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DATA-DRIVEN SELF-OPTIMIZING CONTROL: CONSTRAINED OPTIMIZATION PROBLEM

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ABSTRACT

Self-optimizing control (SOC) is a technique used in selecting controlled variables (CVs) for a process plant control structure with a view to operating the plant optimally in the presence of uncertainties and disturbances. Existing SOC approaches are either local which result to large losses or too cumbersome to be applicable to real systems. In this work, a novel method of CV selection based on data was developed. In the method, a compressed reduced gradient of a constrained optimization problem was proposed to be estimated using finite difference scheme. The CV function was then used to approximate the necessary condition of optimality (NCO) using data only in a single regression step. The new approach was applied to a simplified case study and its performance was compared to an existing SOC methodology. An excellent goodness of fit was obtained during the regression with a R^2-value of 1.0 associated with one of the designed CVs. The formulated CVs were found to be very robust with performance similar to that of NCO approximation method. A zero loss was incurred with one of the CVs.

Keywords: compressed reduced gradient, constrained problems, controlled variable, data-driven, necessary condition of optimality, self-optimizing control.

INTRODUCTION

One of the most important stages in any control structural design is the selection of controlled variables (CVs) and manipulated variables (MVs) and establishment of linkage between the two. This activity has a great effect on the overall safety and economy of any plant operation (Umar *et al.*, 2012). A CV which is sometimes called a process variable is the quantity which is to be controlled. A MV on the other hand is a quantity which can be adjusted directly to influence the output in a favourable way. This is also called a control variable or control input (Janert, 2013).

A method of CV selection called self-optimizing control (SOC) that places an emphasis on the optimal operation of plant was first proposed by Skogestad (2000). *Self-optimizing control is when we can achieve an acceptable loss with constant setpoint values for the controlled variables (without the need to reoptimize when disturbances occur) (Skogestad, 2000).* The main idea of SOC is to select CVs which when controlled in the presence of uncertainties and disturbances can keep the operation of the process at optimal or near-optimal level. That is to say, the process becomes 'self-optimizing' with the control of the selected CVs at constant setpoints (Umar *et al.*, 2012).

Various researchers have shown interest in the development of SOC methodologies which led to its improvement over the years. These methods can be broadly classified into two; local and global methods which are applicable to either static or dynamic processes. The local methods depend heavily on linearizing a non-linear model around a nominal operating point and quadratic approximation of the loss function, and hence only a local solution is obtainable (Umar et al., 2012). A method based on minimum singular value (MSV) rule was one of the pioneer local SOC approaches (Skogestad and Postlethwaite, 1996; Halvorsen *et al.,* 2003). The method works by selecting the CV that maximizes the MSV of a scale gain matrix (Halvorsen, *et al.,* 2003; Umar *et al.,* 2012). But according to Hori and Skogestad

(2008), the method can lead to wrong identification of CVs. As such, Halvorsen *et al.* (2003) developed 'exact local method' to overcome the shortcoming of MSV rule which is based on the assumption that the setpoint around which linearization is done to obtain the approximate model is optimal. In the approach, obtained loss expressions were used to screen CVs (Umar *et al.,* 2012). Other local methods include null space method (Alstad and Skogestad, 2007), and branch and bound methods (Kariwala and Cao, 2010, 2009; Cao and Kariwala, 2008).

Ye *et al.*(2013a) have defined two expressions for loss in objective function that are used as criteria for CV selection. These are worst case and average losses for uniformly distributed disturbance given as follows

$$L_{worst} = \frac{1}{2}\sigma_{max}^2(M) \quad (1)$$

$$L_{average} = \frac{1}{6(n_y + n_d)}\|M\|_F^2 \quad (2)$$

where $\sigma_{max}(\cdot)$ and $\|\cdot\|_F$ are the maximum singular value and Frobenius norm of a matrix, respectively. The matrix M was defined as

$$M = \left[J_{uu}^{1/2}(J_{uu}^{-1}J_{ud} - G^{-1}G_d)W_d \quad J_{uu}^{1/2}G^{-1}HW_n\right] \quad (3)$$

where $G = HG^y$, $G_d = HG_d^y$
and the Hessian matrices are given as (Ye *et al.,* 2013a)

$$J_{uu} = \frac{\partial^2 J}{\partial u^2}$$
$$J_{ud} = \frac{\partial^2 J}{\partial u \partial d} \quad (4)$$

Equation (1) and Equation (2) are used to select the right CV candidate as a subset of measurements (Umar *et al.,* 2012). The CV selection procedure involves minimizing the loss expressions with respect to H (Ye *et al.,* 2013a).

In global methods, gradient functions were proposed to be used as the CVs directly in order to obtain a global optimal operation (Cao, 2005, 2003). The main challenge to this approach is the difficulty in obtaining the analytical expressions of most processes; and when such exist, it may be nonlinear in state and unknown disturbances. In a related work by Cao (2004), chain rule differentiation was proposed to explicitly express the gradient as a function of system's Jacobian. Other methods were also developed for CV selection which approximate necessary condition of optimality (NCO) to achieve near optimal operation globally (Ye *et al.,* 2013a, 2012). However, global optimal operation was achievable with these methods, system model is still necessary for NCO evaluation, which is a short coming to systems with unknown or complicated model.

The above local and global methods were developed for steady state continuous operations. Approaches for dynamic problems were also developed (Dahl-Olsen *et al.,* 2008; Dahl-Olsen and Skogestad, 2009; Hu *et al.,* 2012; Ye *et al.,* 2013b). In the work of Ye *et al.* (2013b), the technique for NCO approximation (formulated for continuous processes) was extended to approximating invariants. Again, the method may not be applicable to complex processes or those with unknown models. In this work, a method that works solemnly based on either operational or simulated data is proposed for constrained optimization problems to approximate the NCO and hence, model equations are not required. This is applied to a simplified problem so that readers may find the concept easier in understanding.

METHODOLOGY

Most processes are constrained in one way or the other (Walter, 2014). However, the methodology presented in the first part of this work does not consider constraints directly but are satisfied during data collection. This might be time consuming for large scale problems. In the present work, the method is extended to solve constrained optimization problems where the constraint equations are considered explicitly in the formulation. For this method, the NCO does not need to be determined analytically but evaluated using simulated or operational data through finite difference scheme.

Algorithm Development

The optimization problem can be written as

$$\min_{\boldsymbol{u}} J(\boldsymbol{u}, \boldsymbol{d})$$
$$subject\ to \quad \boldsymbol{g}(\boldsymbol{u}, \boldsymbol{d}) = 0. \quad (5)$$

For this type of problem, the NCO was split into two parts by Ye *et al.* (2013a) which are, the active constraint, $\boldsymbol{g_a}$ (constraint with strict equality) and reduced gradients $\nabla_r J$. These are respectively given as

$$\boldsymbol{g_a} = 0, \quad \boldsymbol{g_a} \in \mathbb{R}^{n_a} \tag{6}$$

and

$$\nabla_r J = \frac{\partial J}{\partial \boldsymbol{u}}\left[I - \left(\frac{\partial \boldsymbol{g_a}}{\partial \boldsymbol{u}}\right)^+ \frac{\partial \boldsymbol{g_a}}{\partial \boldsymbol{u}}\right] = 0. \tag{7}$$

The reduced gradient has n_u components which can be compressed to $n_u - n_a$ dimensions using singular value decomposition. This was obtained by Ye *et al.* (2013a) as

$$\nabla_{cr} J = \frac{\partial J}{\partial \boldsymbol{u}} V_2 = 0, \quad \nabla_{cr} J \in \mathbb{R}^{n_u - n_a} \tag{8}$$

where $\nabla_{cr} J$ is the compressed reduced gradient, and V_2 are $n_u - n_a$ right singular vectors.

We proposed in this work to approximate the compressed reduced gradient given in Equation (8) using finite difference scheme. The regression CV function is therefore given by

$$C(\boldsymbol{y}_{k,}\boldsymbol{\theta}) = \nabla_{cr} J|_k. \tag{9}$$

In which case θ is to be determined through regression. The following steps are followed to carry out the optimization process:

1. A set of data is collected by sampling the whole space of manipulated variables and disturbance.
2. At each reference point, n_u gradients of the objective function against n_u manipulated variables $\frac{\partial J}{\partial u}$ and $n_a \times n_u$ Jacobian matrix of n_a constraints against n_u manipulated variables, $\frac{\partial J}{\partial u}$ are calculated
3. Singular value decomposition approach is used to calculate $n_u - n_a \nabla_{cr} J$ at each reference point.

Regression is used to fit $n_u - n_a$ controlled variables to approximate the $n_u - n_a \nabla_{cr} J$ for all reference points by minimizing the value of squared 2-norm of the residual as given

$$\min_{\theta} \frac{1}{2} \| C(y_k, \boldsymbol{\theta}) - q \|_2^2 \tag{10}$$

Where q represents the right-hand side of Equation (9).

Illustrations

The toy example (Umar *et al.,* 2012) is modified here to include an equality constraint and two manipulated variables. The objective function is (Grema, 2014)

$$J = u_1^2 + 2u_2^2 + 4u_1u_2d - 2u_1 - 16u_2. \tag{11}$$

The constraint is given as

$$g = u_1 - u_2 - d. \tag{12}$$

It was assumed that there are four available measurements

$$\begin{cases} y_1 = u_1 \\ y_2 = u_2 \\ y_3 = 2u_1 - d \\ y_4 = u_2 - 5d. \end{cases} \tag{13}$$

The disturbance d varies in the range [-0.25, 0.25] while u_1 in the range [-1, 1] and u_2 in [-2, 2] range.

In order to ascertain the robustness of the proposed method, a comparison will be made to NCO approximation techniques reported by Ye et al. (2013a). To use NCO method, the analytical equation of the compressed reduced gradient given in Equation (8) is employed.

Analytical Solution

To derive the necessary condition of optimality for this problem analytically, the following steps are taken:

- The Jacobian of the constraint is computed.
- Singular value decomposition isused to obtain V_2.
- The Jacobian of the objective function with respect to control is computed.
- Using Equation (8), the NCO is

computed.

Data-Driven SOC Solution

Here, both first- and second-order polynomials are used to fit the reduced gradient. For the first-order polynomial, we have, for four measurements

$$C_{LR} = \theta_1 y_{1k} + \theta_2 y_{2k} + \theta_3 y_{3k} + \theta_4 y_{4k} + \theta_5 \quad (14)$$

and for the second-order

$$C_{PR} = \theta_1 y_{1k}^2 + \theta_2 y_{2k}^2 + \theta_3 y_{3k}^2 + \theta_4 y_{4k}^2 + \theta_5 + \theta_6 y_{1k} y_{3k} + \theta_7 y_{1k} y_{4k} + \theta_8 y_{2k} y_{3k} + \theta_9 y_{2k} y_{4k} + \theta_{10} y_{3k} y_{4k} + \theta_{11} y_{1k} + \theta_{12} y_{2k} + \theta_{13} y_{3k} + \theta_{14} y_{4k} + \theta_{15}. \quad (15)$$

After regression was conducted, performances of different CVs were evaluated numerically using the steady-state loss function defined by Ye *et al.* (2013a) as

$$L = J(u_{fb}, d) - J_{opt}(d) \quad (16)$$

where the $J(u_{fb}, d)$ is the value of the objective function which would be obtained when the feedback control law is implemented to maintain the CV at zero while $J_{opt}(d)$ is the actual optimal, J. A Monte Carlo simulation was then carried out using 1000 randomly generated disturbances that vary within its range of values.

If u_1, u_2 and d are divided into N, n and m parts respectively, the number of data points for central difference scheme is given by

$$N_p = [(N-2)(n-2)]m. \quad (17)$$

For this illustrative example, central difference scheme was employed with $N = 41$, $n = 41$ and $m = 11$. Therefore, $N_p = 16731$.

RESULTS AND DISCUSSIONS

The results for the analytical solution to the constrained optimization problem considered are given below:

The Jacobian of the constraint function was found to be

$$\frac{\partial g}{\partial \boldsymbol{u}} = [1 \quad -1]. \quad (18)$$

The value of V_2 obtained using singular value decomposition is

$$\boldsymbol{V}_2 = \begin{bmatrix} 0.7071 \\ 0.7071 \end{bmatrix}. \quad (19)$$

For the objective function, its Jacobian with respect to control is

$$\frac{\partial J}{\partial \boldsymbol{u}} = [2u_1 + 4u_2 d - 2 \quad 4u_2 + 4u_1 d - 16]. \quad (20)$$

The NCO is therefore found to be

$$\nabla_{cr} J = 0.7071(2u_1 + 4u_2 + 4(u_1 + u_2)d - 18). \quad (21)$$

Table 1 gives the regression parameters and losses for both data-driven SOC and NCO approximation. By observing regression coefficient values, it is evidenced that the measurement y_1 is not relevant in the CV formulae of Equations (14) and (15). This is because the coefficients of y_1 or its product with any other measurement is zero for both data-driven SOC and NCO approximation methods. Also, the square of y_3 is not significant for the second-order polynomials.

Table 1: Constrained Data-Driven SOC and NCO Approximation Methods

		Data-Driven SOC		**NCO Approx.**	
		C_{LR}	C_{PR}	C_{LR}	C_{PR}
Coefficients	θ_1	0	0	0	0
	θ_2	0.3111	0.3111	2.9698	0.6222
	θ_3	0.3536	0	0.7071	0
	θ_4	-0.0707	0.0283	-0.1414	0.0566
	θ_5	-6.3640	0	-12.7278	0
	θ_6	-	0	-	0
	θ_7	-	0	-	0
	θ_8	-	0.1414	-	0.2828
	θ_9	-	-0.3394	-	-0.6788
	θ_{10}	-	-0.1414	-	-0.2828
	θ_{11}	-	0	-	0
	θ_{12}	-	1.4849	-	2.9698
	$\theta_{1}3$	-	0.3536	-	0.7071
	θ_{14}	-	-0.0707	-	-0.1414
	θ_{15}	-	-6.3640	-	-12.7278
Losses	**Minimum**	$1.4490\text{x}10^{-6}$	0	$5.8247\text{x}10^{-8}$	0
	Average	1.1083	0	1.0648	0
	Maximum	4.3743	0	4.3393	0
	Std. Dev.	1.0734	0	1.0383	0
	R^2	0.9714	1.0000	0.9714	1.0000

The R^2-values obtained for first-and second-order polynomials are respectively 0.9714 and 1.0000. This indicates that no higher polynomial or more rigorous model is needed to fit the compressed reduced gradient.

By using a central difference scheme in approximating the Jacobians of the objective and constraint functions, the losses associated with both approaches are zero for second-order polynomial. Losses incurred for the case of data-driven SOC when first-order polynomial was used as the CV averaged at a value of1.1083 and standard deviation of 1.0734. The corresponding values for NCO approximation method are 1.0648 and 1.0383 respectively. This indicates that even when we do not have the gradient information of a process, we can use measurements alone to optimize it.

CONCLUSIONS AND RECOMMENDATIONS

This work presented a novel method for controlled variable selection using only data without the need for analytical expression of the NCO. The method was developed for constrained optimization problems where proposed CV expressions were used to approximate compressed reduced gradient using measurement. The compressed reduced gradient was estimated using finite difference scheme.

To illustrate the efficacy of the method in a simplest way possible, the approach was applied to a hypothetical case study. This was also compared to an existing method which requires the explicit expression of the NCO to be derived. Excellent regression performance was obtained owing to the finite difference scheme selected (central difference) and the numerical experiment performed. The proposed approach was shown to be efficient with performance comparable to NCO approximation method.

It is recommended that the method is applied to a large scale problem and its efficiency compared. Although, the objective used in the illustration is a function of only the manipulated variables and the disturbance, the methodology is also applicable when the objective is a function of states, provided it (objective) can be computed.

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