry, 1991)

The resistance change in strain gauge is of the order of 0.1W. To measure such a small resistance change, it is necessary to first convert the resistance change to a voltage. Since the change in resistance is small, the corresponding voltage will be small. This signal will be rather difficult to measure. If however this voltage change is amplified by a certain factor (of about 1000), its magnitude will increase and this can easily be measured by using a millivoltmeter. Therefore, what is required in strain gauge measurements is a circuit that allows the amplification of only the voltage change across the strain gauge caused by a change in its resistance. The circuit that is suitable for this is a bridge resistance circuit. (Coughlin and Driscoll, 1992)

The strain gauge of unstrained resistance R_4 is placed in one arm of a resistance bridge circuit as shown in Fig. 1. The other arms of the bridge are three resistors R_1, R_2 and R_3, all of equal value R. Under this condition $E_1=E_2=E/2$ and the bridge is said to be balanced. If the element of the strain gauge is expanded, R increases by ΔR.

$$E_2 = \frac{E}{2}\left(1+\frac{\Delta R}{2R}\right) \qquad (7a)$$

and

$$E_1 = \frac{E}{2} \qquad (7b)$$

The differential voltage E_2-E_1 is given by:

$$E_1 - E_2 = \frac{E\Delta R}{4R} \tag{8}$$

This approximation is valid because $\Delta R << R$ for strain gauges. Equations 7a and 7b show that E should be made large (about +12V) to maximise the output voltage of the bridge circuit. The output voltage is given by:

$$V_{out} = \frac{E\Delta R}{4R} \tag{9}$$

where A is the gain of the differential amplifier.(Coughlin and Driscoll, 1992)

2. Materials and methods

(*a*) Setting-up the apparatus:

The materials employed for this research work included four 120 – SRA foil type bonded strain gauge arrangement aluminium beam, an instrumentation amplifier circuit with a single three – way rotary ganged switch, a high precision micrometer screw gauge, and a digital multi-meter. An aluminium bar with dimensions 0.23m *x* 0.07m *x* 0.003m was cleaned of all points, rust, grease and dirt and finally smoothened with fine-grade send blasting paper to provide a smooth and sound bounding surface. The surface was degreased with alcohol and neutralised with a weak detergent solution. The surface was then rinsed with distilled water and finally cleaned and wiped with tissue paper until the bonding surface was free of all stains dirt as well as specks. The final cleaning took place immediately prior to the installation of the strain gauges.

Due to the sensitivity and fragility of the gauges, a long nose pliers was used in lifting the gauges which were carefully cleaned with fine cotton wool and placed on the aluminium bar and thereafter, fastened permanently to the surface of the aluminium bar by an epoxy gum (araldite) as the adhesive. A flat screw driver was then used to pass the cover of the gauge for firm contact between the gauge and the surface for about two hours. In all four strain gauges were gummed to the bar (two on the top surface and two on the lower surface). The fragile lead-out wires of the strain gauges were carefully soldered to the connecting wires. The connecting wires from the strain gauges were then connected to the balancing network and powered with electricity. The 10 kΩ-potentiometer and the 100 kΩ-resistors formed a signal conditioning circuit for the proper adjustment of the E_1 and E_2 outputs as required.

An aluminium container for receiving the loads was then arranged at one end of the aluminium bar while the other of the bar was rigidly fixed to a wooden support. The strain gauge arrangement, the aluminium bar, the load container and the wooden support are shown in Fig. 4 while Fig. 5 shows how these gauges were connected electrically in a Wheatstone bridge circuit configuration with an instrumentation amplifier. Under strain, the resistance of R_1, R_2, R_3 and R_4 becomes R+ΔR, R-ΔR, R-ΔR, and R+ΔR respectively.

(*b*) Increasing the strain-gauge bridge output voltage:

Even when the bridge circuit of Fig. 1 was balanced at one temperature, it did not stay balanced when the temperature changed. This was because any change in the temperature of the strain gauge caused a change in its resistance which is superimposed on that caused by a strain. Eliminating the effect of temperature change was done by mounting an identical strain gauge (called the dummy gauge) immediately adjacent to the working strain gauge so that they both share the same thermal environment. The dummy gauge has its axis perpendicular to the direction of the strain. The new bridge circuit is shown in Fig.2. Therefore as the temperature changes, the resistance of the dummy gauge changes exactly as the resistance of the active gauge. When a strain is applied, the resistance of the active gauge increased while that of the dummy gauge decreased exactly by the same amount of the active gauge resistance. The dummy gauge provides automatic temperature compensation. Once the bridge was balanced, the resistance of the dummy gauge and working gauge tracked each other as the temperature changed. An unbalanced condition only occurred, when the resistance of the working gauge changed.

The differential voltage E_2-E_1 in Fig. 2 was small and needed further amplification. A typical circuit is shown in Fig. 2 where R_1 and R_2 are the resistances of the bridge resistors. When strain was not applied, the active strain gauge (R_1) and the dummy strain gauge (R_2) all had equal resistances. The bridge was balanced and the output voltage was zero. When strain was applied, the resistance of the active strain gauge changed while the resistance of the other resistor including the dummy gauge in the bridge did not change.

The implication of eq.9 is that the output of the differential amplifier in Fig. 2 increases with increasing value of DR One way of increasing V_0 further is by doubling both the dummy and the working gauges. The circuit

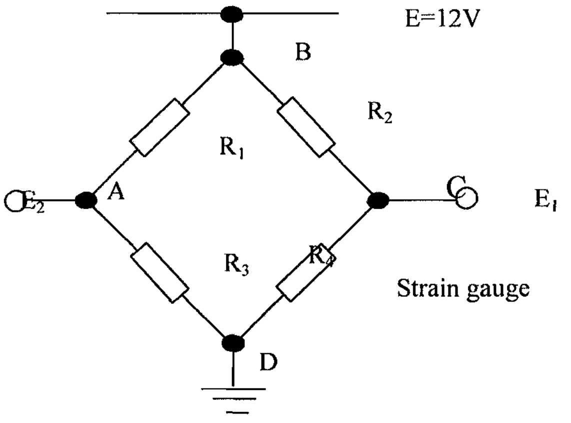


Fig. 1 Strain gauge arrangement

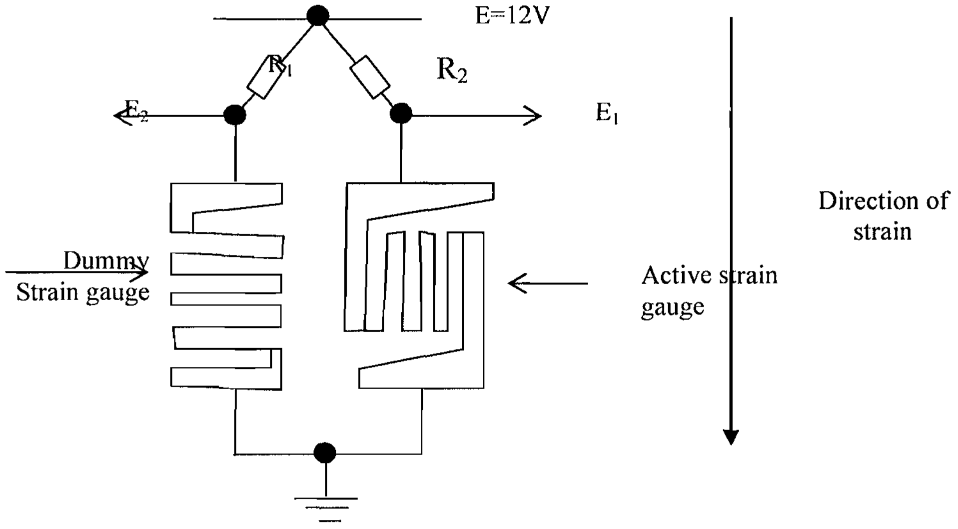


Fig. 2 Dummy and working strain gauges

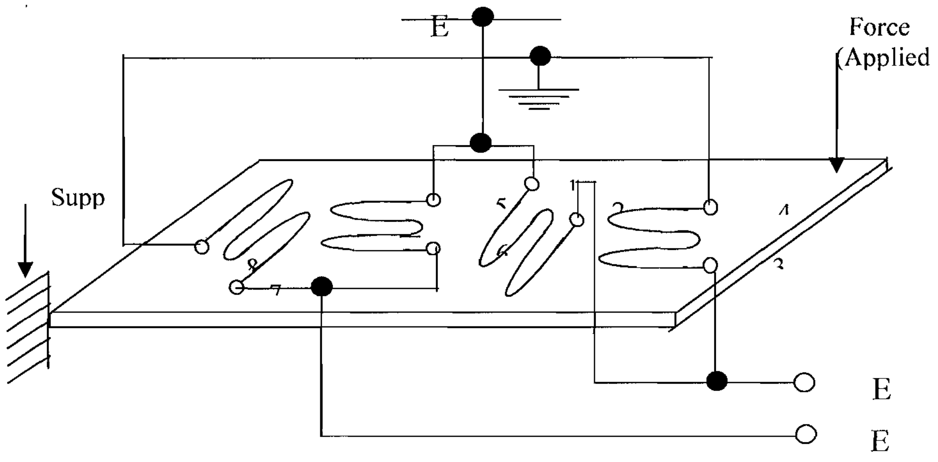


Fig. 3(a): Four working gauges on top of the beam doubles the $E_1 - E_2$ output of Fig. 2. (Gauges 3-4 and 5- 6 are active gauges while gauges 1-2 and 7-8 are temperature gauges)

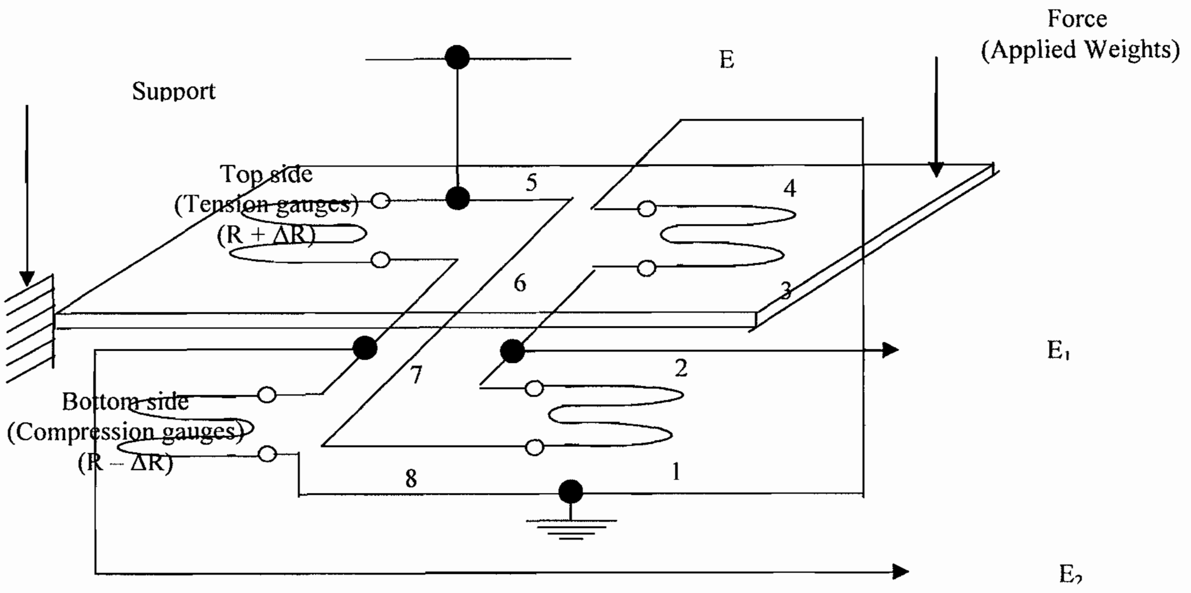
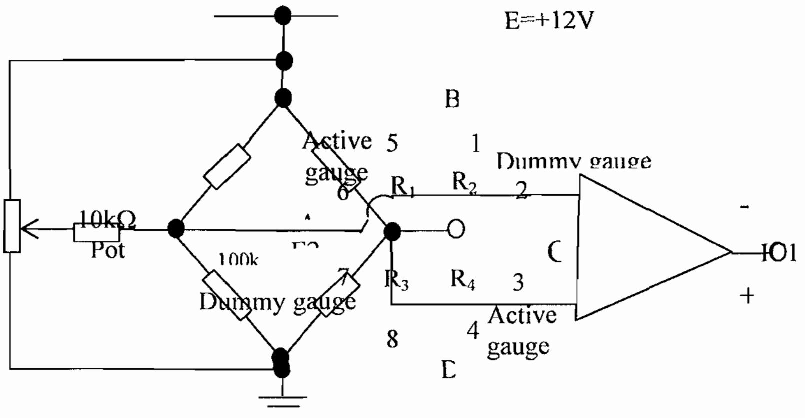


Fig. 3(b): Four working gauges quadruples the E_1 –f E_2 output voltage of Fig. 2. (Active gauges 3-4 and 5- 6 are on top of the beam while the temperature gauges 1- 2 and 7- 8 are at the bottom of the beam)



$$V_{Out}= E\frac{\Delta R}{2R}$$

Fig.3(c) Increasing strain gauge sensitivity

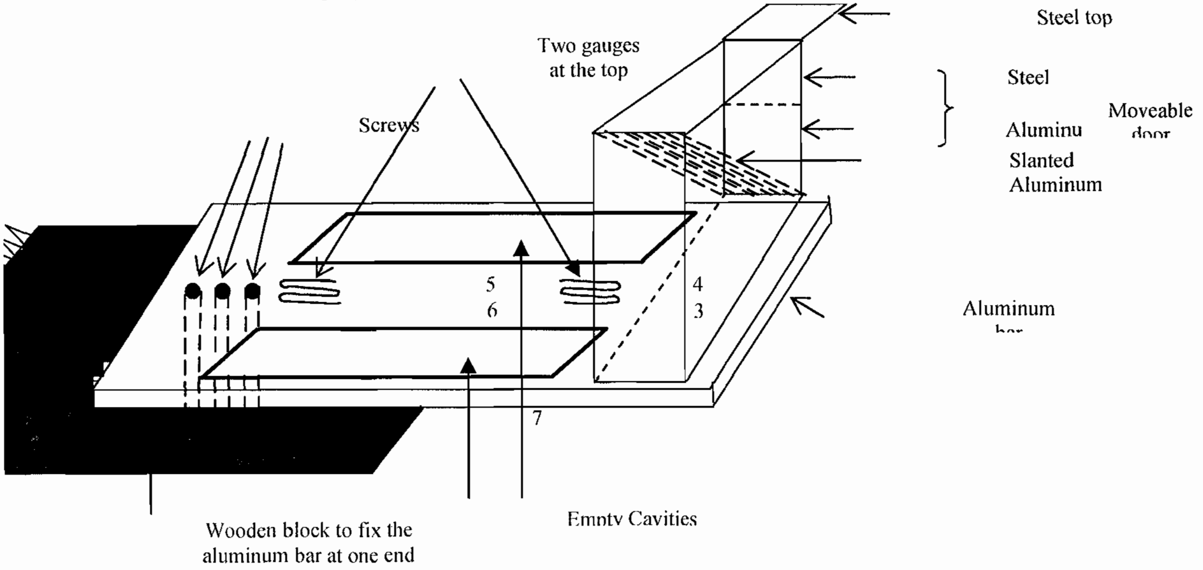


Fig. 4: Aluminium bar and strain gauge set-up with cavities for greater sensitivity.

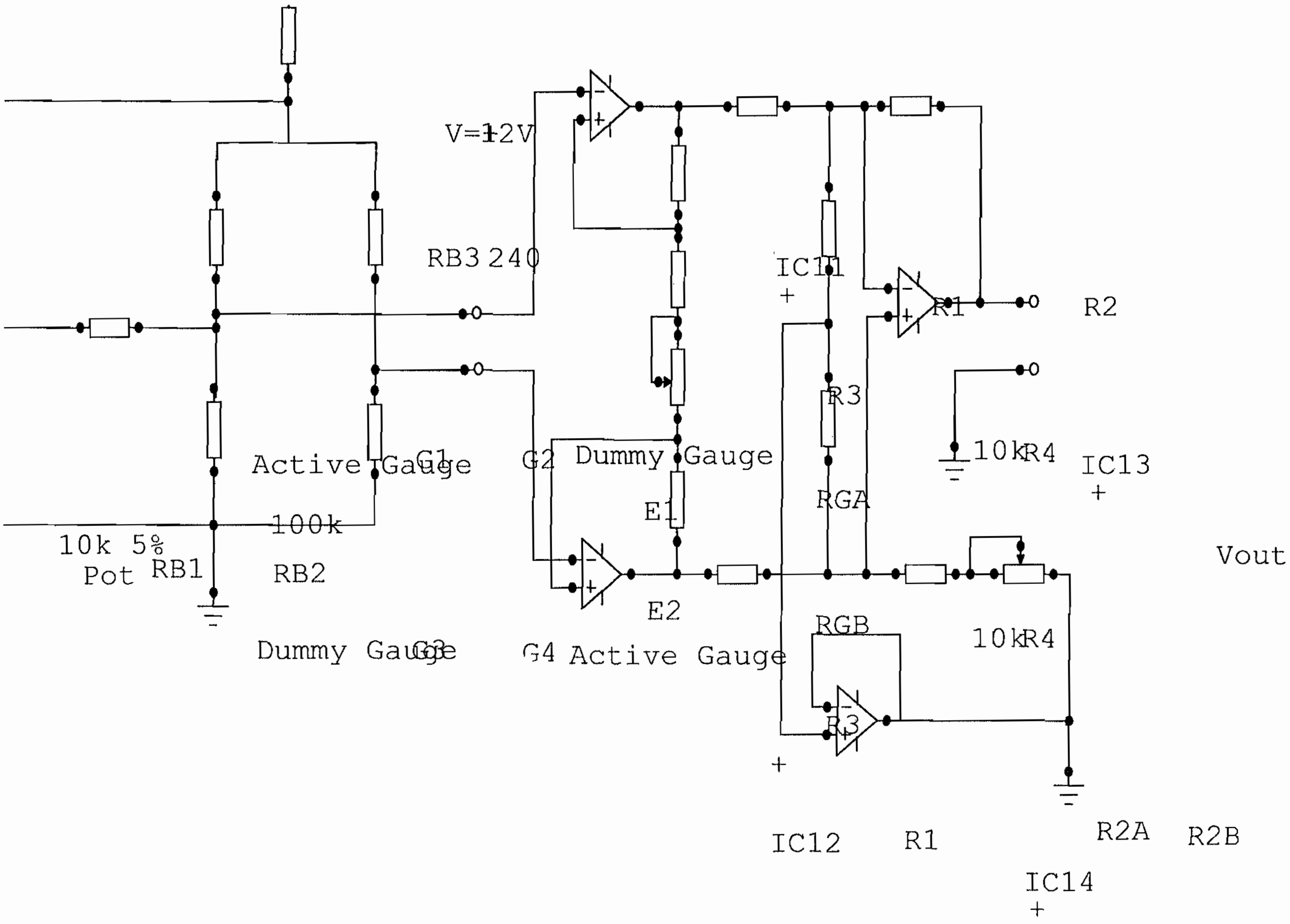


***Fig. 5**: Circuit showing strain gauges in Wheatstone bridge network with the instrumentation amplifier*

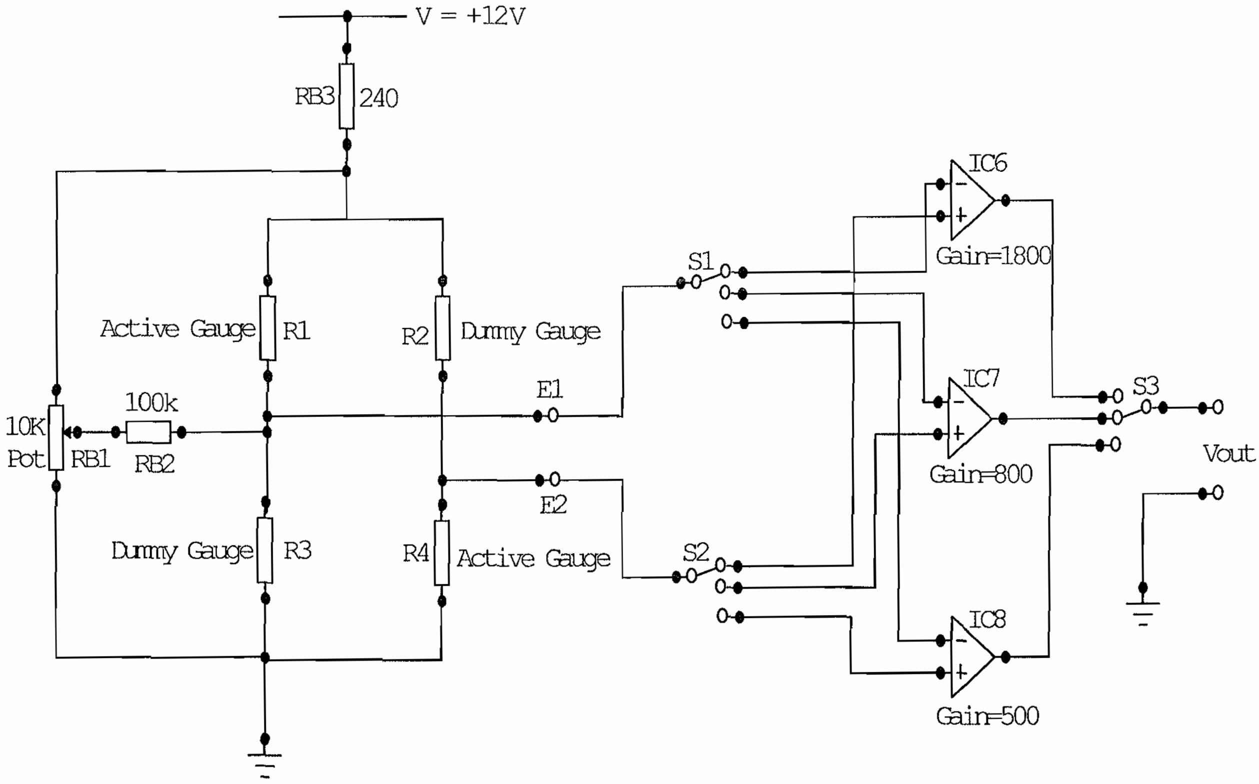


Table 1: Gauge materials with their respective μ and k

Material	μ	K
Zinc	0.21	1.42
Brass	0.35	1.70
Steel	0.29	1.58

Table 2: Resistors and their corresponding gains

Gain	R_{GA}	R_{GB}	R_1	R_2	R_3
500	390Ω	100Ω(Pot)	1.9kΩ	47.0kΩ	4.7kΩ
800	390Ω	100Ω(Pot)	1.5kΩ	47.0kΩ	5.9kΩ
1800	390Ω	100Ω(Pot)	1.0kΩ	47.0kΩ	9.6kΩ

Table 3: The readings of some parameters obtained for a single active strain gauge using Fig. 6 with a gain of 500 for loads in steps of 100 grams.

Load (g)	R(Ω)	? R(Ω)	*l*(mm)	? *l*(mm)	d(mm)	? d(mm)
000	166	0.0	5.346	0.00	0.891	0.00
100	167	1.0	5.348	0.02	0.890	0.001
200	168	2.0	5.350	0.04	0.889	0.002
300	169	3.0	5.352	0.06	0.888	0.003
400	170	4.0	5.355	0.09	0.887	0.004
500	171	5.0	5.357	0.11	0.886	0.005
600	172	6.0	5.359	0.13	0.885	0.006
700	173	7.0	5.361	0.15	0.884	0.007
800	174	8.0	5.363	0.17	0.883	0.008
900	175	9.0	5.365	0.19	0.882	0.009
1000	176	10.0	5.368	0.22	0.881	0.010

Table 4: Readings for the graphical analysis of the strain parameters.

Load (g)	R (Ω)	? R (Ω)	*l*? d x 10^{-3} (m^{2})	d? *l* x 10^{-3} (m^{2})	*l*? R x 10^{-3} (Ωm)	R? *l* x 10^{-3} (Ωm)	G x 10^{-3}	RG (Ω)
000	166	0.0	0.000	0.00	0.00	0.00	0.00	0.00
100	167	1.0	5.348	17.80	5.35	3.34	3.74	0.63
200	168	2.0	10.70	35.56	10.70	6.72	7.48	1.26
300	169	3.0	16.06	53.28	16.06	10.14	11.21	1.90
400	170	4.0	21.42	79.83	21.42	15.30	16.80	2.86
500	171	5.0	26.79	97.46	26.79	18.81	20.53	3.51
600	172	6.0	32.79	115.05	32.15	22.36	24.26	4.17
700	173	7.0	37.53	132.60	37.53	25.95	27.98	4.84
800	174	8.0	42.90	150.11	42.90	29.58	31.70	5.52
900	175	9.0	48.29	167.58	48.29	33.25	35.41	6.20
1000	176	10.0	53.68	193.82	53.68	38.72	40.98	7.21

that implements this is shown in Fig. 3. The two active strain gauges (R_1 and R_4) are arranged in opposite arms of the bridge while the dummy gauges (R_2 and R_3) are arranged in the opposite arms. The new output voltage, given by Eqn. 10, is double that shown in Fig. 1 under different load conditions.

$$V_{out} = E\frac{\Delta R}{2R} \qquad (10)$$

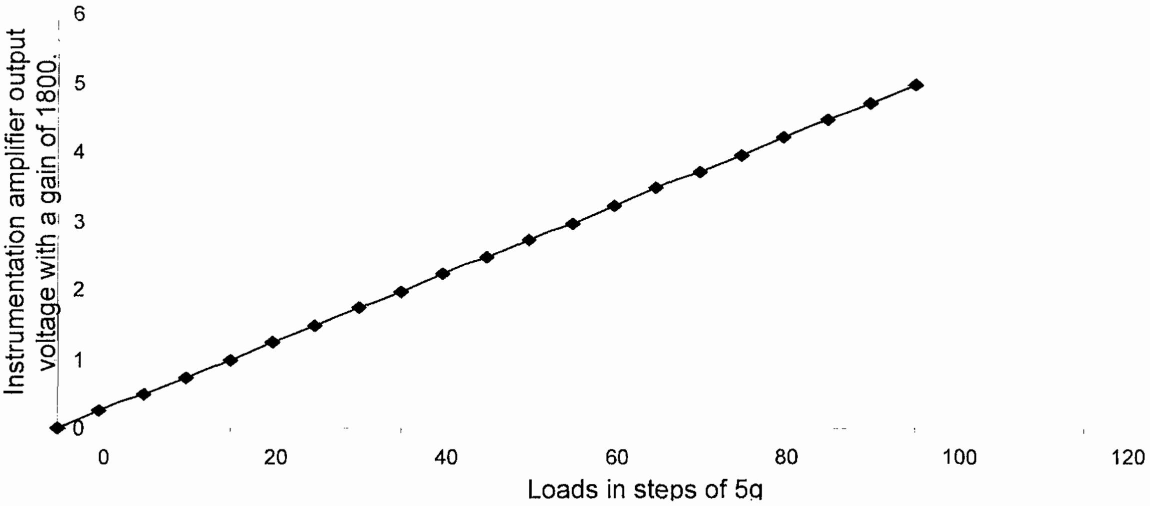


Fig. 7 A graph of Instrumentation amplifier output voltage against load for Table 3.

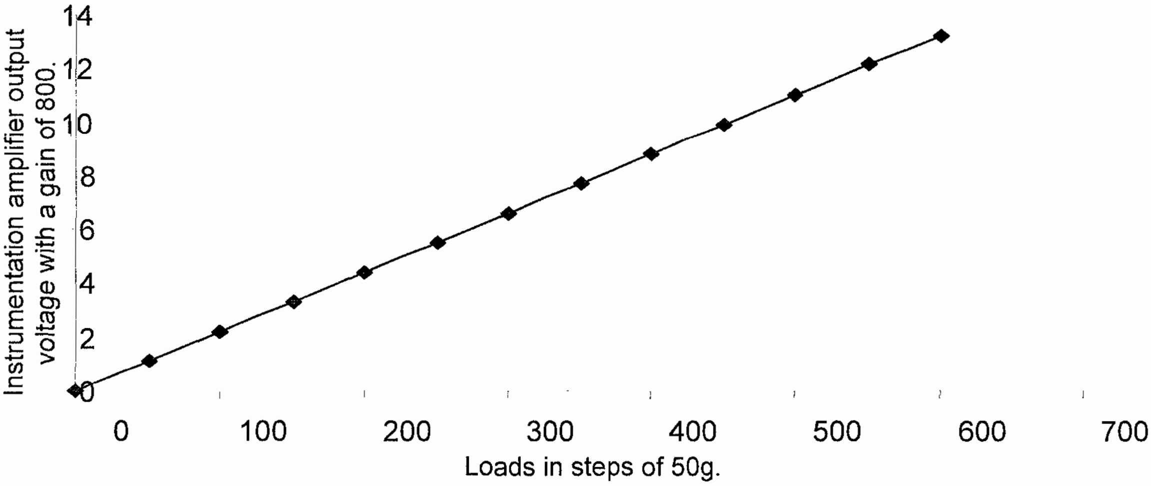


Fig. 8 A graph of instrumentation amplifier output voltage against loads for Table 4.

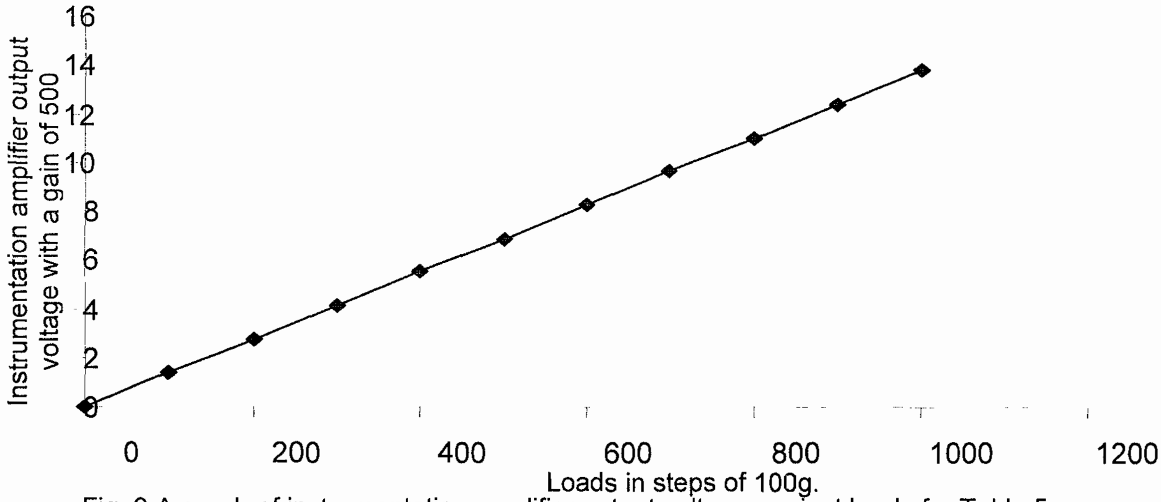


Fig. 9 A graph of instrumentation amplifier output voltage against loads for Table 5.

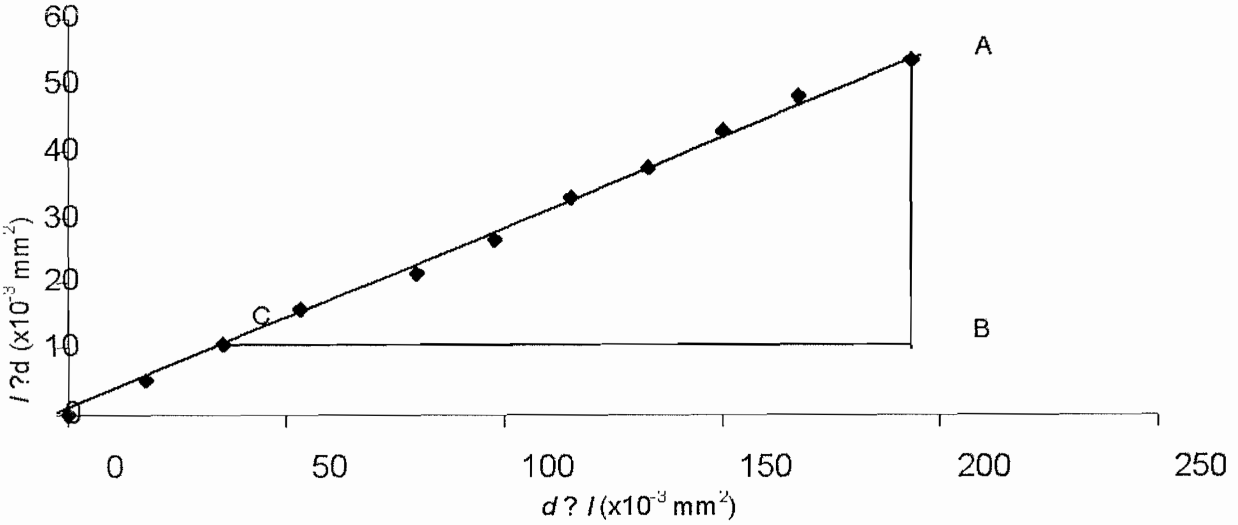


Fig. 10 A graph of l ?d against d?l

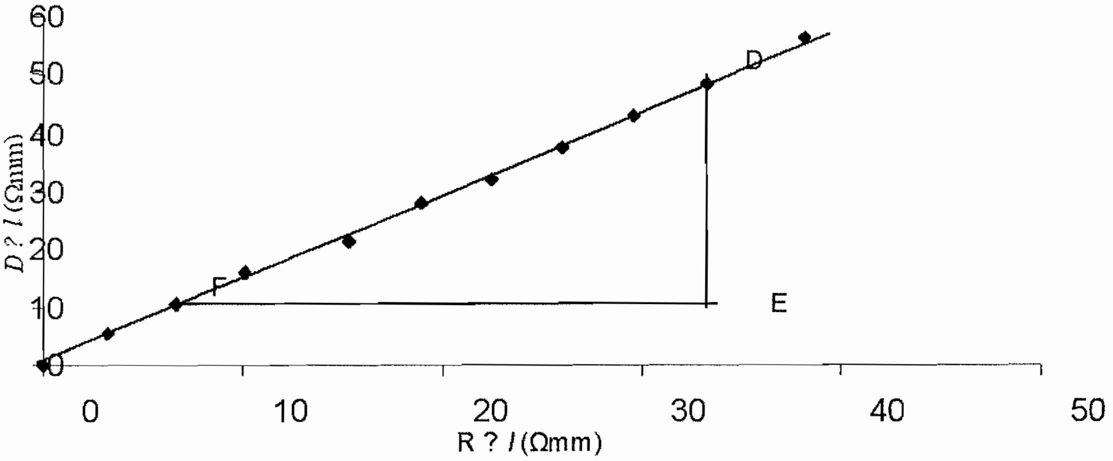


Fig. 11 A graph of l ? R against R ? l

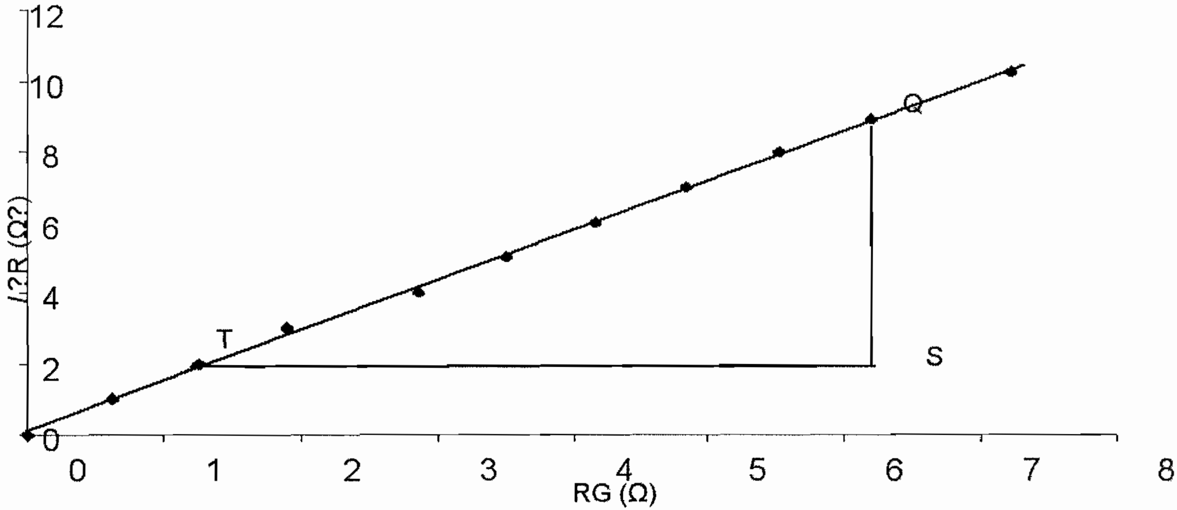


Fig. 12 A graph of R (Ω) against RG (Ω).

Eqn. 10 was obtained when the four strain gauges were mounted on top of the aluminium bar as shown in Fig. 3(a).

However, when the four strain gauges were mounted as shown in Fig. 3(b), i.e. the active gauges on top and the dummy gauges at the bottom of the aluminium bar, the aluminium bar suffered a lateral elongation in length per weight added during the loading process. Further bending (i.e. more sensitivity) was achieved by carving both sides of the lateral sections of the aluminium bar as shown in Fig. 4.

The upper (top) side of the bar lengthened (tension) to increase the resistance of the working strain gauges by (+)ΔR while the lower (bottom) side of the bar shortened (compression) to decrease the resistance of the dummy strain gauges by (–) ΔR. The tension gauges 3-4 and 5-6 are connected in opposite arms of the bridge. Compression gauges 1-2 and 7-8 are connected in the remaining opposite arms of the bridge. The gauges also temperature – compensates one another. The output voltage of the four strain-gauge arrangement of Fig. 4, given by Eqn. 11, is quadrupled over the voltage given by the single-gauge bridge of Fig. 1 (Eqn. 9).

$$V_{out} = E_1 - E_2 = E\frac{\Delta R}{R} \qquad (11)$$

The four strain gauges were mounted using the configuration of Fig. 3(b).

(*c*) Amplifying the strain gauge output voltage:

Due to the low output voltage from the strain gauge, three instrumentation amplifiers (Fig. 6) were design and constructed with gains of 500, 800, and 1800 to accommodate the different load ranges with the aid of a single three – way range rotary switch. The overall instrumentation amplifier's gain, A, are given from (Horowitz and Hill, 1990)

$$A_1 = 1 + \frac{2R_3}{R_G} \qquad (12)$$

and

$$A_2 = \frac{R_2}{R_1} \qquad (13)$$

$$\therefore A = A_1 A_2 = \left(1 + \frac{2R_2}{R_G}\right)\left(\frac{R_2}{R_1}\right) \qquad (14)$$

The following resistor values were used in the design of the instrumentation amplifiers for the various gains shown in Table 2.

Due to the vibration of the aluminium bar during the loading conditions, there was a significance vibration between the voltage level at the input of IC2 and IC3; and the output voltage oscillated for a relatively very short period of time whenever a load was dropped unto the strain gauge system. This was eliminated by introducing the bootstrapping network (IC5). In addition, IC5 buffered the common-mode signal level, driving the common terminal of the split supply for IC2 and IC3 and thus these two op-amps see no swing at their inputs relative to the vibration of the arrangement as well as their power supplies. IC4 serves as a buffer to amplify the differential common – mode signal from IC2 and IC3.

Using a single three-way rotary ganged switch (**S**), the various amplifiers were selected according to the load range to be measured. For loads between 0 and 100 grams (g) with an increment of 5g, the instrumentation amplifier (IC6) with a gain of 1800 (Table 3) was suitable and this corresponded to the first range of the ganged rotary switch. For loads in steps of 50g to a maximum of 600g, IC7 was suitable with a gain of 800 (Table 4) and for loads in steps of 100g to a maximum of 1000g (1Kg), the last switch range corresponding to IC8 was suitable with a gain of 500 (Table 5). The maximum load values quoted in this paper is based on the dimensions of the aluminium beam and these values must be adhered to. Exceeding these load conditions by 10%, hampered the elastic limit of the beam, thereby making the beam unable to recover its original shape. Higher load values, however, required a beam of higher dimensions.

IC6, IC7 and IC8 are analogies of the instrumentation amplifier shown in Fig. 5. Their various gains depend on the resistor values shown in Table 2. The circuit diagram of the instrumentation amplifier used in this

work including the strain gauge arrangement wired in the Wheatstone bridge configuration as well as the signal conditioning circuit is shown in Fig. 5. For neatness and avoidance of circuit repetition, this format was employed in this work.
The sensitivity was greatly increased by the two cavities on both sides of the surface of the aluminium bar while the central axis was left to accommodate the strain gauges (Fig. 4). This was necessary to increase the bending capability of the bar with its elastic limit not exceeded beyond deformation. The sensitivity was further increased by using four strain gauges (two active strain gauges on the top central axis and two dummy (temperature) gauges on the other (bottom) side of the aluminium bar). Only the active strain gauges (at the top) are shown in Fig. 4 while the dummy strain gauges (at the bottom) have been omitted in Fig. 4 for the sake of the diagram clarity.

3. Results

The results obtained from the various experiments are presented in their respective graphs are shown of Figures 7, 8, and 9. The strain gauges gave similar changes for all the parameters measured in Tables 3 and 4 except that the resistance (R) of the active gauges increased proportionally to the load while the resistance of the dummy gauges reduced by the same proportion as that of the active gauges. The results of the readings obtained for a single active strain gauge are shown in Table 3 for different load conditions in steps of 100g. A step of 100g was chosen since this value gave an appreciable elongation of both the beam and the strain gauges. This appreciable elongation was as a result of the higher sensitivity of the aluminium beam thereby creating an avenue for a high precision measurement using the micrometer screw gauge.

4. Analysis

The sensitivity of the strain gauges is expressed by the Figures 7, 8 and 9. The graphs show the linearity of the strain gauge system with various loads. The sensitivity of the strain gauge system is based on the fact that it could sense and measure very little load of 1g with the dimensions of the aluminium bar used in this research work.
(*a*) The value of the Poisson's ratio (μ) is obtained from Fig. 10 as shown below:

$$\mu = \frac{\Delta AB}{\Delta BC} = \frac{l\Delta d}{d\Delta l} = \frac{53.68 - 10.70}{193.82 - 35.56} = \frac{32.98}{158.26} = 0.21 \qquad (15)$$

(*b*) The value of the gauge factor (k) is calculated from the Poisson's ratio as:

$$\mathrm{k} = 1 + 2\mu = 1 + 0.42 = 1.42 \qquad (16)$$

(*c*) The value of the gauge factor (k) is given by the slopes of Figures 11 and 12 as shown below:
(i) From Fig. 11, the value of k is:

$$k = \frac{\Delta DE}{\Delta EF} = \frac{l\Delta R}{R\Delta l} = \frac{48.29 - 10.70}{33.25 - 6.72} = \frac{37.59}{26.53} = 1.42 \qquad (17)$$

(ii) From Fig. 12, the value of k is:

$$k = \frac{\Delta QS}{\Delta ST} = \frac{\Delta R}{RG} = \frac{9 - 2}{6.20 - 1.26} = \frac{7}{4.94} = 1.42 \qquad (18)$$

It can be seen that the value of k in Equations (16), (17) and (18) are the same i.e. 1.42 and this shows the consistency of the methodology employed in this work. The ¼
value of 0.21 and the k value of 1.42, when compared with the values in Table 1, is an indication that the strain gauges are made of zinc wires.

5. Discussion and Conclusion

The 120-SRA foil type strain gauge is highly sensitive with a K value of 1.42 and a μ value of 0.21. The K and μ values of 1.42 and 0.21 respectively show that these strain gauges are made of zinc wires. The graphs of Figures 7, 8 and 9 reveals the linearity and the sensitivity of the strain gauge system. The techniques described in this work can be adapted for any industrial process control and instrumentation as well as for stress-strain measurement. The comparison of the values of k and μ from Figures 9, 10, and 11 with their values in standard literature shows the consistency of this research methodology.

The principles and the apparatus employed in this research paper were adapted for the design and construction of a low cost automatic electromechanical vending machine. It can also be used for the design of a highly sensitive weighing machine with a range of 0 to 1kg load capabilities while higher load range can be achieved with little modification in the dimensions of the aluminium bar with an instrumentation of appropriate gain.

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